AB Calculus Chapter 7.1 Review – Integration by U-Substitution

1. The POWER Rule: $\int x^n dx =$ _______________ and thinking BACKWARDS:

A $\int x dx =$	B $\int 5x dx =$	C $\int x^5 dx =$	D $\int \sqrt{x} dx =$	E $\int \frac{5}{\sqrt[3]{x^2}} dx =$
F $\int \frac{1}{x^4} dx =$	G $\int 3x^{-2} dx =$	H $\int 3x^{0.7} dx =$	I $\int \sqrt[3]{x} dx =$	J $\int x^e dx =$

and thinking BACKWARDS:

K $\int \sin(x) dx =$	L $\int \sec^2(x) dx =$	M $\int dx =$	N $\int \csc^2(x) dx =$	O $\int \cos(x) dx =$
P $\int \sec(x)\tan(x) dx =$	Q $\int a^x dx =$	R $\int e^x dx =$	S $\int 5^x dx =$	T $\int \csc(x)\cot(x) dx =$

2. Substitution Rule:

A $\int \sin(3x) dx =$	B $\int (2x-1)e^{(x^2-x)} dx =$	C $\int \sec^2(7x) dx =$	D $\int \frac{\cos(\sqrt{x})}{\sqrt{x}} dx =$
E $\int \sin^3(x)\cos(x) dx =$	F $\int \tan(3x) dx =$ Hint: rewrite as ratio	G $\int \frac{\ln(x)}{x} dx =$	H $\int \frac{3x-9}{x^2-6x+7} dx =$
J $\int \frac{1}{1+(2x)^2} dx$	K $\int \frac{1}{\sqrt{1-9x^2}} dx$	L $\int x\sin^3(x^2)\cos(x^2) dx$	M $\int \frac{(x-3)}{\sqrt{x^2-6x+7}} dx =$

3. Definite Integrals with Substitution: $\int_a^b f(x) dx = \ldots \int_{u_a}^{u_b} f(u) du = F(u_b) - F(u_a) \ldots = F(b) - F(a)$

A $\int_0^5 x\sqrt{x^2+4} dx$	B $\int_0^\pi \sin\theta(\cos(\theta)+5)^7 dx$	C $\int_0^1 \frac{x}{5x^2+1} dx$
D $\int_0^2 \left(2x^3 - 6x + \frac{3}{x^2+1}\right) dx$	E $\int_1^9 \frac{2t^2 + t^2\sqrt{t} - 1}{t^2} dt$	

4. Use ALGEBRA:

A $\int x^2(x^2-3)^2 dx$	B $\int \frac{(x+1)^2}{x} dx$	C $\int \frac{3x-5}{\sqrt{x}} dx$

MC - NON-CALCULATOR

7. What is the average value of $y = x^2\sqrt{x^3+1}$ on the interval [0, 2]?

A. $\frac{26}{9}$ B. $\frac{52}{9}$ C. $\frac{26}{3}$ D. $\frac{52}{3}$ E. 24

10. $\int \frac{x\,dx}{\sqrt{3x^2+5}} =$

A. $\frac{1}{9}(3x^2+5)^{3/2} + C$ B. $\frac{1}{4}(3x^2+5)^{3/2} + C$ C. $\frac{1}{12}(3x^2+5)^{1/2} + C$

D. $\frac{1}{3}(3x^2+5)^{1/2} + C$ E. $\frac{3}{2}(3x^2+5)^{1/2} + C$

12. $\int_0^{\frac{\pi}{2}} \frac{\cos\theta}{\sqrt{1+\sin\theta}}\, d\theta =$

A. $-2\left(\sqrt{2}-1\right)$ B. $-2\sqrt{2}$ C. $2\sqrt{2}$ D. $2\left(\sqrt{2}-1\right)$ E. $2\left(\sqrt{2}+1\right)$

14. $\int_2^3 \frac{x}{x^2+1}\, dx =$

A. $\frac{1}{2}\ln\frac{3}{2}$ B. $\frac{1}{2}\ln 2$ C. ln 2 D. 2 ln 2 E. $\frac{1}{2}\ln 5$

19. $\int_0^{\sqrt{3}} \frac{dx}{\sqrt{4-x^2}} =$ Hint: can you make this look like $\int \frac{dx}{\sqrt{1-x^2}}$

A. $\frac{\pi}{3}$ B. $\frac{\pi}{4}$ C. $\frac{\pi}{6}$ D. $\frac{1}{2}\ln 2$ E. $-\ln 2$

22. $\int_0^{\frac{\pi}{4}} \frac{e^{\tan x}}{\cos^2 x}\, dx$ is

A. 0 B. 1 C. $e - 1$ D. e E. $e + 1$

MC Answers: 7. A 10. D 12. D 14. B 19. A 22. C

AP FRQ 1991 #1

Let f be the function that is defined for all real numbers x and that has the following properties.

i) $f''(x) = 24x - 18$

ii) $f'(1) = -6$

iii) $f(2) = 0$

a) Find each x such that the line tangent to the graph of f at $(x, f(x))$ is horizontal.

b) Write an expression for $f(x)$.

c) Find the average value of f on the interval $1 \le x \le 3$.

AP FRQ 2004 #1

Traffic flow is defined as the rate at which cars pass through an intersection, measured in cars per minute. The traffic flow at a particular intersection is modeled by the function F defined by

$$F(t) = 82 + 4\sin\left(\frac{t}{2}\right) \quad \text{for} \quad 0 \le t \le 30,$$

where *F(t)* is measured in cars per minute and t is measured in minutes.

(a) To the nearest whole number, how many cars pass through the intersection over the 30-minute period?

(b) Is the traffic flow increasing or decreasing at $t = 7$? Give a reason for your answer.

(c) What is the average value of the traffic flow over the time interval $10 \le t \le 15$? Indicate units of measure.

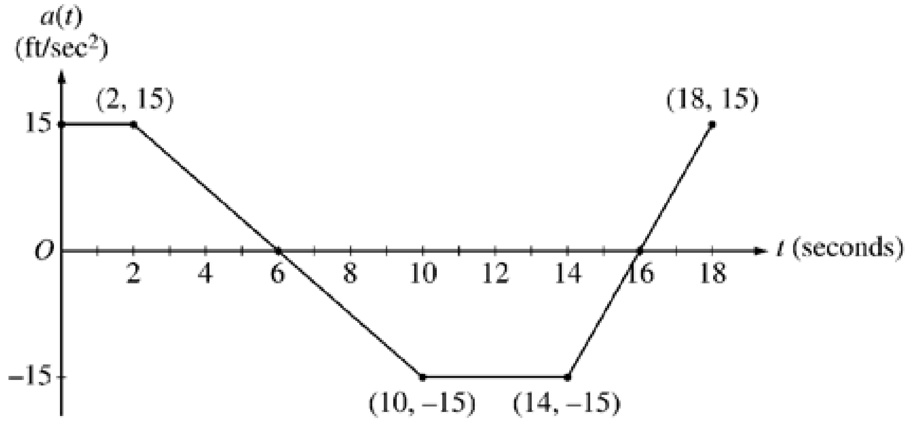
(d) What is the average rate of change of the traffic flow over the time interval $10 \le t \le 15$? Indicate units of measure.

AP FRQ 2002 #2 **Calculator Active**

The rate at which people enter an amusement park on a given day is modeled by the function E defined by $E(t) = \frac{15600}{(t^2 - 24t + 160)}$. The rate at which people leave the same amusement park on the same day is modeled by L defined by the function $L(t) = \frac{9890}{(t^2 - 38t + 370)}$. Both $E(t)$ and $L(t)$ are measured in people per hour and time t is measured in hours after midnight. These functions are valid for $9 \le t \le 23$, the hours during which the park is open. At time $t = 9$, there are no people in the park.

a. How many people have entered the park by 5:00 P.M. $(t = 17)$? Round your answer to the nearest whole number.
b. The price of admission to the park is $15 until 5:00 P.M. $(t = 17)$. After 5:00 P.M. the price of admission to the park is $11. How many dollars are collected from admissions to the park on the given day? Round your answers to the nearest whole number.
c. Let $H(t) = \int_9^t E(x) - L(x)\,dx$ for $9 \le x \le 23$. The value of $H(17)$ to the nearest whole number is 3725. Find the value of $H'(17)$, and explain the meaning of $H(17)$ and $H'(17)$ in the context of the amusement park.
d. At what time, t, for $9 \le t \le 23$, does the model predict that the number of people in the park is a maximum?

AP FRQ 2001 #3



3. A car is traveling on a straight road with velocity 55 ft/sec at time $t = 0$. For $0 \le t \le 18$ seconds, the car's acceleration $a(t)$, in ft/sec^2, is the piecewise linear function defined by the graph above.

   (a) Is the velocity of the car increasing at $t = 2$ seconds? Why or why not?

   (b) At what time in the interval $0 \le t \le 18$, other than $t = 0$, is the velocity of the car 55 ft/sec? Why?

   (c) On the time interval $0 \le t \le 18$, what is the car's absolute maximum velocity, in ft/sec, and at what time does it occur? Justify your answer.

   (d) At what times in the interval $0 \le t \le 18$, if any, is the car's velocity equal to zero? Justify your answer.