§3.6 Derivative of Inverse Function ANSWER KEY HW DAY 55

6) The following figure shows $f(x)$ and $f^{-1}(x)$. Using the given table, find:

#6

a) $f(2), f^{-1}(2), f'(2), (f^{-1})'(2)$.

$f(2)=4$ $f^{-1}(2)=1$

$f'(2)=2.8 = \frac{14}{5}$ $(f^{-1})'(2) = \frac{1}{f'(1)} = \frac{1}{1.4} = \frac{5}{7}$

b) The equation of the tangent line at the points $P(3, 8)$ and $Q(8, 3)$.

$f'(3)=5.5$ $P(3,8)$ $(f^{-1})'(8) = \frac{1}{f'(3)} = \frac{1}{5.5} = \frac{2}{11}$

$y = \frac{11}{2}(x-3)+8$ $y = \frac{2}{11}(x-8)+3$

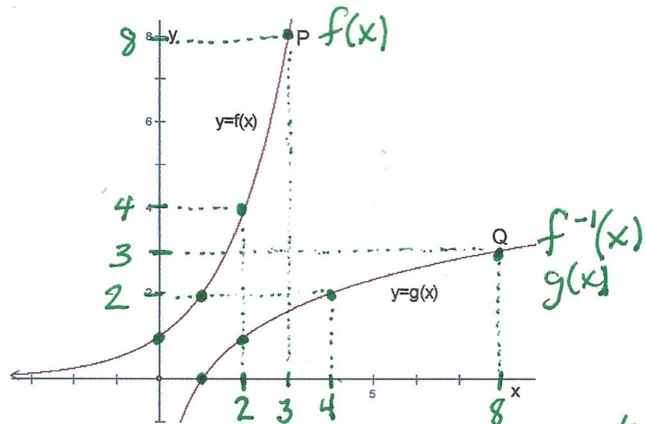
c) What is the relationship between the two tangent lines?

c) These tangent lines are inverse functions

x	$f(x)$	$f'(x) = g(x)$
0	1	0.7
1	2	1.4
2	4	2.8
3	8	5.5

$1.4 \rightarrow \frac{14}{10} = \frac{7}{5}$

$5.5 \rightarrow \frac{55}{10} = \frac{11}{2}$



$f(x) = 2^x$

$f'(x) = (\ln 2)2^x \Big|_{x=3}$

$f'(3) = 2^3 \cdot \ln 2 = 8\ln 2$

$g(x) = f^{-1}(x) = \log_2(x)$

$g(x) = f^{-1}(x) = \frac{\ln(x)}{\ln(2)}$

$g'(x) = (f^{-1})'(x) = \frac{1}{x \cdot \ln(2)} \Big|_{x=8}$

$g'(2) = \frac{1}{8\ln 2}$

7) Calculate $g'(1)$, where $g(x)$ is the inverse of the function $f(x) = x + e^x$ without solving for $g(x)$.

Ⓐ $f(x) = x + e^x = 1 \longrightarrow$ $g(1) = 0$ Ⓑ

$x = 0$ Ⓒ $f'(x) = 1 + e^x$

$\therefore f(0) = 1$ $f'(0) = 1+1 = 2$ $\xrightarrow{\text{reciprocal}}$ Ⓓ $g'(1) = \frac{1}{2}$

8) Calculate $g'(x)$, where $g(x)$ is the inverse of the function $f(x) = x^3 + 1$ without solving for $g(x)$.

$f(x) = x^3 + 1 \longrightarrow g(x) = f^{-1}(x)$

$f'(x) = 3x^2 \xrightarrow{\text{RECIPROCAL}} g'(x) = \frac{1}{3x^2}$

9) Let $f(x) = \frac{1}{4}x^3 + x - 1$. Assume that $f(x)$ is one-to-one. ← meaning f(x) has an inverse that is a function.

a. What is the value of $f^{-1}(x)$ when $x = 3$?

$f^{-1}(3) = ?$ $\therefore$ Solve $f(x) = 3 = \frac{1}{4}x^3 + x - 1$ $\therefore \frac{1}{4}x^3 + x - 4 = 0$ TI $x = 2$

b. Find the slope of the tangent line to the curve $y = f^{-1}(x)$ at $x = 3$.

$\therefore f(2) = 3$ $f^{-1}(3) = 2$

$f'(x) = \frac{3}{4}x^2 + 1$ $(f^{-1})'(3) = \frac{1}{4}$

$f'(2) = 4$

$y = \frac{1}{4}(x-3) + 2$

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Keys to Properly Solving Derivative of an Inverse Problems:

- First, identify the point (a, b) on the function f using whatever information is given.
- Differentiate f.
- Take the reciprocal of the derivative of f. This is the derivative of f^{-1}.
- Evaluate the derivative of f^{-1} at the point (b, a).

Practice:

Given the following values for differentiable functions f and g.

f(x)   g(x)

x	f	f'	g	g'
1	2	½	-3	5
2	3	1	0	4
3	4	2	2	3
4	6	5	3	½

a. If $h(x) = f^{-1}(x)$, what is $h'(4)$?

$f(?)=4$ $(3,4)$ on $f(x)$ $f'(3)=2$
$(4,3)$ on $h(x)$ $\rightarrow h'(4)=\frac{1}{f'(3)}=\frac{1}{2}$

b. If $h(x) = f^{-1}(x)$, what is $h'(2)$?

$f(?)=2$ $(1,2)$ on $f(x)$ $f'(1)=\frac{1}{2}$
$(2,1)$ on $h(x)$ $\rightarrow h'(2)=\frac{1}{f'(1)}=\frac{1}{\frac{1}{2}}=2$

c. If $d(x) = g^{-1}(x)$, what is $d'(-3)$?

$g(?)=-3$ $(1,-3)$ on $g(x)$ $g'(1)=5$
$(-3,1)$ on $h(x)$ $\longrightarrow h'(-3)=\frac{1}{g'(1)}=\frac{1}{5}$

And these are not exactly on derivatives of inverses, but they are good practice nonetheless:

d. If $p(x) = g^2(x)$, what is $p'(3)$?

$p'(x)=2(g(x))^1\cdot g'(x)$ $\therefore p'(3)=2(g(3))^1\cdot g'(3)$
$=2(2)(3)=12$

e. If $b = f \bullet g$ what is $b'(2)$?

$b'=f'\cdot g+f\cdot g'$ $\therefore b'(2)=f'(2)g(2)+f(2)\cdot g'(2)$
$=(1)(0)+(3)(4)=0+12=12$

f. If $n(x) = f(x^3)$, what is $n'(1)$?

$n'(x)=f'(x^3)(3x^2)$ $\therefore n'(1)=f'(1^3)\cdot(3(1)^2)$
$=f'(1)\cdot(3)$
$=(\frac{1}{2})(3)=\frac{3}{2}$