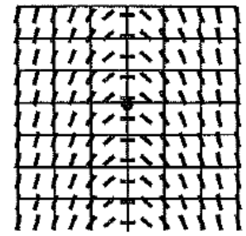
Problems with * are CALCULATOR active**

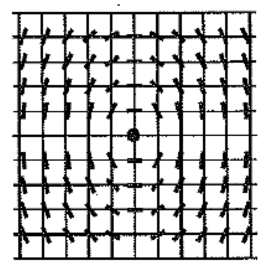


$[-3,3]\times[-4,-3]$

1. Which of the following differential equations goes with the slope field shown?

a) $y'=-x$ b) $y'=-x^2$ c) $y'=-2x$

d) $y'=2x$ e) $y'=x^2$

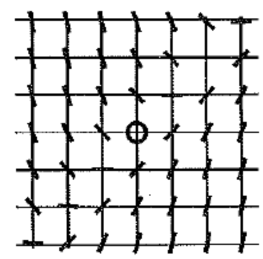


$[-5,5]\times[-5,5]$

2. Which of the following differential equations goes with the slope field shown?

a) $y'=4x^2+y^2$ b) $y'=x+y$ c) $y'=x^2+4y^2$

d) $y'=-\dfrac{x}{2y}$ e) $y'=-\dfrac{2x}{y}$

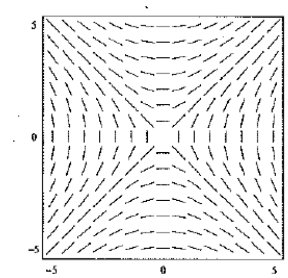


$[-3,3]\times[-3,3]$

3. Which of the following differential equations goes with the slope field shown?

a) $y'=x-y$ b) $y'=x+y$ c) $y'=x^2+y^2$

d) $y'=\dfrac{x}{y}$ e) $y'=-\dfrac{x}{y}$



4. Which differentiable equation corresponds to the slope field shown at right?

(A) $\dfrac{dy}{dx}=x^2-y^2$ (B) $\dfrac{dy}{dx}=y^2-x^2$ (C) $\dfrac{dy}{dx}=\dfrac{x}{y}$

(D) $\dfrac{dy}{dx}=\dfrac{y}{x}$ (E) $\dfrac{dy}{dx}=-\dfrac{x}{y}$

5. Find the equation of a family of curves which are defined by $f'(x)=\dfrac{-2}{(x+1)^3}$.

a) $f(x)=(x+1)^2+C$ b) $f(x)=\dfrac{1}{(x+1)^2}+C$ c) $f(x)=\dfrac{3}{(x+1)^2}+C$

d) $f(x)=\tan^{-1}(x+1)+C$ e) $f(x)=(x+1)^{-1}+C$

6. Solve the differential equation $e^{x-y}dy=-dx$.

a) $e^{-x}+e^{y}=C$ b) $e^{-x}+e^{-y}=C$ c) $e^{x}+e^{-y}=C$ d) $2e^{-x}+e^{-y}=C$ e) $e^{-x}+2e^{-y}=C$

7. Find the solution of the differential equation $\frac{dy}{dx} = \frac{1}{3}x^{-2/3}$ for $f(-8) = 0$.

a) $f(x) = x^{1/3} - 2$ b) $f(x) = x^{1/3} + 5$ c) $f(x) = x^{1/3} + 2$

d) $f(x) = x^{2/3} + 2$ e) $f(x) = x^{2/3} + 1$

8. Find the solution of the differential equation $xy' = y - 2$ for $y > 2,\ x > 0$, given that the point $(3,8)$ is on the curve.

a) $y = x + 2$ b) $y = 2x + 2$ c) $y = 5x - 2$ d) $y = 2x - 2$ e) $y = x - 1$

***9. A mold culture doubles its mass every three days. Find the growth model for a plate seeded with 1.2 grams of mold.

a) $y = 1.2e^{0.10034t}$ b) $y = 1.2e^{0.23105t}$ c) $y = 1.2e^{0.23856t}$

d) $y = 1.2e^{0.38761t}$ e) $y = 1.2e^{0.54931t}$

10. If h is the inverse function of f and if $f(x) = \frac{1}{x}$, then $h'(3) =$

a) -9 b) $-\frac{1}{9}$ c) $\frac{1}{9}$ d) 3 e) 9

***11. The population P of a herd is given by $P = 2000e^{kt}$. Let $t = 0$ correspond to the year 1998 and suppose the population in 1995 was 1,700. Use this model to predict the population in 2005.

a) 2,922 b) 2,517 c) 1,815 d) 3,218 e) 2,623

***12. A radioactive element has a half-life of 50 days. What percentage of the original sample is left after 60 days?

a) 25.00% b) 37.50% c) 40.21% d) 43.53% e) 49.56%

13. Suppose $f(x) = 2x^3 - 3x$. If $h(x)$ is the inverse function of f, then $h'(-1) =$

a) -1 b) $\frac{1}{5}$ c) $\frac{1}{3}$ d) 1 e) 3

14. A stone is thrown at a target so that its velocity after t sec is $(100 - 20t)$ ft/sec. If the stone hits the target in 1 sec, then the distance from the sling to the target is

a) 80 ft b) 90 ft c) 100 ft d) 110 ft e) 120 ft

15. A solution of the differential equation $y\,dy = x\,dx$ is

a) $x^2 - y^2 = 4$ b) $x^2 + y^2 = 4$ c) $y^2 = 4x^2$

d) $x^2 - 4y^2 = 0$ e) $x^2 = 9 - y^2$

16. The general solution of the differential equation $x\,dy = y\,dx$ is a family of
(A) circles (B) hyperbolas (C) parallel lines
(D) parabolas (E) lines passing through the origin

17. The population of a city increases continuously at a rate proportional, at any time, to the population at that time. The population doubles in 50 years. After 75 years, the ratio of the population P to the initial population P_0 is

(A) $\frac{9}{4}$ (B) $\frac{5}{2}$ (C) $\frac{4}{1}$ (D) $\frac{2\sqrt{2}}{1}$ (E) none of these

18. If a substance decomposes at a rate proportional to the amount of the substance present, and if the amount decreases from 40 g to 10 g in 2 hours, then the constant of proportionality is

(A) $-\ln 2$ (B) $-\frac{1}{2}$ (C) $-\frac{1}{4}$ (D) $\ln\frac{1}{4}$ (E) $\ln\frac{1}{8}$

***19. A cup of coffee at temperature $180^\circ F$ is placed on a table in a room at $68^\circ F$. The differential equation for its temperature at time t is $\frac{dy}{dt} = -0.11(y - 68)$; $y(0) = 180$. After 10 minutes, the temperature (in $^\circ F$) of the coffee is
(A) 96 (B) 100 (C) 105 (D) 110 (E) 115

20. According to Newton's law of cooling, the temperature of an object decreases at a rate proportional to the difference between its temperature and that of the surrounding air. Suppose a corpse at a temperature of $32^\circ C$ arrives at a mortuary where the temperature is kept at $10^\circ C$. Then the differential equation satisfied by the temperature T of the corpse t hours later is

(A) $\frac{dT}{dt} = -k(T - 10)$ (B) $\frac{dT}{dt} = -k(T - 32)$ (C) $\frac{dT}{dt} = 32e^{-kt}$

(D) $\frac{dT}{dt} = -kT(T - 10)$ (E) $\frac{dT}{dt} = kT(T - 32)$

***21. If the corpse in Question 20 cools to $27^\circ C$ in one hour, then its temperature (in $^\circ C$) is given by the equation
(A) $T = 22e^{0.205t}$ (B) $T = 10e^{1.163t}$ (C) $T = 10 + 22e^{-0.258t}$
(D) $T = 32e^{-0.169t}$ (E) $T = 32 - 10e^{-0.093t}$

22. The general solution to $y' = \frac{2x}{y}$ is $y = \sqrt{2x^2 + C}$. Find a solution to the initial value problem

$y' = \frac{2x}{y}$, $y(0) = 4$.

(A) $y = 4\sqrt{2x^2 + 1}$ (B) $y = \sqrt{2x^2 + 4}$ (C) $y = 2\sqrt{2x^2 + 4}$ (D) $y = \sqrt{2x^2 + 16}$

23. A differential equation is separable if it can be written in the form (may have more than one answer):

A) $\frac{dy}{dx} = f(x) + g(y)$ (B) $\frac{dy}{dx} = f(g(x,y))$ (C) $\frac{dy}{dx} = \frac{f(x)}{g(y)}$ (D) $\frac{dy}{dx} = f(x)g(y)$

***24. If $\frac{dx}{dt} = \sqrt{1 + \sin(t^2)}$ and, $x(0) = 3$ what is $x(2)$?

(A) 0.349 (B) 0.588 (C) 2.333 (D) 2.413 (E) 5.333

25. If a function $y = f(x)$ satisfies the differential equation $\frac{dy}{dx} = -4y$ and $f(0) = 6$, then $f(x) =$

(A) $-2x^2 + 6$ (B) $-\frac{x}{4} + 6$ (C) $6e^{-4x}$ (D) $e^{-4x} + 5$ (E) $-\frac{1}{4}\ln\left(x + e^{-24}\right)$

26. The solution to the differential equation $\frac{dy}{dx} = \frac{x}{\cos y}$ with the initial condition $y(1) = 0$ is

(A) $y = \sin^{-1}\left(\frac{x^2 - 1}{2}\right)$ (B) $y = \sin^{-1}\left(\frac{x^2}{2}\right)$ (C) $y = \cos^{-1}\left(x^2 - 2\right)$

(D) $y = \ln\left[\cos(x-1)\right] + \pi$ (E) $y = \ln\left[\sin x\right] + \pi$

***27. The rate at which a bacteria population grows is proportional to the number of bacteria present. At time $t = 2$ hours, 500 bacteria were present, and at time $t = 6$, 1500 bacteria were present. Approximately how long does it take for the population to double?

(A) 2.5 hours B) 5.4 hours (C) 7.6 hours

(D) 8.4 hours (E) 9.6 hours

***28. If $\frac{dy}{dx} = \frac{\sin x}{x}$ and $y(1) = 4$, then $y(2) =$

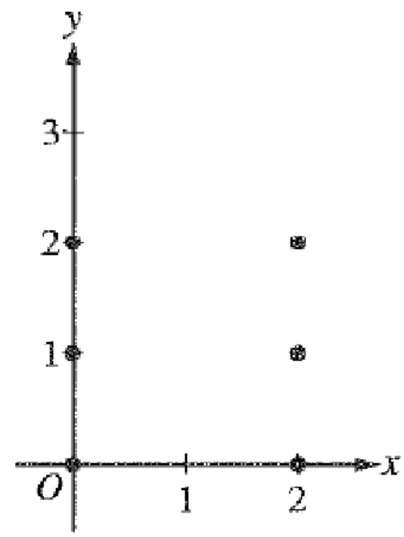
(A) 0.455 B) 0.659 (C) 4.455 (D) 4.659 (E) 5.289

Answers:

1. A	2. E	3. A	4. C	5. B	6. B	7. C	8. B	9. B	10. B	11. A	12. D
13. C	14. B	15. A	16. E	17. D	18. A	19. C	20. A	21. C	22. D	23. D	24. E
25. C	26. A	27. A	28. D								

Complete these FRQ on your own paper, except for graphing the slope fields.

1. Consider the differential equation $\frac{dy}{dx} = \frac{y^2}{x-1}$.

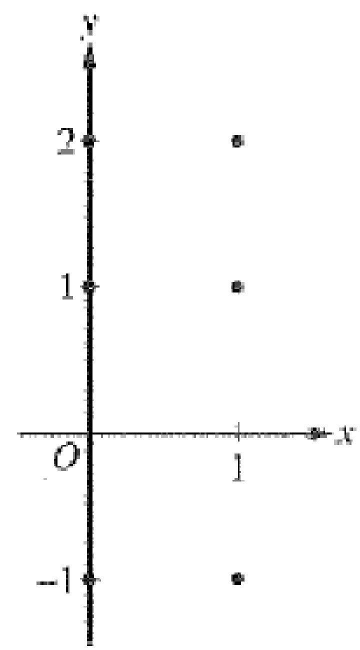


(a) On the axes provided, sketch a slope field for the given differential equation at the six points indicated.

(b) Let $y = f(x)$ be the particular solution to the given differential equation with the initial condition $f(2) = 3$. Write an equation for the tangent line to the graph of $y = f(x)$ at $x = 2$.

(c) Find the particular solution $y = f(x)$ to the given differential equation with the initial condition $f(2) = 3$.

2. Consider the differential equation $\frac{dy}{dx} = 2x - y$.



(a) On the axes provided, sketch a slope field for the given differential equation at the six points indicated.

(b) Find $\frac{d^2y}{dx^2}$ in terms of x and y. Determine the concavity of all solution curves for the given differential equation in Quadrant II. Give a reason for your answer.

(d) Let $y = f(x)$ be the particular solution to the given differential equation with the initial condition $f(2) = 3$. Does $y = f(x)$ have a relative minimum, relative maximum or neither at $x = 2$. Justify your answer.

(c) Find the particular solution $y = f(x)$ to the given differential equation with the initial condition $f(2) = 3$.

(e) Find the values of m and b such that $y = mx + b$ is a solution to the differential equation.