DAY 56 § 3.7
p.p. 164-165 # 3, 6, 9, 12, 15, 18, 23, 25, 26, 31

(3) $x^2+xy-y^3=xy^2$ [Product Rule]

$2x+\left[y+x\frac{dy}{dx}\right]-3y^2\frac{dy}{dx}=\left[y^2+x\,2y\frac{dy}{dx}\right]$

$2x+y-y^2=\left(3y^2-x+2xy\right)\frac{dy}{dx}$

$\frac{dy}{dx}=\frac{2x+y-y^2}{3y^2-x+2xy}$

(6) $x^2y-2y+5=0$

$2x\cdot y+x^2\frac{dy}{dx}-2\frac{dy}{dx}=0$

$(x^2-2)\frac{dy}{dx}=-2xy$

$\frac{dy}{dx}=\frac{-2xy}{(x^2-2)}$

(9) $\sqrt{x}+\sqrt{y}=25$

$\frac{1}{2\sqrt{x}}+\frac{1}{2\sqrt{y}}\frac{dy}{dx}=0$

$\frac{1}{2\sqrt{y}}\frac{dy}{dx}=\frac{-1}{2\sqrt{x}}$

$\frac{dy}{dx}=\frac{-2\sqrt{y}}{2\sqrt{x}}$

$\frac{dy}{dx}=\frac{\sqrt{y}}{\sqrt{x}}$

(12) $ax^2-by^2=c$

$2ax-2by\frac{dy}{dx}=0$

$2ax=2by\frac{dy}{dx}$

$\frac{dy}{dx}=\frac{2ax}{2by}$

(15) $\sin(xy)=2x+5$

$\cos(xy)\left[y+x\frac{dy}{dx}\right]=2$

$y\cos(xy)+x\cos(xy)\frac{dy}{dx}=2$

$x\cos(xy)\frac{dy}{dx}=2-y\cos(xy)$

$\frac{dy}{dx}=\frac{2-y\cos(xy)}{x\cos(xy)}$

(18) $e^{x^2}+\ln y=0$

$(2x)e^{x^2}+\frac{1}{y}\frac{dy}{dx}=0$

$\frac{1}{y}\frac{dy}{dx}=-2xe^{x^2}$

$\frac{dy}{dx}=\left(-2xe^{x^2}\right)(y)$

(23) $\sin(xy)=x$ @ $(1,\frac{\pi}{2})$

$\cos(xy)\left[y+x\frac{dy}{dx}\right]=1$

$y\cos(xy)+x\cos(xy)\frac{dy}{dx}=1$

$x\cos(xy)\frac{dy}{dx}=1-y\cos(xy)$

$\frac{dy}{dx}=\left.\frac{1-y\cos(xy)}{x\cos(xy)}\right|_{(1,\frac{\pi}{2})}=\frac{1-\frac{\pi}{2}\cos(\frac{\pi}{2})}{1\cos(\frac{\pi}{2})}$

$\frac{dy}{dx}=\frac{1-0}{0}$ undefined so slope is infinite. tangent to curve at $(1,\frac{\pi}{2})$ is vertical tangent.

DAY 56 §3.7

(25) $x^3 + 5x^2y + 2y^2 = 4y + 11$ @ $(1,2)$

$$3x^2 + 5\left(2xy + x^2\frac{dy}{dx}\right) + 4y\frac{dy}{dx} = 4\frac{dy}{dx}$$

$$\cancel{3x^2 + 10xy} + 5x^2\frac{dy}{dx} + 4y\frac{dy}{dx} - 4\frac{dy}{dx} = -(3x^2 + 10xy)$$

$$(5x^2 + 4y - 4)\frac{dy}{dx} = -(3x^2 + 10xy)$$

$$\frac{dy}{dx} = \left.\frac{-(3x^2 + 10xy)}{(5x^2 + 4y - 4)}\right|_{(1,2)} = \frac{-(3+20)}{(5+8-4)} = \frac{-23}{9}$$

(26) $xy^2 = 1$

$$1 \cdot y^2 + x\,2y\frac{dy}{dx} = 0$$

$$2xy\frac{dy}{dx} = -y^2$$

$$\frac{dy}{dx} = \frac{-y^2}{2xy}$$

$$\frac{dy}{dx} = \left.\frac{-y}{2x}\right|_{(1,-1)} = \frac{1}{2}$$

TANGENT LINE:

$$y = \tfrac{1}{2}(x-1) - 1$$

(31) $x^2 + y^2 - 4x + 7y = 15$

$$2x + 2y\frac{dy}{dx} - 4 + 7\frac{dy}{dx} = 0$$

$$(2y+7)\frac{dy}{dx} = 4 - 2x$$

$$\frac{dy}{dx} = \frac{4-2x}{2y+7}$$

- The tangent line to the curve will be horizontal when $y \neq \frac{-7}{2}$ & $x = 2$
- The tangent line to the curve will be vertical when $y = \frac{-7}{2}$ & $x \neq 2$