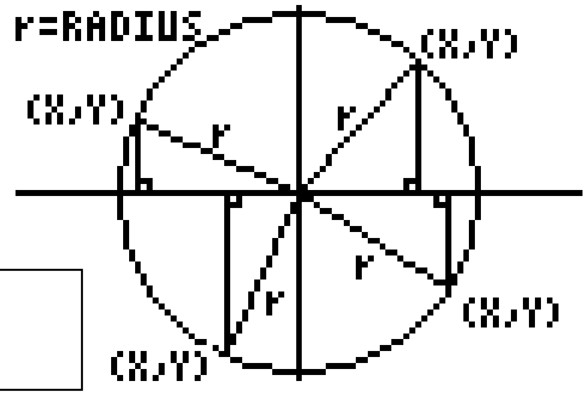
Conic Sections — Name______________________ Monday 21 May

Circles Classwork & HW — period 2 3 4 5

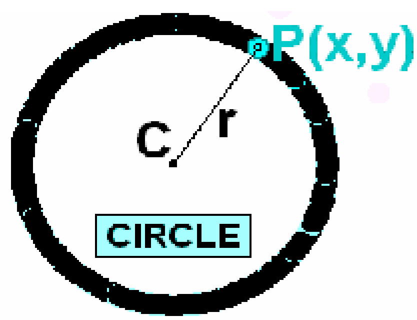
DUE at end of period TODAY! (period.)

Warm-Up:



For any (x, y) point on a circle of radius r whose center is at the origin $(0,0)$, use the distance formula or the Pythagorean Identity to write an equation for the distance between the origin and the point (x, y).

For any (x, y) point on a circle of radius r whose center is at the point (h,k), use the distance formula to write an equation for the distance between the point (h,k) and the point (x, y).



Circles:

A circle is the set of all points equidistant (a fixed **radius** distance **r**) from a **center** point.

(h,k) form: $\quad (x-h)^2 + (y-k)^2 = r^2 \qquad \frac{(x-h)^2}{r^2} + \frac{(y-k)^2}{r^2} = 1$

expanded form derived from (h,k) form:

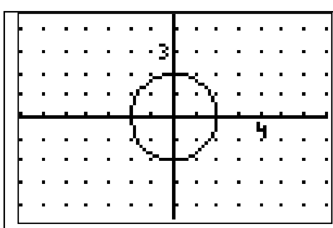
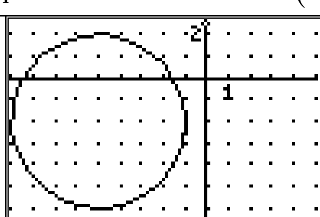
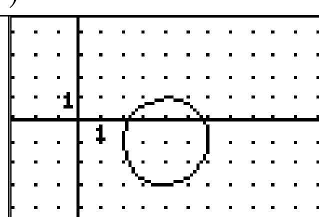
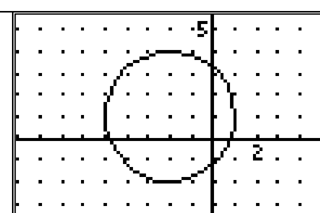
$$\left(x^2 - 2hx + h^2\right) + \left(y^2 - 2ky + k^2\right) = r^2$$

$$x^2 + y^2 - (2h)x - (2k)y + \left(h^2 + k^2 - r^2\right) = 0$$

$$(A)x^2 + (C)y^2 + (D)x + (E)y + (F) = 0$$

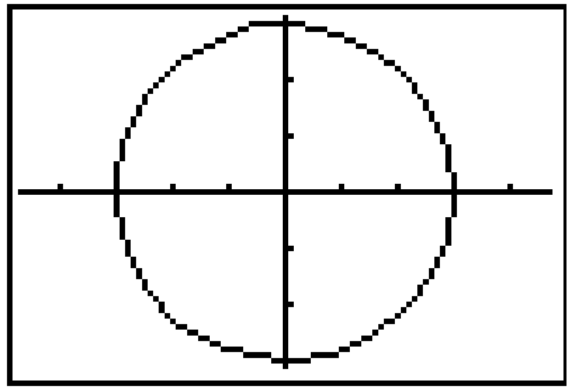
To graph the circle if its equation is in standard form, complete the square to write the equation in (h,k) form.

Examples: Write the equations of the circles in (h,k) form

 1.	 2.	 3.	 4.

H.W. Exercises:

5. In the desert, irrigation sprinklers rotate forming circular fields for crops. If the field has an area of $748,225\pi$ square yards (or approximately $\approx 2,350,618$ square yards),



a) Write an equation for the circular boundary of the field assuming the center of the field is at the origin. Show work to justify your answer.

b) Calculate the circumference of the circular field. Show work to justify your answer.

c) If you are out surveying the crops and your current position is given, are you
   i) inside the circular field?
   ii) on the perimeter (circumference) of the field?
   iii) outside the field? Show work to justify your answer.

A) $800\ yds\ East$ $100\ yds\ North$	B) $850\ yds\ West$ $100\ yds\ South$	C) $860\ yds\ West$ $150\ yds\ North$

6. Write the equation of the circle in (h,k) form with the given characteristics

A	center: $(-7,3)$ radius: $r=8$	
B	center: $(-2,5)$ area: 81π	
C	diameter with endpoints at $(3,8)$ and $(9,16)$	
D.	center at the point located $\frac{1}{3}$ of the way from pt A:$(6,-1)$ toward pt B:$(30,-13)$ with radius half the length of the $\overline{AB}$. **HINT: Sketch a diagram**	

8. Complete the square to find the equation of the circle in (h,k) form.

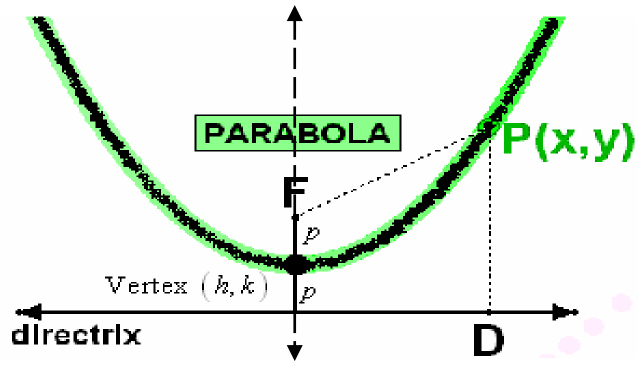
A. $x^2+y^2+4x-18y+69=0$ Identify Center (____,____) and radius: ____	B. $3x^2+3y^2+6x-24y-93=0$ Identify Center (____,____) and radius: ____

9. Sketch all of the possible scenarios and state how many intersection points exist for each. You may need more or fewer spaces to show all scenarios.

A line and a parabola				
A line and a circle				
A parabola and a circle				

Parabolas:

A parabola is the set of all points equidistant from a line called the **directrix** and a point called the **focus**.



$y = a(x-h)^2 + k$ $\qquad$ $x =$ ____________________

$(y-k) = a(x-h)^2$ $\qquad$ $(x -$ _____ $)^2 =$ ____________

The value p represents the distance from **vertex to focus** and **vertex to directrix line**. In the parabola equation p is determined by the coefficient in front of the $(x-h)$ term or the $(y-k)$ term as follows:

$$(y-k) = \left[\ \frac{1}{4p}\ \right](x-h)^2 \qquad\qquad [\ 4p\ ](y-k) = (x-h)^2$$

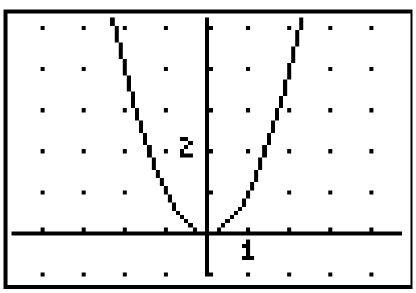
For the graphs in the warm-up find the value of p, the focus and directrix line. Add these to the graphs.

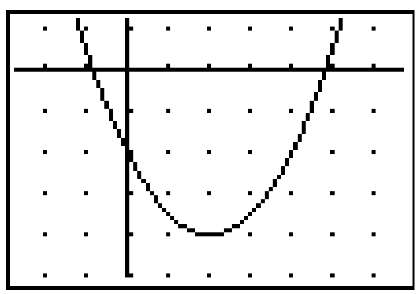
1) Write equations for each parabola in (h,k) form: $\quad y = a(x-h)^2 + k$

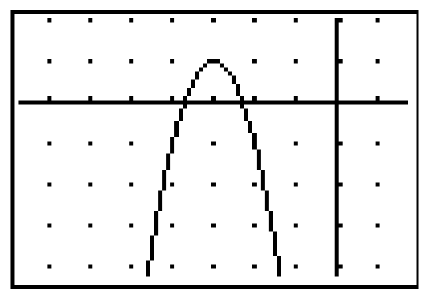
A) ____________________
$a =$ ___ $\therefore p =$ ___
Focus : _________
Vertex : _________
Directrix : _______

B) ____________________
$a =$ ___ $\therefore p =$ ___
Focus : _________
Vertex : _________
Directrix : _______

C) ____________________
$a =$ ___ $\therefore p =$ ___
Directrix : _______
Vertex : _________
Focus : _________





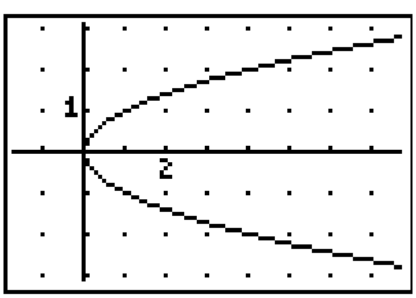


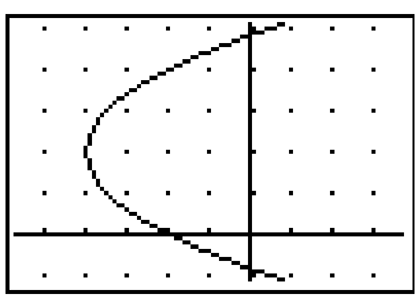
2) Write equations for each parabola in (h,k) form: ____________________________

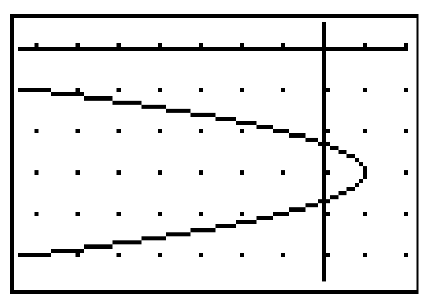
C) ____________________
$a =$ ___ $\therefore p =$ ___
Directrix : _______
Vertex : _________
Focus : _________

D) ____________________
$a =$ ___ $\therefore p =$ ___
Directrix : _______
Vertex : _________
Focus : _________

E) ____________________
$a =$ ___ $\therefore p =$ ___
Focus : _________
Vertex : _________
Directrix : _______





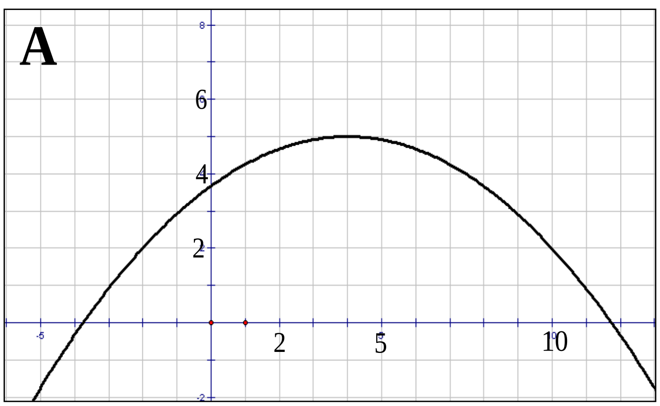


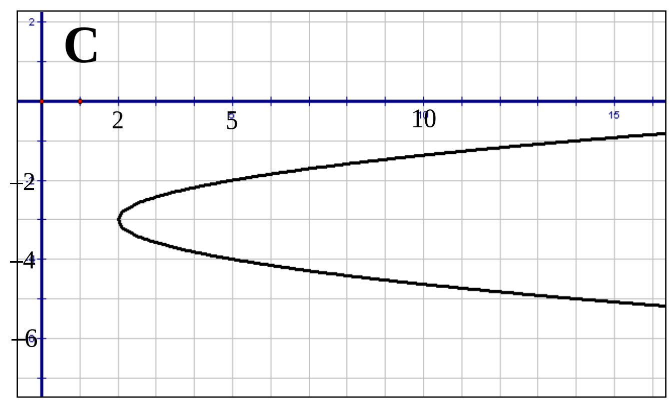
H.W. Exercises:

1. Complete the square to write each parabola in (h,k) form. Identify Vertex, Focus, Directrix line.

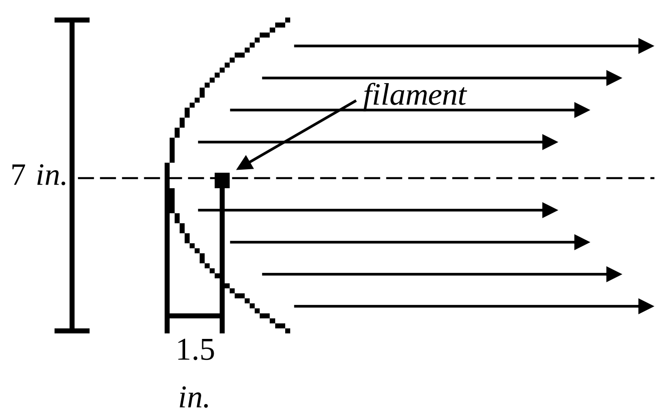
A. $x=3y^2+18x+29$	B. $x=\frac{-1}{4}y^2-2y-7$	C. $y=\frac{-1}{12}x^2+\frac{2}{3}x+\frac{11}{3}$

2. Identify the vertex and the focal point then write an equation for the parabola and the directrix line.

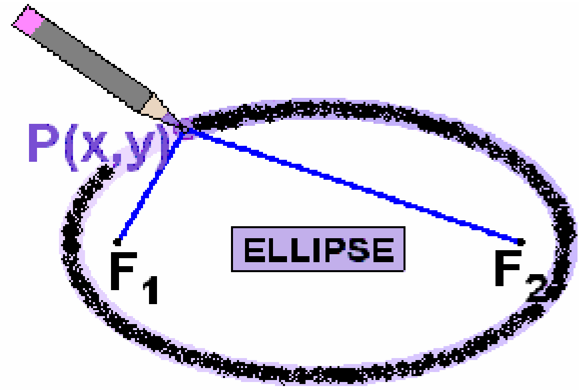




3. The filament of a light bulb is a thin wire that glows when electricity passes through it. The filament of a car headlight is at the focus of the parabolic reflector, which sends light out in a straight beam (parallel to the axis of symmetry of the parabolic cross section of the reflector). Given that the filament is 1.5 inches from the vertex, find an equation for the cross section of the reflector. If the reflector is 7 inches wide, how deep is it? (page 599 #79)



Ellipses: An ellipse is the set of all points



$$\boxed{F_1P + F_2P = constant}$$

(h,k) form: $\dfrac{(x-h)^2}{a^2}+\dfrac{(y-k)^2}{b^2}=1 \quad where \quad a>b$

or $\dfrac{(x-h)^2}{b^2}+\dfrac{(y-k)^2}{a^2}=1 \quad where \quad a>b$

expanded form derived from (h,k) form:

$$b^2(x-h)^2+a^2(y-k)^2=a^2b^2$$

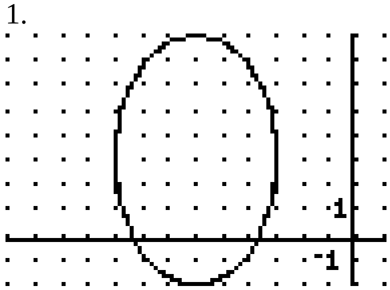
$$b^2\left(x^2-2hx+h^2\right)+a^2\left(y^2-2ky+k^2\right)=a^2b^2$$

$$b^2x^2+a^2y^2-b^2(2h)x-a^2(2k)y+\left(b^2h^2+a^2k^2-a^2b^2\right)=0$$

$$(A)x^2+\ (C)y^2+\ (D)x+\ \ (E)y\ \ +\ \ (F)\ \ =\ \ 0$$

(h,k) = coordinate of center
a = distance from center to major axis vertices
b = distance from center to minor axis co-vertices
c = distance from center to foci ("fo-sigh") which are on the major axis

1.



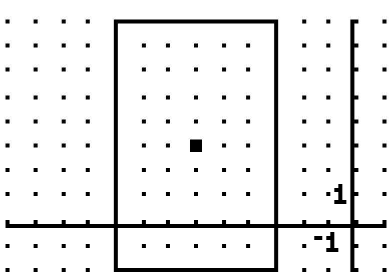
Center:

a =
Vertices:

c =
Foci:

b=
Co-Vertices:

Equation:________________________________

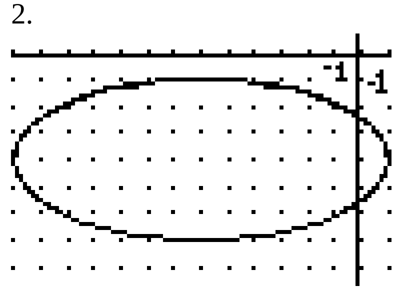


The ellipse above can be inscribed in this rectangle.

Write the equations of the diagonal lines of the rectangle:

Equations:________________________________

2.



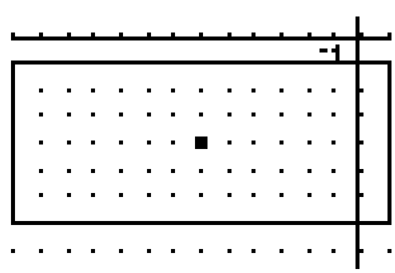
Center:

a =
Vertices:

c =
Foci:

b =
Co-Vertices:

Equation:________________________________



The ellipse above can be inscribed in this rectangle.

Write the equations of the diagonal lines of the rectangle:

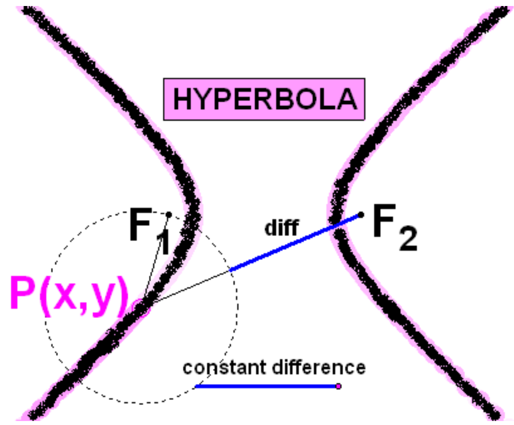
Equations:________________________________

Complete the square to identify all key features of the ellipse.

3. $225x^2 + 144y^2 - 450x + 864y - 30879 = 0$	4. $25x^2 + 169y^2 + 200x - 676y - 3149 = 0$
Graph. Label center, vertices, co-vertices, and foci.	Graph. Label center, vertices, co-vertices, and foci.

Hyperbolas: A hyperbola is the set of all points in a plane whose distances from two fixed points **differ** by a constant.

$$\boxed{F_1P - F_2P = \text{constant}}$$



(h,k) form: $$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$$

or $$\frac{(y-k)^2}{b^2} - \frac{(x-h)^2}{a^2} = 1$$

expanded form derived from (h,k) form:

$$b^2(x-h)^2 - a^2(y-k)^2 = a^2b^2$$

$$b^2\left(x^2 - 2hx + h^2\right) - a^2\left(y^2 - 2ky + k^2\right) = a^2b^2$$

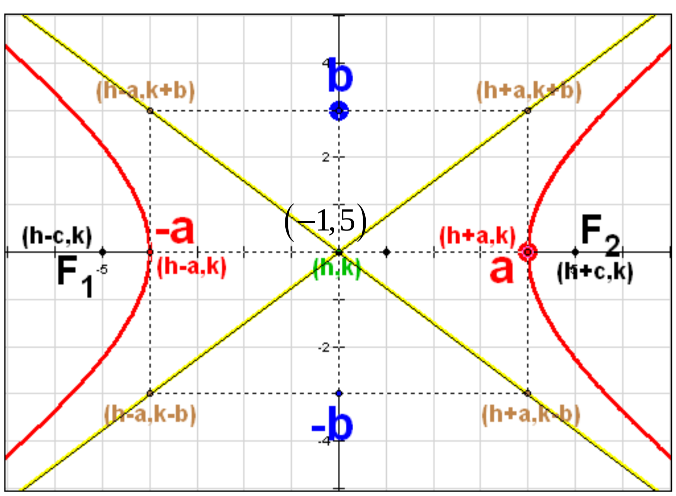
$$b^2x^2 - a^2y^2 - b^2(2h)x + a^2(2k)y + \left(b^2h^2 - a^2k^2 - a^2b^2\right) = 0$$

$$(A)x^2 + (C)y^2 + (D)x + (E)y + (F) = 0$$

(h,k) = coordinate of center
a = distance from center to transverse axis vertices
b = distance along the conjugate axis
c = distance from center to foci ("fo-sigh") which are on the transverse axis

Hyperbolas

1.



Equation:______________________________

Center:____________ **a =**___ **b =**___ **c =**___

Relationship between a, b, & c:___________

Transverse axis:__________

Conjugate Axis:__________

Vertices on the Transverse Axis:

Foci (pronounced "fo-sigh"; plural of focus)
Focal points along the transverse axis:

Asymptotes:

2. **Equation:**_________________________

Center:____________ **a =___ b =___ c =___**

Relationship between a, b, & c:___________

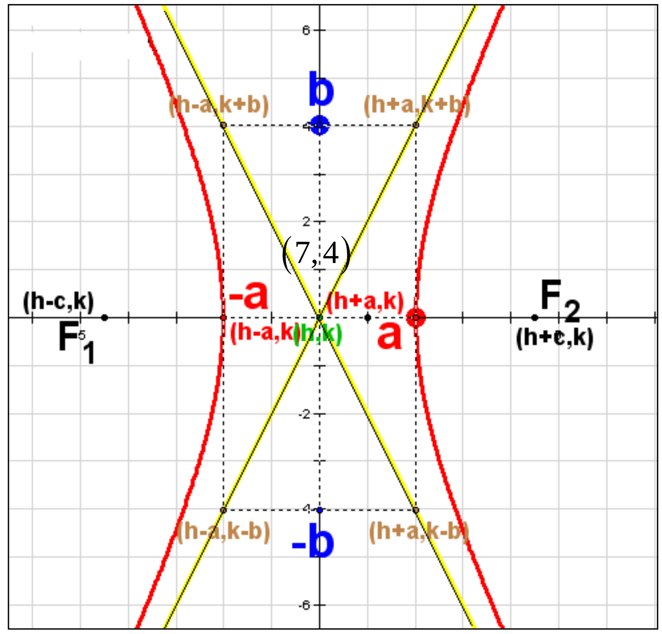
Transverse axis:__________

Conjugate Axis:__________

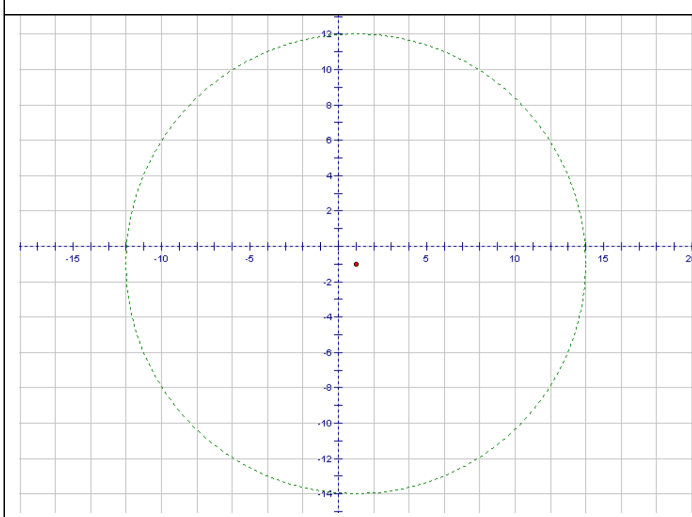
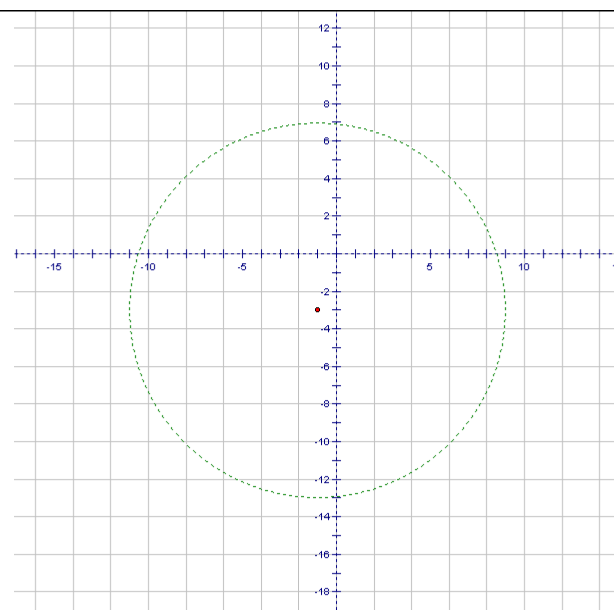
<u>**Vertices**</u> on the Transverse Axis:

<u>**Foci**</u> (pronounced “fo-sigh”; plural of focus)
Focal points along the transverse axis:

<u>**Asymptotes**</u>**:**



Complete the square to identify all key features of the hyperbola.

3. $144x^2 - 25y^2 - 288x - 50y - 3481 = 0$	4. $-36x^2 + 64y^2 - 72x + 384y - 1764 = 0$
 Graph. Label center, vertices, foci and asymptotes.	 Graph. Label center, vertices, foci and asymptotes.