DAY 54 §3.6 DERIVATIVES OF INVERSE FUNCTIONS.

p. 159 #57–59, 63–68

ESTIMATE GRAPHICALLY:

57 $(f^{-1})'(5) \longrightarrow f(x)=5\,?$

$x=14$

$f'(14)=.38$

$\Downarrow$

$\approx \dfrac{1}{.38}=2.631$

58 $(f^{-1})'(10) \longrightarrow f(x)=10\,?$

$x=24$

$f'(24)=.63$

$\Downarrow$

$\approx \dfrac{1}{.63}=1.587$

59 $(f^{-1})'(15) \longrightarrow f(x)=15\,?$

$x=30$

$f'(30)=.73$

$\Downarrow$

$\approx \dfrac{1}{.73}=1.369$

GIVEN: $f(x)$ & $g(x)$ are inverse fns.

$\therefore (a, f(a)) = (a,b)$ & $(b, g(b)) = (b,a)$

$f'(a) = \dfrac{1}{g'(b)}$ & $g'(b) = \dfrac{1}{f'(a)}$

EQUIVALENT OTHER NOTATION:

$f^{-1}(x) = g(x) \;\therefore\; (f^{-1})'(x) = \dfrac{1}{f'(f^{-1}(x))}$

63 a) Given $f(x)=x^3$ $f'(2)=3(2)^2=12$

b) $f^{-1}(x)=\sqrt[3]{x}$

c) $(f^{-1})'(x)=\dfrac{1}{3x^{2/3}}$ $(f^{-1})'(8)=\dfrac{1}{3(8)^{1/3})^2}=\dfrac{1}{12}$

d) $(f^{-1})'(8)=\dfrac{1}{f'(2)}=\dfrac{1}{12}$

64 $f(x)=2x^5+3x^3+x$ a) $f'(x)=10x^4+9x^2+1$

$\cancel{=x(2x^4+3x^2+1)}$

$\cancel{=x(2x^2+1)(x^2+1)}$

b) b/c $f'(x)>0$ for all x-values we know that $f(x)$ is a strictly increasing function verifying that its inverse will also be a function & therefore $f(x)$ is <u>invertible</u>.

c) $f(1)=2+3+1=\boxed{6}$

d) $f'(1)=10+9+1=20$

e) $(f^{-1})'(6)=\dfrac{1}{f'(1)}=\dfrac{1}{20}$

$(f^{-1})'(6)=\dfrac{1}{f'(f^{-1}(1))}$

DAY 54

65

x	$f(x)$	$f'(x)$
3	1	7
6	2	10
9	3	5

$(f^{-1})'(3) \longrightarrow f(x)=3$

$x=9$

$f'(9)=5$

$(f^{-1})'(3)=\frac{1}{5}$

66 $f(p)$ is the # of gallons of gas sold when the price is p dollars per gallon.

THINK $\left\{ \begin{array}{l} (p, f(p)) \rightarrow \text{units} \left(\frac{\$p}{\text{gallons}}, \text{\# of gallons sold} \right) \\ f'(p) = \frac{\Delta f(p)}{\Delta p} = \frac{\text{\# gallons sold}}{\$/\text{gallon}} = \frac{\text{Qty Gallons}}{\$/\text{gal}} \end{array} \right.$

a) $f(2)=4023$

When gas costs $\frac{\$2}{\text{gallon}}$ 4023 gallons are sold.

b) $f^{-1}(4023)$

This quantity represents the $ cost of gas per gallon.

According to part (a) we can say

→ When 4023 gallons of gas are sold, the gas costs \$2/gallon.

c) $f'(2)=-1250$ When gas costs $\frac{\$2}{\text{gal}}$, the quantity of gas sold is decreasing at a rate of $\left(\frac{1250 \text{ gallons}}{\$1/\text{gallon}}\right)$.

d) $(f^{-1})'(4023) = \frac{1}{f'(2)} = \frac{1}{-1250} = \frac{-1}{1250} \frac{\$1/\text{gal}}{\text{gallons}}$

THINK $\left\{ \begin{array}{l} (f(p), p) \\ (q, f^{-1}(q)) = (q, P) \text{ w/ units } \left(\text{Qty of gallons}, \frac{\$P}{\text{gallons}} \right) \\ \frac{\Delta f^{-1}(q)}{\Delta q} = \frac{\$/\text{gallon}}{\text{Qty gallons sold}} \end{array} \right.$

Interpret: When 4023 gallons of gas are sold, the price per gallon is decreasing at a rate of $\frac{\$1/\text{gal}}{\text{gallon}}$.

DAY 54

"WHEN" the moment of x INPUT. "WHAT" is the function $f(x)$ output "BEHAVIOR" increasing or decreasing at a RATE of $f'(x)$ VALUE "ALWAYS" positive $\frac{\text{units output}}{\text{units input}}$

(67) $P = f(t)$ is the US population in millions in year t.

a) $f(2005) = 296$: In 2005, the US population was 296 million.

b) $f^{-1}(296) = 2005$: When the US population was 296 million, the year was 2005.

c) $f'(2005) = {}^{+}2.65$: In 2005, the US population was increasing at a rate of $\frac{2.65 \text{ million}}{\text{year}}$.

d) $(f^{-1})'(296) = \dfrac{1}{f'(2005)} = \dfrac{1}{2.65} = 0.377 \dfrac{\text{year}}{\text{million people}}$.

What? NEED TO CHECK ON THIS!

When the US population was 296 million (in 2005) the year is increasing at rate of $0.377 \dfrac{\text{years}}{\text{million people}}$.

(68) # of motor vehicles $f(t)$, in millions, registered in the world t years after 1965. (see graph) (years, millions of registered vehicles)

a) $f(20) = 500$ million motor vehicles were registered in 1985.

b) $f'(20) \approx \dfrac{100}{5} = 20$ million vehicles per year

In 1985 the # of motor vehicles registered is increasing at a rate of $\dfrac{20 \text{ million vehicles}}{\text{year}}$.

c) $f^{-1}(500) = 20$.

20 years after 1965 ⇒ 1985 approximately 500 million vehicles are registered in the US.

d) $(f^{-1})'(500) = \dfrac{1 \text{ year}}{20 \text{ million of vehicles}}$.

When 500 million vehicles are registered

reciprocals