

1. What is the area of the region between the graphs of $y = x^2$ and $y = -x$ from $x = 0$ to $x = 2$?

- a) $\frac{2}{3}$ b) $\frac{8}{3}$ c) 4 d) $\frac{14}{3}$ e) $\frac{10}{3}$

2. The area of the region enclosed by the graph of $y = x^2 + 1$ and the line $y = 5$ is

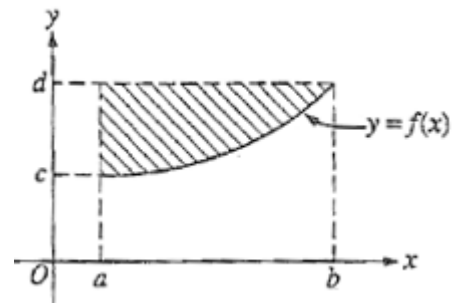
- a) $\frac{14}{3}$ b) $\frac{16}{3}$ c) $\frac{28}{3}$ d) $\frac{32}{3}$ e) 8π

3. If the region enclosed by the y-axis, the line $y = 2$, and the curve $y = \sqrt{x}$ is revolved about the y-axis, the volume of the solid generated is

- a) $\frac{32\pi}{5}$ b) $\frac{16\pi}{3}$ c) $\frac{16\pi}{3}$ d) $\frac{8\pi}{3}$ e) π

4. Which of the following represent the area of the shaded region in the figure above?

- a) $\int_c^d f(y)dy$ b) $\int_a^b (d - f(x))dx$
 c) $f'(b) - f'(a)$ d) $(b - a)[f(b) - f(a)]$
 e) $(d - c)[f(b) - f(a)]$



5. The area of the region enclosed by the curve $y = \frac{1}{x-1}$, the x-axis, and the lines $x = 3$

- and $x = 4$ is a) $\frac{5}{36}$ b) $\ln \frac{2}{3}$ c) $\ln \frac{4}{3}$ d) $\ln \frac{3}{2}$ e) $\ln 6$

6. The region enclosed by the x-axis, the line $x = 3$, and the curve $y = \sqrt{x}$ is rotated about the x-axis. What is the volume of the solid generated?

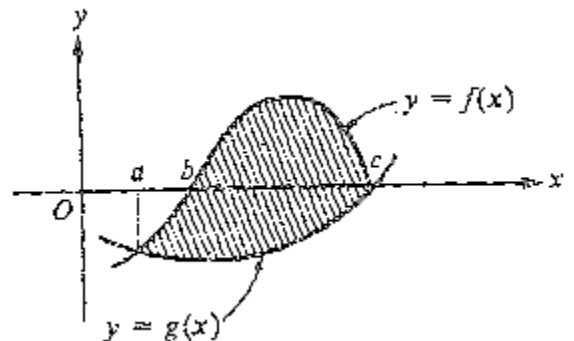
- a) 3π b) $2\sqrt{3}\pi$ c) $\frac{9}{2}\pi$ d) 9π e) $\frac{36\sqrt{3}}{5}\pi$

7. The area of the region enclosed by the graphs of $y = x$ and $y = x^2 - 3x + 3$ is

- a) $\frac{2}{3}$ b) 1 c) $\frac{4}{3}$ d) 2 e) $\frac{14}{3}$

8. The area of the shaded region in the figure above is represented by which of the following integrals?

- a) $\int_a^c (|f(x)| - |g(x)|)dx$ b) $\int_b^c f(x)dx - \int_a^c g(x)dx$
 c) $\int_a^c (g(x) - f(x))dx$ d) $\int_a^c (f(x) - g(x))dx$
 e) $\int_a^b (g(x) - f(x))dx + \int_b^c (f(x) - g(x))dx$



9. Find the area of the region whose boundaries are $y^2 = x$ and $x + y = 2$.

- a) $\frac{5}{2}$ b) $\frac{3}{2}$ c) $\frac{11}{6}$ d) $\frac{9}{2}$ e) $\frac{29}{6}$

10. Find the area of the region whose boundaries are the parabolas $x = y^2 - 5y$ and $x = 3y - y^2$.

- a) $\frac{32}{3}$ b) $\frac{139}{6}$ c) $\frac{64}{3}$ d) $\frac{128}{3}$ e) none of these

11. Find the area of the region whose boundaries are in the first quadrant, bounded below by the x-axis and above by the curves of $y = \sin x$ and $y = \cos x$.

- a) $2 - \sqrt{2}$ b) $2 + \sqrt{2}$ c) 2 d) $\sqrt{2}$ e) $2\sqrt{2}$

12. Find the area of the region which is bounded above by $y = \sin x$ and below by $y = \cos x$ from $x = \frac{\pi}{4}$ to

- $x = \frac{5\pi}{4}$. a) $2\sqrt{2}$ b) $\frac{2}{2\sqrt{2}}$ c) $\frac{1}{2\sqrt{2}}$ d) $2(\sqrt{2} - 1)$ e) $2(\sqrt{2} + 1)$

13. The area of the region bounded by the curve of $y = x^3 - 2x^2 - 3x$ and the x-axis is equal to

- (A) $\frac{28}{3}$ (B) $\frac{79}{6}$ (C) $\frac{45}{4}$ (D) $\frac{71}{6}$ (E) None of these

14. The total area bounded by the cubic $x = y^3 - y$ and the line $x = 3y$ is equal to

- (A) 4 (B) $\frac{16}{3}$ (C) 8 (D) $\frac{32}{3}$ (E) 16

For #17 – 20, the region whose boundaries are given is rotated about the line indicated. Choose the answer which gives the volume of the solid generated.

15. $y = x^2$, $x = 2$, and $y = 0$ about the x-axis.

- (A) $\frac{64\pi}{3}$ (B) $\frac{32\pi}{5}$ (C) $\frac{8\pi}{3}$ (D) $\frac{128\pi}{5}$ (E) 8π

16. $y = x^2$, $x = 2$, and $y = 0$ about the y-axis.

- (A) $\frac{16\pi}{3}$ (B) 4π (C) $\frac{32\pi}{5}$ (D) 8π (E) $\frac{8\pi}{3}$

17. $y = x^2$, $x = 2$, and $y = 0$ about the line $x = 2$.

- (A) 4π (B) $\frac{4\pi}{3}$ (C) $\frac{88\pi}{3}$ (D) $\frac{16\pi}{3}$ (E) 8π

18. The first quadrant region bounded by $y = x^2$, the y-axis, and $y = 4$; about the y-axis.

- (A) 8π (B) 4π (C) $\frac{64\pi}{3}$ (D) $\frac{32\pi}{3}$ (E) $\frac{16\pi}{3}$

19. $y = 3x - x^2$ and $y = x$ about the x-axis.

- (A) $\pi \int_0^{3/2} [(3x - x^2)^2 - x^2] dx$ (B) $\pi \int_0^2 (9x^2 - 6x^3) dx$ (C) $\pi \int_0^2 [(3x - x^2)^2 - x^2] dx$
(D) $\pi \int_0^3 [(3x - x^2)^2 - x^4] dx$ (E) $\pi \int_0^3 (2x - x^2)^2 dx$

20. The base of a solid is a circle of radius a , and every plane section perpendicular to a diameter is a square. The solid has volume

- (A) $\frac{8}{3}a^3$ (B) $2\pi a^3$ (C) $4\pi a^3$ (D) $\frac{16}{3}a^3$ (E) $\frac{8\pi}{3}a^3$

21. The base of a solid is the region bounded by the parabola $x^2 = 8y$ and the line $y = 4$, and each plane section perpendicular to the y-axis is an equilateral triangle. The volume of the solid is

- (A) $\frac{64\sqrt{3}}{3}$ (B) $64\sqrt{3}$ (C) $32\sqrt{3}$ (D) 32 (E) None of these

Calculator Section:

22. The area bounded by $y = e^x$, $y = 1$, $y = 2$ and $x = 3$ is equal to

- (A) $3 + \ln 2$ (B) $3 - 3 \ln 3$ (C) $4 + \ln 2$ (D) $3 - \frac{1}{2} \ln^2 2$ (E) $4 - \ln 4$

23. The base of a solid is a region in the first quadrant bounded by the x-axis, the y-axis, and the line $x + 2y = 8$. If cross sections of the solid perpendicular to the x-axis are semicircles, what is the volume of the solid?

- (A) 12.566 (B) 14.661 (C) 16.775 (D) 67.021 (E) 134.041

24. What is the area of the region in the first quadrant enclosed by the graphs of $y = \cos x$, $y = x$, and the y-axis?

- (A) 0.127 (B) 0.385 (C) 0.400 (D) 0.600 (E) 0.947

25. The base of a solid S is the region enclosed by the graph of $y = \sqrt{\ln x}$, the line $x = e$, and the x-axis. If the cross sections of S perpendicular to the x-axis are squares, then the volume of S is

- (A) $\frac{1}{2}$ (B) $\frac{2}{3}$ (C) 1 (D) 2 (E) $\frac{1}{3}(e^3 - 1)$

Multiple Choice Answers:

- | | | | | | | |
|-------|-------|-------|-------|-------|-------|-------|
| 1. D | 2. D | 3. A | 4. B | 5. D | 6. C | |
| 7. C | 8. D | 9. D | 10. C | 11. A | 12. A | |
| 13. D | 14. C | 15. B | 16. D | 17. C | 18. A | |
| 19. C | 20. D | 21. B | 22. E | 23. C | 24. C | 25. C |

- Find the area bounded by $y = \frac{1}{x^3}$, $x = 4$ and $x = 5$.
- Find the area bounded by $x = y^2 + 1$ and $x = y + 3$.
- Find the area bounded by $y = e^x$, $y = e^2$ and $x = 0$.
- Let R be the region bounded by $y = \sqrt{x}$, $y = 0$ and $x = 9$. Find the value of k such that the line $x = k$ divides the region R into two regions of equal area.
- Find the volume of the solid bounded by the region $y = x$, $y = 0$ and $x = 4$ revolved about
 - the x -axis
 - the y -axis
 - the line $x = 4$
- Find the volume of the solid bounded by the region $y = \sqrt{x}$, $y = 2$ and $x = 0$ revolved about
 - the x -axis
 - the line $y = 2$
- Find the volume of the solid bounded by the region $y = e^{-x}$, $y = 0$, $x = 0$ and $x = 1$ revolved about the x -axis.
- Find the volume of the solid whose base is bounded by $y = 2 \sin x$, $y = 0$ for $0 \leq x \leq \pi$, with the indicated cross sections taken perpendicular to the x -axis.
 - squares
 - semicircles
 - equilateral triangles

Chapter 8 Review Answers:

1. $9/800$ 2. 4.5 3. $e^2 + 1$ 4. $\left(\frac{27}{2}\right)^{\frac{2}{3}} = \frac{9}{\sqrt[3]{4}}$ 5. a. $\frac{64\pi}{3}$
5. a. ? b. $\frac{128\pi}{3}$ c. $\frac{64\pi}{3}$ 6. a. 8π b. $\frac{8\pi}{3}$ 7. $\frac{\pi}{2} - \frac{\pi}{2e^2}$
8. a. 2π b. $\frac{\pi^2}{4}$ c. $\frac{\sqrt{3}}{2}\pi$