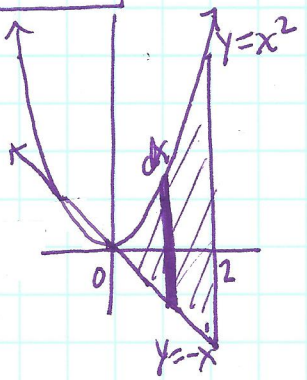
ANSWERS W/ WORK SHOWN.

CH 8 REVIEW

① $y=x^2$, $y=-x$, $x\in[0,2]$

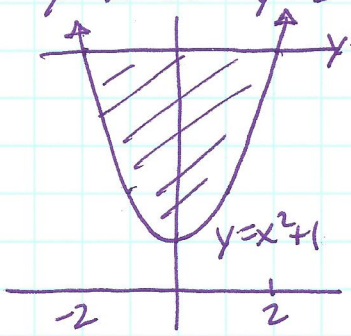


$A=\int_0^2(x^2--x)\,dx=\int_0^2(x^2+x)\,dx$

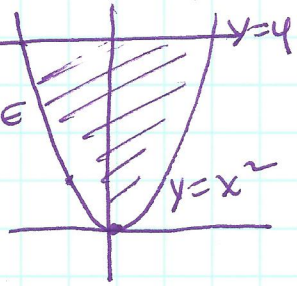
$\frac{1}{3}x^3+\frac{1}{2}x^2\Big|_0^2=x^2\left(\frac{x}{3}+\frac{1}{2}\right)\Big|_0^2=(4)\left(\frac{2}{3}+\frac{1}{2}\right)-0$

$=4\left(\frac{4}{6}+\frac{3}{6}\right)=\frac{28}{6}=\frac{14}{3}$ **D**

② $x=x^2+1$ $y=5$



SAME ⇒



$2\int_0^2 5-(x^2+1)\,dx=2\int_0^2(4-x^2)\,dx$

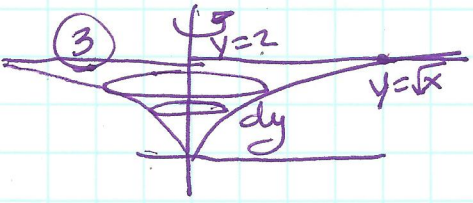
$2\left(4x-\frac{1}{3}x^3\Big|_0^2\right.$

$2(2)^3\left(1-\frac{1}{3}\right)=16\left(\frac{2}{3}\right)=\frac{32}{3}$ **D**

④ $y=d$ upper, $y=f(x)$ lower

$\int_a^b(d-f(x))\,dx$ **B** or $\int_c^d(f(y)-a)\,dy$ **B** ④

③



$r=x=y^2$

$A_1=\pi(y^2)^2$

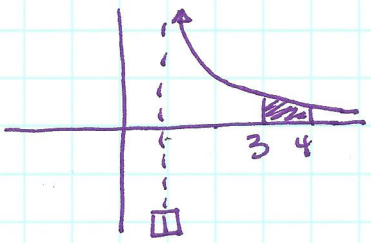
$V=\pi\int_0^2 y^4\,dy=\frac{\pi}{5}y^5\Big|_0^2$

$\frac{\pi}{5}(32-0)=\frac{32\pi}{5}$ **A** ③

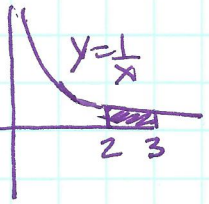
⑤ $y=\frac{1}{1-x}$

x-axis

$x\in[3,4]$



SAME AS ⇒



$\int_3^4\frac{1}{1-x}\,dx=\int_2^3\frac{1}{x}\,dx$

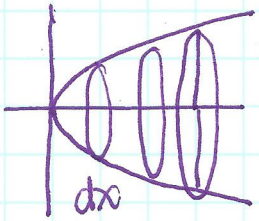
$\ln|1-x|\Big|_3^4=\ln|x|\Big|_2^3$

$\ln|3|-\ln|2|=\ln\left|\frac{3}{2}\right|$ **D**

⑥ $x=3$

$y=\sqrt{x}$

x-axis



$\pi(\sqrt{x})^2$

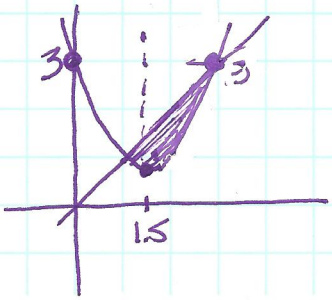
$V=\pi\int_0^3 x\,dx=\frac{\pi}{2}x^2\Big|_0^3=\frac{9\pi}{2}$ **C**

⑦ $y=x^2-3x+3$

$y=\left(x-\frac{3}{2}\right)^2+3-\frac{9}{4}$

$y=\left(x-\frac{3}{2}\right)^2+\frac{3}{4}$

$V\left(\frac{3}{2},\frac{3}{4}\right)$



$\int_1^3 x-(x^2-3x+3)\,dx$

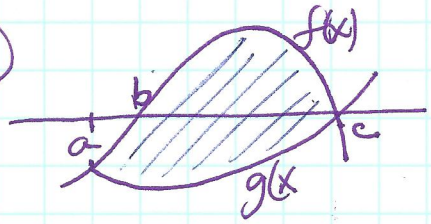
$\int_1^3 -x^2+4x-3\,dx=-\frac{1}{3}x^3+2x^2-3x\Big|_1^3$

$9\left(-1+2-1\right)-\left(-\frac{1}{3}+2-3\right)$

$F(3)-F(1)$

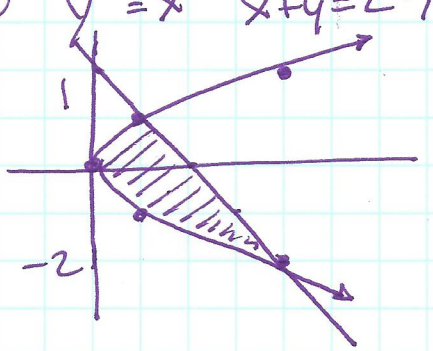
$0-\left(-\frac{4}{3}\right)=\frac{4}{3}$ **C**

Ch 8 Review

(8) 

$$\int_a^c \big(f(x) - g(x)\big)\,dx \quad \boxed{D}$$

(9) $y^2 = x \qquad x + y = 2 \to y = 2 - x$



$$\int_{-2}^{1} (\text{Rt} - \text{Left})\,dy$$

$$\int_{-2}^{1} (2-y) - (y^2)\,dy = \int_{-2}^{1} (2 - y - y^2)\,dy$$

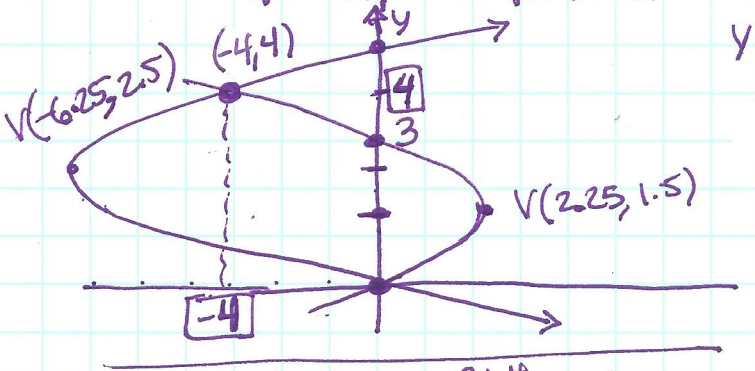
$$2y - \tfrac{1}{2}y^2 - \tfrac{1}{3}y^3 \Big|_{-2}^{1}$$

$$\left(2 - \tfrac{1}{2} - \tfrac{1}{3}\right) - \left(-4 - 2 + \tfrac{8}{3}\right)$$

$$2 - \tfrac{1}{2} - \tfrac{1}{3} + 6 - \tfrac{8}{3} = 8 - \tfrac{1}{2} - 3 = 4.5 = \tfrac{9}{2} \quad \boxed{D}$$

(10) $x = y^2 - 5y \qquad x = 3y - y^2$

$y(y-5)$ opens right $\qquad y(3-y)$ opens left



$y = 4 \to x = 4^2 - 5(4) \quad x = -4$ $\qquad$ & $\qquad y = 4 \to x = 3(4) - 4^2 \quad x = 12 - 16 \quad x = -4$

$$\int_0^4 (\text{Rt} - \text{Left})\,dy$$

$$\int_0^4 (3y - y^2) - (y^2 - 5y)\,dy$$

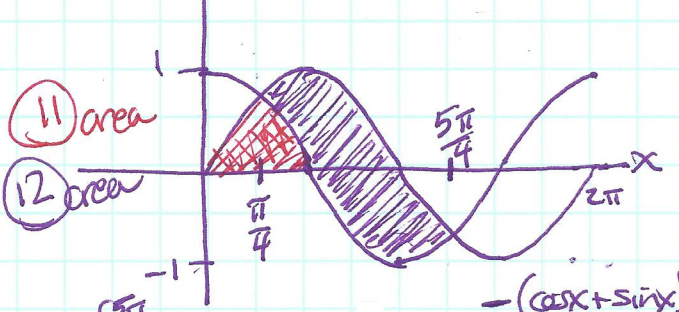
$$\int_0^4 (3y - y^2 - y^2 + 5y)\,dy$$

$$\int_0^4 (8y - 2y^2)\,dy$$

$$= 4y^2 - \tfrac{2}{3}y^3 \Big|_0^4$$

$$= 4^3\left(1 - \tfrac{2}{3}\right) = 4^3\left(\tfrac{1}{3}\right) = \tfrac{64}{3} \quad \boxed{C}$$

(12) $y = \sin x$ & $y = \cos x$



$$\int_{\pi/4}^{5\pi/4} (\sin x - \cos x)\,dx = -\cos x - \sin x \Big|_{\pi/4}^{5\pi/4} \qquad -(\cos x + \sin x)$$

$$= -\left[\left(\cos\tfrac{5\pi}{4} + \sin\tfrac{5\pi}{4}\right) - \left(\cos\tfrac{\pi}{4} + \sin\tfrac{\pi}{4}\right)\right]$$

$$= -\left[\left(-\tfrac{\sqrt{2}}{2} - \tfrac{\sqrt{2}}{2}\right) - \left(\tfrac{\sqrt{2}}{2} + \tfrac{\sqrt{2}}{2}\right)\right] = +2\sqrt{2} \quad \boxed{A}$$

(11) see graph for (12)

$$\int_0^{\pi/4} \sin x\,dx + \int_{\pi/4}^{\pi/2} \cos x\,dx$$

$$-\cos x \Big|_0^{\pi/4} + \sin x \Big|_{\pi/4}^{\pi/2}$$

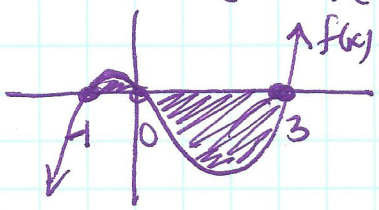
$$\left[-\cos\left(\tfrac{\pi}{4}\right) + \cos(0)\right] + \left[\sin\left(\tfrac{\pi}{2}\right) - \sin\left(\tfrac{\pi}{4}\right)\right]$$

$$\left(-\tfrac{\sqrt{2}}{2} + 1\right) + \left(1 - \tfrac{\sqrt{2}}{2}\right)$$

$$2 - \sqrt{2} \quad \boxed{A}$$

Ch8 Review

(13) $y = x^3 - 2x^2 - 3x$
$= x(x^2 - 2x - 3)$
$= x(x-3)(x+1)$



$$\int_{-1}^{0}(x^3-2x^2-3x)\,dx + \int_{0}^{3}\left(0-(x^3-2x^2-3x)\right)dx$$

$$\frac{1}{4}x^4 - \frac{2}{3}x^3 - \frac{3}{2}x^2\Big|_{-1}^{0} + \int_{0}^{3}(3x+2x^2-x^3)\,dx$$

$$\frac{1}{12}\left(3x^4 - 8x^3 - 18x^2\right)\Big|_{-1}^{0} + \frac{3}{2}x^2 + \frac{2}{3}x^3 - \frac{1}{4}x^4\Big|_{0}^{3}$$

$$\frac{1}{12}\left(0-(3+8-18)\right) + \frac{1}{12}\left(6\cdot 3x^2 + 8\cdot x^3 - 3x^4\right)\Big|_{0}^{3}$$

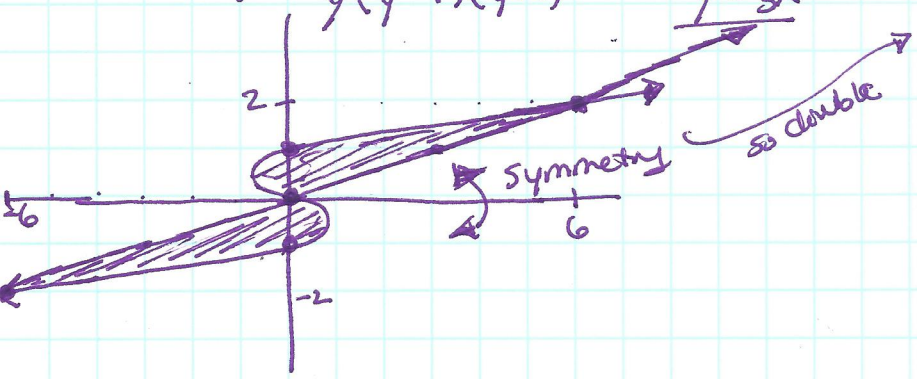
$$\frac{1}{12}\left(-(-7)\right) + \frac{3^3}{12}\left((6+8-9)-0\right)$$

$$\frac{7}{12} + \frac{27}{12}(5)$$

$$\frac{7}{12} + \frac{135}{12} = \frac{142}{12} = \frac{71}{6}$$ **D**

(14) $x = y^3 - y$
$x = y(y-1)(y+1)$

$x = 3y$
$y = \frac{1}{3}x$



$$2\int_{0}^{2}(3y)-(y^3-y)\,dy$$

$$2\int_{0}^{2}(3y-y^3+y)\,dy$$

$$2\int_{0}^{2}(4y-y^3)\,dy$$

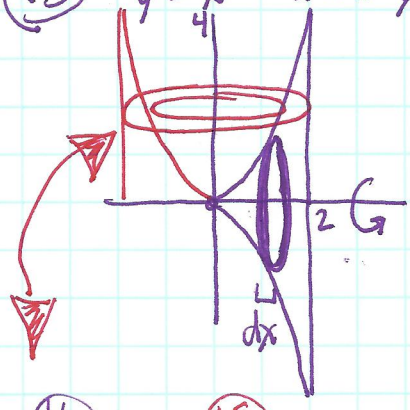
$$2\left(2y^2 - \frac{1}{4}y^4\Big|_{0}^{2}\right)$$

$$(2)\left(\frac{1}{4}\right)\left(8y^2 - y^4\Big|_{0}^{2}\right)$$

$$\left((2)\left(\frac{1}{4}\right)(2^4)\right)(2-1)$$

$$2^3(1) = 8$$ **C**

(15) $y = x^2$ $x = 2$ $y = 0$ → x-axis



$$\pi\int_{0}^{2}(x^2)^2\,dx = \pi\int_{0}^{2}x^4\,dx = \frac{\pi}{5}x^5\Big|_{0}^{2} = \frac{32\pi}{5}$$ **B**

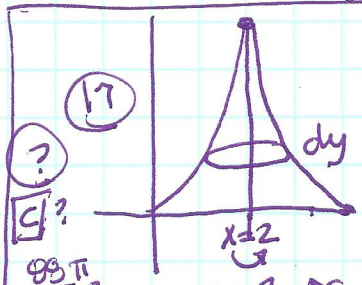
(16) see (15)

$r = x = \sqrt{y}$
$R = 2$

$$\pi\int_{0}^{4}2^2 - (\sqrt{y})^2\,dy = \pi\int_{0}^{4}4-y\,dy$$

$$\pi\left(4y - \frac{1}{2}y^2\Big|_{0}^{4}\right) = 16\pi\left(1-\frac{1}{2}\right) = 8\pi$$ **D**

(17)



$r = 2 - x$
$r = 2 - \sqrt{y}$

$$\pi\int_{0}^{2}(2-\sqrt{y})^2\,dy$$

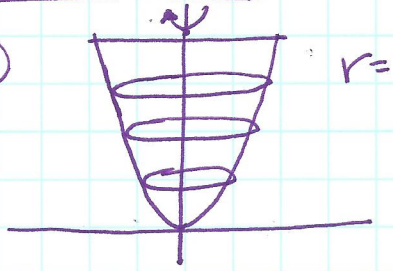
$$\pi\int_{0}^{2}(4-4\sqrt{y}+y)\,dy$$

$$\pi\left(4y - \frac{8}{3}y^{\frac{3}{2}} + \frac{1}{2}y^2\Big|_{0}^{2}\right.$$

$\pi(8$

? ? ? ?

Ch 8 Review

(18) 

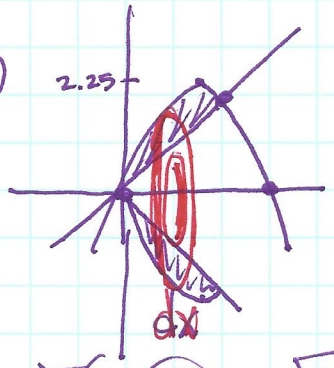
$r = x = \sqrt{y}$ $\quad \pi\int_0^4 (\sqrt{y})^2\,dy = \pi\int_0^4 y\,dy = \frac{\pi}{2}y^2\Big|_0^4$

$= 8\pi - 0$ **A**

(19) $y = 3x - x^2$

$y = x(3-x)$

& $y = x$



$R = y = 3x - x^2$

$r = y = x$

$\pi(R^2 - r^2)\,dx$

$\pi\int_0^2 (3x-x^2)^2 - (x)^2\,dx$

$\pi\int_0^2 (9x^2 - 6x^3 + x^4 - x^2)\,dx$

$\pi\int_0^2 (8x^2 - 6x^3 + x^4)\,dx$

$\pi\left(\frac{8}{3}x^3 - \frac{3}{2}x^4 + \frac{1}{5}x^5\Big|_0^2\right.$

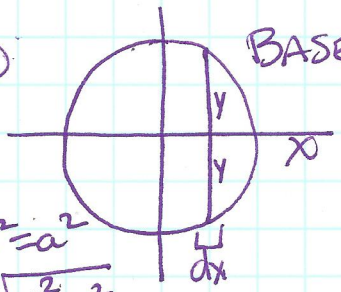
$2^4\pi\left(\frac{4}{3} - \frac{3}{2} + \frac{2}{5}\right)$

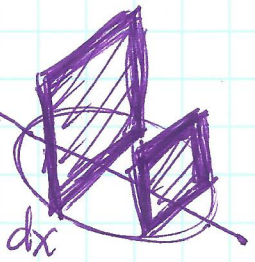
$16\pi\left(\frac{40-45+12}{30}\right) = \frac{16\pi\,7}{30\,15}$

$= \frac{56\pi}{15}$ ✓

Choose from ~~A~~ or C **C**

$\pi\int_0^2 (3x-x^2)^2 - (x)^2\,dx$

(20) 



$y^2 + x^2 = a^2$

$y = \sqrt{a^2 - x^2}$

$s = 2y$

$s = 2\sqrt{a^2 - x^2}$ → $A = (2\sqrt{a^2-x^2})^2 = 4(a^2-x^2)$

$V = \int_{-a}^{a} 4(a^2-x^2)\,dx$

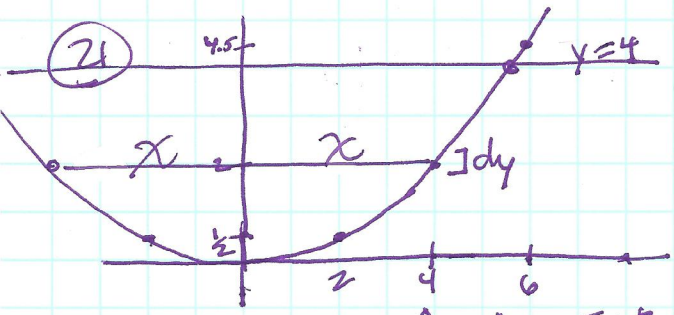
$= 2\int_0^a (4(a^2-x^2)\,dx$

$8\int_0^a (a^2-x^2)\,dx$

$8\left(a^2x - \frac{1}{3}x^3\Big|_0^a\right.$

$8a^3\left(1 - \frac{1}{3}\right) = 8a^3\left(\frac{2}{3}\right)$

D $= \frac{16a^3}{3}$

(21) 

$y = \frac{1}{8}x^2$

$8y = x^2$

$\sqrt{8y} = x$ → $s = 2x = 2\sqrt{8y}$

△ $A = \frac{\sqrt{3}}{4}(s)^2 = \frac{\sqrt{3}}{4}(2\sqrt{8}\,y)^2$

$A = \frac{\sqrt{3}}{4}(4 \cdot 8\,y^2)$

$A = 8\sqrt{3}\,y^2$

$V = 8\sqrt{3}\int_0^4 y^2\,dy = \frac{8\sqrt{3}}{3}y^3\Big|_0^4$

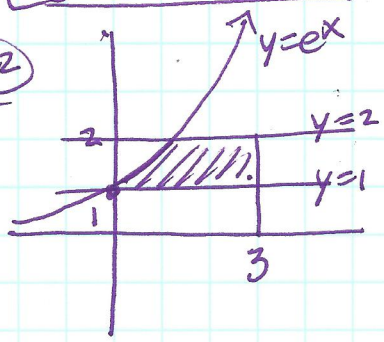
$= \frac{8\sqrt{3}}{3}(64-0)$

$= \frac{512\sqrt{3}}{3}$

I did not get **B** $64\sqrt{3}$

CH 8 REVIEW — CALCULATOR

(22)



$\int_1^2 (\text{Rt} - \text{Left})\,dy = \int_1^2 |3 - \ln(y)|\,dy$

$= 2.613705$

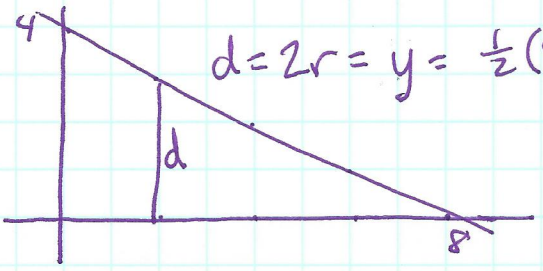
E $\quad 4 - \ln(4) \approx 2.613705$

Why are answer choices not decimals? hmmm... Fix this!

(23) $x + 2y = 8$

$(8,0)$

$(0,4)$



$d = 2r = y = \frac{1}{2}(8-x) \quad \therefore \quad r = \frac{1}{4}(8-x)$

$A_{\text{semicircle}} = \frac{\pi}{2}(r)^2$

$= \frac{\pi}{2}\left(\frac{1}{4}(8-x)\right)^2$

$\frac{\pi}{32}(64 - 16x + x^2)$

$V = \frac{\pi}{32}\int_0^8 (64 - 16x + x^2)\,dx$

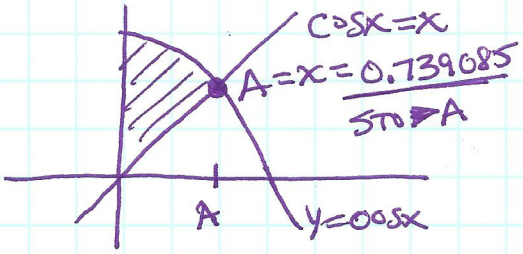
$= \frac{\pi}{32}\left(64x - 8x^2 + \frac{1}{3}x^3\right)\Big|_0^8 = \frac{\pi}{32}\left(64(8) - 8(64) + \frac{8}{3}(64) - 0\right)$

$= 2\pi\left(8 - 8 + \frac{8}{3}\right) = 2\pi\left(\frac{8}{3}\right) = \frac{16\pi}{3} \approx 16.755$ C

(24) $y = \cos x$

$y = x$

y-axis

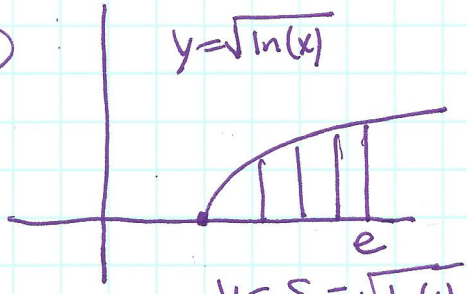


$\int_0^A (\cos x - x)\,dx \approx 0.40048$

$\sin x - \frac{1}{2}x^2\Big|_0^A$ C

$\left(\sin(A) - \frac{1}{2}A^2\right) - (0)$

(25)



$y = s = \sqrt{\ln(x)}$

$A = s^2 = (\ln x)$

$V = \int_1^e (\ln x)\,dx = 1$

$V = 1$ C