TWO MC REVIEW FIXES & FRQ ANSWERS →

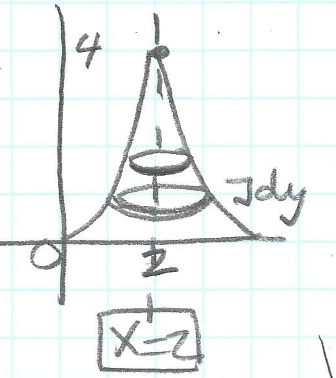
#17

$y = x^2$

$\sqrt{y} = x$

$x = 2$

$y = 0$



$R = 2 - x = 2 - \sqrt{y}$

$A = \pi(2-\sqrt{y})^2 = \pi(4 - 4\sqrt{y} + y)$

$V = \pi \int_0^4 (4 - 4\sqrt{y} + y)\, dy$

$V = \pi \left(4y - \frac{8}{3}y^{3/2} + \frac{1}{2}y^2\right)\Big|_0^4$

$V = \pi\left(\left(4^2 - 4^2 \cdot \frac{4}{3} + 4^2\left(\frac{1}{2}\right)\right) - 0\right)$

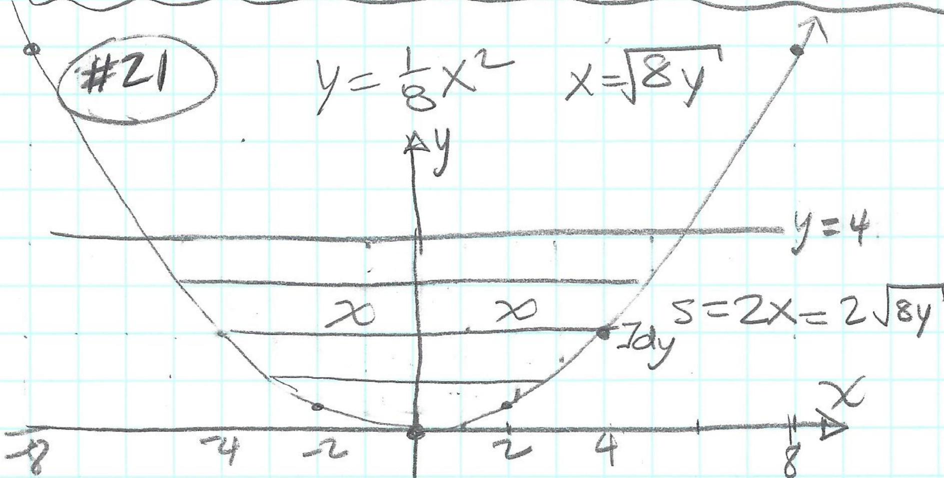
$V = 16\pi\left(1 - \frac{4}{3} + \frac{1}{2}\right)$

$\frac{16\pi}{6}(6 - 8 + 3) = \frac{16\pi}{6} = \frac{8\pi}{3}$

CHOICE $\boxed{C}$

TYPO $\cancel{\frac{88\pi}{3}} \Rightarrow \boxed{\frac{8\pi}{3}}$

#21 $y = \frac{1}{8}x^2$ $x = \sqrt{8y}$



$s = 2x = 2\sqrt{8y}$

EQUILATERAL TRIANGLES $A = \frac{\sqrt{3}}{4}s^2$

$A = \frac{\sqrt{3}}{4}\left(2\sqrt{8y}\right)^2$

$A = \sqrt{3}(8y)$

my earlier mistake: $A = \sqrt{3}(8y^2)$ WRONG.

$V = 8\sqrt{3}\int_0^4 y\, dy$

$V = 8\sqrt{3}\left(\frac{y^2}{2}\right)\Big|_0^4$

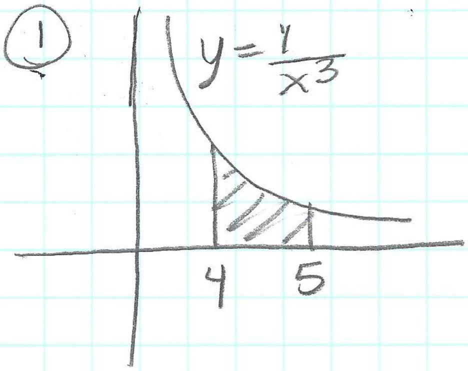
$V = 4\sqrt{3}\left(y^2\Big|_0^4 = 4\sqrt{3}(16 - 0)\right.$

$= 64\sqrt{3}$

YAY! THAT'S CORRECT

FRQ — CH 8 REVIEW

(1)



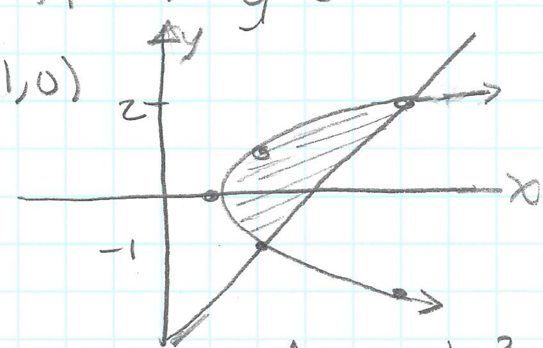
$$A = \int_4^5 \frac{1}{x^3}\,dx = \int_4^5 x^{-3}\,dx$$

$$A = \frac{x^{-2}}{-2}\Big|_4^5 = \frac{-1}{2}\left(\frac{1}{x^2}\right)\Big|_4^5$$

$$A = -\frac{1}{2}\left(\frac{1}{25} - \frac{1}{16}\right) = \frac{1}{2}\left(\frac{1}{16} - \frac{1}{25}\right) = \frac{9}{800}$$

(2) $x = y^2 + 1 \quad x = y + 3$

$V(1,0)$



$$A = \int_{-1}^{2} (\text{Rt} - \text{Left})\,dy$$

$$A = \int_{-1}^{2} (y+3) - (y^2+1)\,dy$$

$$A = \int_{-1}^{2} (y + 3 - y^2 - 1)\,dy$$

$$A = \int_{-1}^{2} (-y^2 + y + 2)\,dy$$

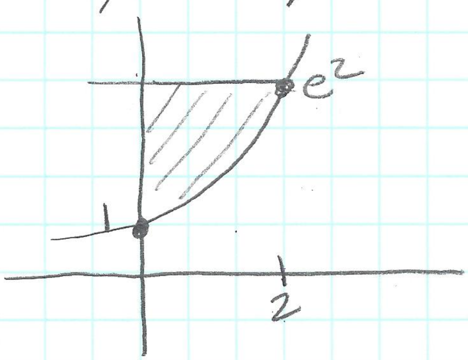
$$A = -\frac{1}{3}y^3 + \frac{1}{2}y^2 + 2y \Big|_{-1}^{2} = F(2) - F(-1)$$

$$\left[2^2\left(-\frac{1}{3}(2) + \frac{1}{2} + 1\right)\right] - \left[\frac{1}{3} + \frac{1}{2} - 2\right]$$

$$\frac{2^2}{6}\left(-4 + 3 + 6\right) - \frac{1}{6}\left(2 + 3 - 12\right)$$

$$= \frac{20}{6} - \left(\frac{-7}{6}\right) = \frac{27}{6} = \frac{9}{2}$$

(3) $y = e^x \quad y = e^2 \quad x = 0$



$$\int_0^2 (\text{upper} - \text{lower})\,dx \quad \boxed{\text{or}} \quad \int_1^{e^2} (\text{Rt} - \text{Left})\,dy$$

$$\int_0^2 (e^2 - e^x)\,dx \qquad \int_1^{e^2} \underbrace{(\ln y - 0)}\,dy$$

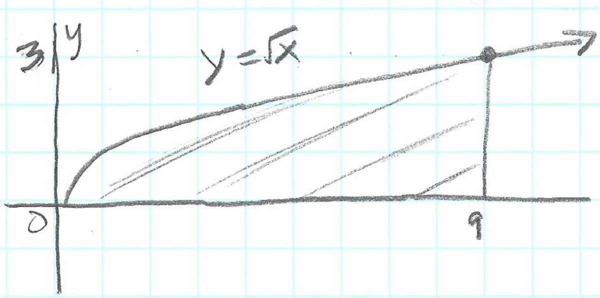
but can't integrate by hand (yet)

$$= e^2 x - e^x \Big|_0^2$$

$$= 2e^2 - e^2 - (0 - e^0)$$

$$= e^2 - (-1) = \boxed{e^2 + 1}$$

(4)



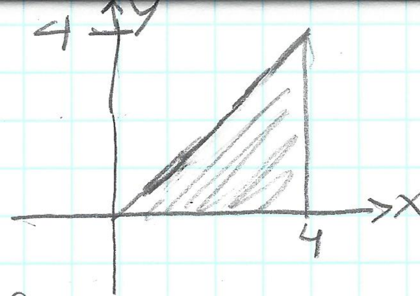
$$A = \int_0^9 \sqrt{x}\,dx = \frac{2}{3}x^{3/2}\Big|_0^9 = \frac{2}{3}(27 - 0) = 18$$

$$\frac{1}{2}A = 9 = \int_0^k \sqrt{x}\,dx = \int_k^9 \sqrt{x}\,dx$$

$$9 = \frac{2}{3}x^{3/2}\Big|_0^k \Rightarrow 9 = \frac{2}{3}k^{3/2} \Rightarrow \frac{27}{2} = k^{3/2}$$

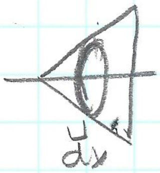
$$k = \left(\frac{27}{2}\right)^{2/3} = \frac{9}{\sqrt[3]{4}} = \frac{9\sqrt[3]{2}}{2}$$

CH8 FRQ REVIEW



(5) $y=x \quad y=0 \quad x=4$

a) about x-axis



$A = \pi y^2 dx$

$A = \pi x^2 dx$

$V = \pi \int_0^4 x^2 dx = \frac{\pi}{3} x^3 \Big|_0^4 = \frac{64\pi}{3}$

CHECK: $V_{cone} =$

$h=4$

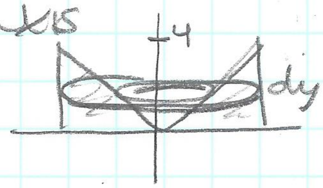
$r=4$

$V = \frac{\pi}{3}(4)^2(4) = \frac{64\pi}{3}$ ✓

b) about y-axis

$R=4$

$r = x = y$



$V = \pi \int_0^4 (16 - y^2) dy$

$V = \pi \left(16y - \frac{1}{3}y^3 \Big|_0^4\right.$

$V = \pi \left(16(4) - \frac{1}{3}(4)^3\right)$

$4^3\pi \left(1 - \frac{1}{3}\right) = 64\pi\left(\frac{2}{3}\right) = \frac{128\pi}{3}$

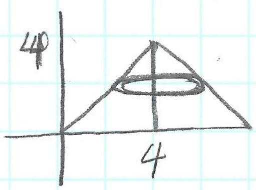
CHECK: $V_{cyl} - V_{cone}$

$\pi r^2 h - \frac{1}{3}\pi r^2 h$

$\pi(4)^2(4) - \frac{\pi}{3}(4)^2(4)$

$64\pi\left(1 - \frac{1}{3}\right) = 64\pi\left(\frac{2}{3}\right) = \frac{128\pi}{3}$ ✓

c) about line $x=4$



$r = 4 - x$

$r = 4 - y$

$V = \pi \int_0^4 (4-y)^2 dy$

$-\pi \int_0^4 (4-y)^2 (-1\,dy)$

$-\frac{\pi}{3} (4-y)^3 \Big|_0^4$

$-\frac{\pi}{3}\left(0^3 - 4^3\right) = \frac{64\pi}{3}$

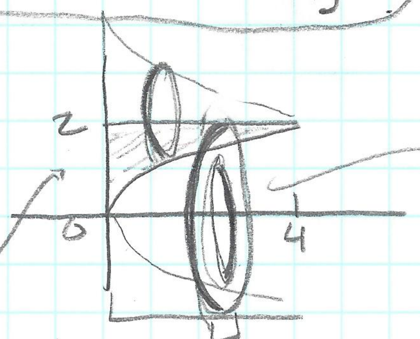
CHECK: Volume of Cone $V = \frac{\pi}{3}(4)^2(4)$

$= \frac{64\pi}{3}$

(6) $y = \sqrt{x}$

$y = 2$

$x = 0$



a) x-axis

$R = 2$

$r = y = \sqrt{x}$

$V = \pi \int_0^4 4 - (x)\, dx$

$= \pi\left(4x - \frac{1}{2}x^2 \Big|_0^4\right.$

$= \pi(16 - 8) = 8\pi$

b) about $y=2$

$R = 2 - y = 2 - \sqrt{x}$

$V = \pi \int_0^4 (2-\sqrt{x})^2 dx = \pi \int_0^4 (4 - 4\sqrt{x} + x)\, dx = \pi\left(4x - \frac{8}{3}x^{3/2} + \frac{1}{2}x^2 \Big|_0^4\right.$

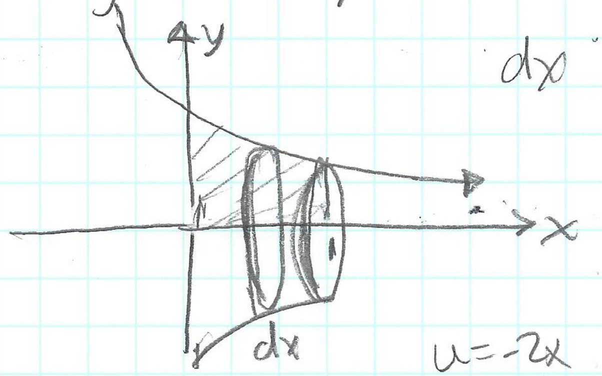
16·4

$V = \pi\left(4^2 - \frac{8}{3}(8) + \frac{4^2}{2}\right) = 16\pi\left(1 - \frac{4}{3} + \frac{1}{2}\right)$

$16\pi\left(\frac{6-8+3}{6}\right) = \frac{16\pi}{6} = \frac{8\pi}{3}$

CH8 FRQ REVIEW

(7) $y=e^{-x}$ $y=0$ $x=0$ $x=1$ about x-axis



dx $R=e^{-x}$

$V=\pi\int_0^1 (e^{-x})^2\,dx$

$\pi\int_0^1 e^{-2x}\,dx$

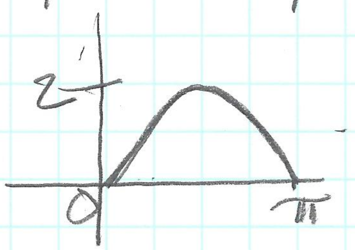
$u=-2x$
$-\frac{1}{2}du=dx$

$-\frac{\pi}{2}\int_0^{-2} e^u\,du$

$-\frac{\pi}{2}\left(e^u\Big|_0^{-2}\right) = -\frac{\pi}{2}(e^{-2}-1)$

$\frac{\pi}{2}(1-e^{-2}) = \boxed{\frac{\pi}{2}\left(1-\frac{1}{e^2}\right)}$

(8) $y=2\sin x$ $y=0$ $x\in[0,\pi]$ cross-sections $\perp$ x-axis



(A) SQUARES $A=s^2=(2\sin x)^2=4\sin^2 x$
$s=2\sin x$ FLASHCARD REWRITE!

$4\int_0^\pi \sin^2 x\,dx$

$4\int_0^\pi \frac{1}{2}(1-\cos(2x))dx$

$2\int_0^\pi (1-\cos 2x)dx$

$2\left(x-\frac{1}{2}\sin 2x\right)\Big|_0^\pi$

$2((\pi-0)-(0-0)) = 2\pi$

U-SUB
$\int\cos 2x$
$\frac{1}{2}\int\cos 2x(2dx$
$\frac{1}{2}\sin 2x$

(B) SEMICIRCLES:
$r=\frac{1}{2}(2\sin x)=\sin x$
$A=\frac{\pi}{2}r^2=\frac{\pi}{2}\sin^2 x$

$V=\frac{\pi}{2}\int_0^\pi \sin^2 x\,dx=$

$\frac{\pi}{4}\int_0^\pi 1-\cos 2x\,dx$

$\frac{\pi}{4}\left(x-\frac{1}{2}\sin 2x\right)\Big|_0^\pi$

$\frac{\pi}{4}(\pi-0) = \frac{\pi^2}{4} = \left(\frac{\pi}{2}\right)^2$

(C) EQUILATERAL TRIANGLES:
$s=2\sin x$ $A=\frac{\sqrt{3}}{4}(2\sin x)^2=\sqrt{3}\sin^2 x$

$V=\sqrt{3}\int_0^\pi \sin^2 x\,dx$

$V=\frac{\sqrt{3}}{2}\int_0^\pi (1-\cos(2x))dx$

$V=\frac{\sqrt{3}}{2}\left(x-\frac{1}{2}\sin 2x\right)\Big|_0^\pi$

$V=\frac{\sqrt{3}}{2}(\pi-0)=\frac{\sqrt{3}\pi}{2}$

YAY!! Got 'em 1st TRY!