ANSWER KEY

NOTES

Derivative of Inverse Function Theorem

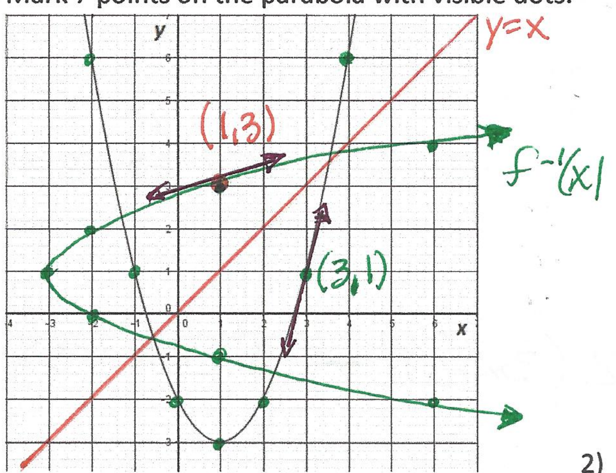
Function and Inverse Pre-requisites:

Given each function, identify key points on the function that fall on lattice points of the coordinate grid.

LINE OF REFLECTION.

Mark 7 points on the parabola with visible dots.

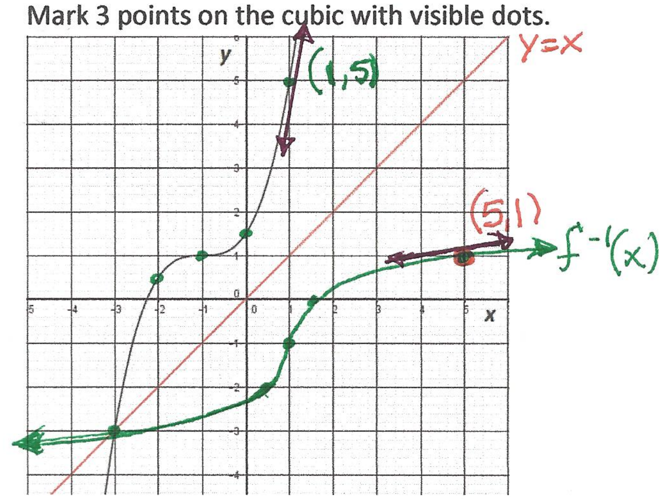
1)



LINE OF REFLECTION.

Mark 3 points on the cubic with visible dots.

2)



a) Write the equation of each function in (h,k) form and evaluate the function at the given point.

V(1,-3)
a=1

Quadratic function $f(x) = 1(x-1)^2 - 3$	$(3, f(3)) =$ $(3,1)$	Cubic function $a=\frac{1}{2}$ $(-1,1)$ $f(x) = \frac{1}{2}(x+1)^3 + 1$	$(1, f(1)) =$ $(1,5)$

b) For each function, list the operations on *x* that yield y.

Parabola x: −1, SQUARE, −3   x: +3, $\pm\sqrt{\ }$, +1

Cubic x: +1, CUBE, $\times\frac{1}{2}$, +1   x: −1, ×2, $\sqrt[3]{\ }$, −1

INVERSE

c) Write inverse equations by using the list in (b) & applying *inverse operations in reverse order on x*.
State the corresponding inverse coordinate from the point on the function in part (a)

$f^{-1}(x) = \pm\sqrt{(x+3)} + 1$	$(x, f^{-1}(x)) = (1,3)$	$f^{-1}(x) = \sqrt[3]{2(x-1)} - 1$	$(x, f^{-1}(x)) = (5,1)$

d) Accurately, sketch the inverse function on the coordinate grid using the key lattice points. $\boxed{\text{Label } f^{-1}(x)}$ ✓

e) Find the derivative of the function at the specified point.
Find the derivative of its inverse at the corresponding point on the inverse.

$f'(x) = 2(x-1)$ (3,1)	$\left.\frac{dy}{dx}\right\|_{x=3} = 2(2) = 4$	$f'(x) = \frac{3}{2}(x+1)^2$ (1,5)	$\left.\frac{dy}{dx}\right\|_{x=1} = \frac{3}{2}(4) = 6$
$(f^{-1})'(x) = \frac{1}{2\sqrt{x+3}}$ (1,3)	$(f^{-1})'(\underline{1}) = \frac{1}{2\sqrt{4}} = \frac{1}{4}$	$(f^{-1})'(x) = \frac{2}{3(2(x-1))^{2/3}}$ (5,1)	$(f^{-1})'(\underline{5}) = \frac{2}{3(\sqrt[3]{8})^2} = \frac{2}{3(4)} = \frac{1}{6}$

f) What is the relationship between the derivative value of the function at the point and its inverse at the corresponding inverse point? ____

§3.6 NOTES

AB Calculus – Supplement
Derivative of the Inverse of a Function

Name: ANSWER KEY

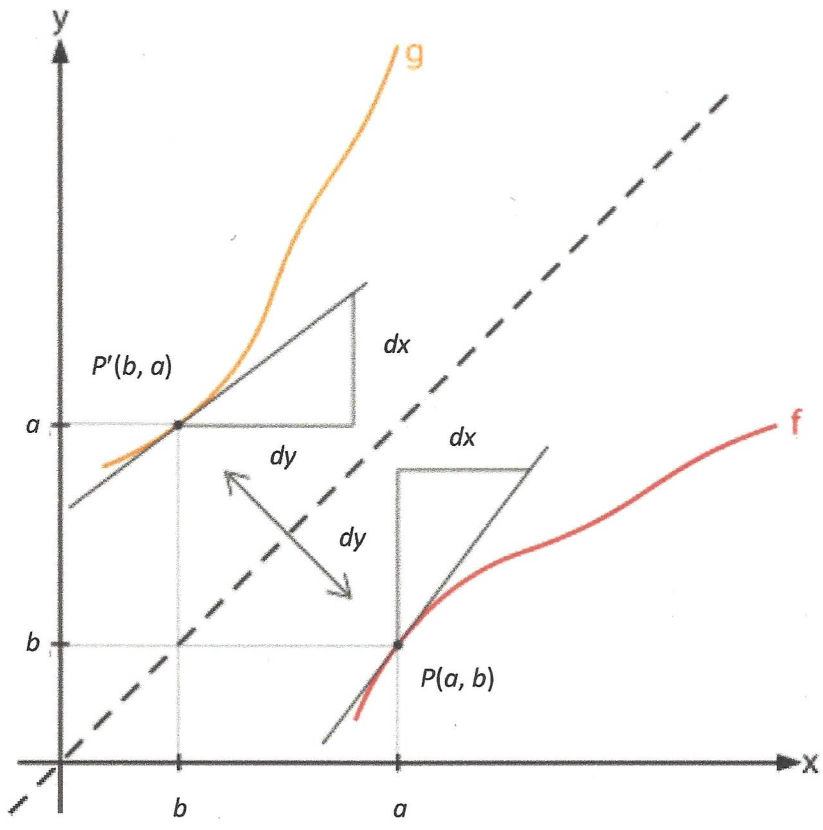
Class/Period: ________ Date: ____________

Suppose that f and g are inverse functions. What is the relationship between their derivatives?

- Algebraically: inverses are obtained by interchanging the x and the y coordinates and solving for y.
- Graphically: inverses are a reflection of the graph on the line $y = x$.

If f passes through the point (a, b), then the slope of the curve at $x = a$ is represented by $f'(a)$ and is represented by the ratio of the change in y over the change in x, $\dfrac{\Delta y}{\Delta x}$.

When this figure is reflected on the line $y = x$, we obtain the graph of the inverse f^{-1} and this passes through the point (b, a), with the horizontal and vertical sides of the slope triangle interchanged. So the slope of the line tangent to the graph of f^{-1} at $x = b$ is represented by the change in x over the change in y, $\dfrac{\Delta x}{\Delta y}$. This is the reciprocal of the slope of f at $x = a$.



http://demo.activemath.org/ActiveMath2/LeAM_calculusPics/DerivInverseFunction.png?lang=en

Given (a, b) is a point on f, and g is the inverse of f, ∴ Point on $g(x)$ (b,a)

If $f'(a) = m$, then $g'(b) = \dfrac{1}{m}$.

The derivative of the inverse of a function at a point is the reciprocal of the derivative of the function at the corresponding point.

§3.6 Derivative of Inverse Function ANSWER KEY NOTES.

Examples: DECODE

1) If $f(7) = 1$ and $f'(7) = 5$, and g is the inverse of f, then what is $g'(1)$?

$f(x):(7,1) \longrightarrow g(x):(1,7)$

$f'(7)=5 \qquad g'(x)=\frac{1}{5}$

RECIPROCALS

$\frac{8}{7} \leftrightarrow \frac{7}{8}$

2) Given $f(-2) = 5$, $f'(-2) = 6$, ~~$f'(5) = -3$~~ and g is the inverse of f, what is $g'(5)$?

$f(x):(-2,5) \longrightarrow g(x):(5,-2)$

$f'(-2)=6 \qquad g'(5)=\frac{1}{6}$

3) A function f and its derivative are shown on the table. If g is the inverse of f, find $g'(4)$ and $g'(-1)$. Reciprocal

x	$f(x)$	$f'(x)$
-3	4	0.25
2	-1	$-\frac{2}{3}$

$f(x):(-3,4)$

$f'(-3)=\frac{1}{4}$

$f(x):(2,-1)$

$f'(2)=-\frac{2}{3}$

$g(x):(4,-3)$

$g'(4)=\frac{1}{\frac{1}{4}}=4$

$g(x):(-1,2)$

$g'(-1)=-\frac{3}{2}$

4) Let $f(x) = \sqrt{x}$, and let g be the inverse function. Evaluate $g'(3)$.

$f(x)=\sqrt{x} \xrightarrow{\text{INVERSES}} g(x)=x^2$

$f(9)=3 \quad f(x):(9,3) \longleftarrow g(3)=9 \quad g(x):(3,9)$

$f'(x)=\left.\frac{1}{2\sqrt{x}}\right|_{x=9} \qquad g'(x)=2x\Big|_{x=3}$

$f'(9)=\frac{1}{6} \xrightarrow{\text{RECIPROCAL}} g'(3)=6$

5) If $f(2) = -3$, $f'(2) = \frac{3}{4}$, and g is the inverse of f, what is the equation of the tangent line to $g(x)$ and $x = -3$?

$f(2)=-3$

$f(x):(2,-3) \longrightarrow g(x):(-3,2)$

$f'(2)=\frac{3}{4} \qquad g'(-3)=\frac{4}{3}$

RECIPROCAL

$\boxed{\text{ATQ}}$ EQN of TAN. LINE on $g(x)$ @ $x=-3$: $\boxed{y=\frac{4}{3}(x+3)+2}$