CH 6 REVIEW

FRQ = SHOW WORK | MC = SHOW WORK

For your learning benefit SHOW WORK for all questions.
... or NO CREDIT.

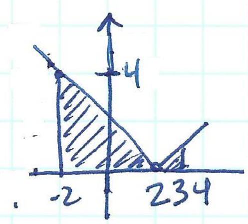
① a) $\int_0^2 f(x)\,dx = \frac{1}{2} + \frac{1}{2}(1)(1+4) = \frac{1}{2} + \frac{5}{2} = 3$

b) $\int_5^2 f(x)\,dx = -\int_2^5 f(x)\,dx = \frac{1}{2}(4)(2+3) = 10$

c) avg value $f(x)$ on $[1,4]$ $= \frac{1}{4-1}\int_1^4 f(x)\,dx = \frac{1}{3}\left(\frac{5}{2}+8\right) = \frac{1}{3}\left(\frac{21}{2}\right) = \frac{7}{2}$

② $\int_1^2 (e^x + 2x - 3)\,dx = e^x + x^2 - 3x \Big|_1^2 = (e^2+4-6) - (e^1+1-3)$
$= (e^2-2) - (e-2) = e^2 - e^1$

③ $\int_{-2}^3 |x-2|\,dx = \int_{-2}^2 (-x+2)\,dx + \int_2^3 (x-2)\,dx$



$= 8 + \frac{1}{2}$
$= \frac{17}{2}$

$= \left(-\frac{1}{2}x^2 + 2x\right)\Big|_{-2}^2 + \left(\frac{1}{2}x^2 - 2x\right)\Big|_2^3$

$[(\cancel{-2}+4) - (\cancel{-2}-4)] + [(\frac{9}{2}-6) - (2-4)]$
$8 \quad + \quad (-\frac{3}{2}+2) = 8 + \frac{1}{2} = \frac{17}{2}$

④ A) $\frac{d}{dx}\left(\int_2^x \sqrt{t^3-3t^2+4}\,dt\right) = \sqrt{x^3-3x^2+4}$

B) $\frac{d}{dx}\left(\int_2^{x^3} \frac{1-\sin t^2}{2t}\,dt\right) = \frac{1-\sin(x^6)}{2x^3} \cdot 3x^2 = \frac{3(1-\sin(x^6))}{2x}$

C) $\frac{d}{dx}\left(\int_1^{\sin^2(3x)} \frac{2t+1}{e^t}\,dt\right) = \frac{(2\sin^2(3x)+1)}{e^{\sin^2(3x)}} \cdot \overbrace{2\sin(3x)\cos(3x)}^{\sin(6x)} \cdot 3$

$= \frac{[2\sin^2(3x)+1][3\sin(6x)]}{e^{\sin^2(3x)}}$

MC - NONCALC - SHOW WORK

① $\int_1^2 \frac{1}{x^2}\,dx = \int_1^2 x^{-2}\,dx = -1x^{-1}\Big|_1^2 = \frac{-1}{x}\Big|_1^2 = -\frac{1}{2} + 1 = \boxed{\frac{1}{2}}$ C

② $\int_0^x \sin t\,dt = -\cos t\Big|_0^x = -\cos(x) - (-\cos 0)$
$= -\cos x + 1 = \boxed{1-\cos(x)}$ E

CH6 REVIEW (continued) SHOW WORK

(3) $\int_1^e \left(\frac{x^2-1}{x}\right) dx = \int_1^e \left(x - \frac{1}{x}\right) dx = \frac{1}{2}x^2 - \ln|x| \Big|_1^e$

$= \left(\frac{1}{2}e^2 - 1\right) - \left(\frac{1}{2} - 0\right) = \boxed{\frac{1}{2}e^2 - \frac{3}{2}}$ (E)

(4) f is linear
∴ $f' =$ constant
∴ $f'' = 0$

$\int_a^b f''(x)\,dx = c\Big|_a^b = c - c = \boxed{0}$ (A)

(5) $F(x) = \int_0^x \sqrt{t^3+1}\,dt$

$F'(x) = \frac{d}{dx}\left(\int_0^x \sqrt{t^3+1}\,dt\right) = \sqrt{x^3+1}$

$F'(2) = \sqrt{2^3+1} = \boxed{3}$ (D)

(6) $\int_{-3}^k x^2\,dx = 0$

$\frac{1}{3}x^3 \Big|_{-3}^k = \frac{1}{3}(k^3+27) = 0$ → or $k^3+27=0$, $k^3=-27$, $k=-3$

$\frac{1}{3}(k+3)(k^2-3k+9) = 0$

$\boxed{k=-3}$ (A)

(8) $v(t) = e^t$ $\boxed{\int_0^2 e^t\,dt} = e^t\Big|_0^2 = e^2 - e^0 = \boxed{e^2-1}$ (A)

Total change in distance the particle has moved on $t \in [0,2]$

(9) $\int \sec^2 dx = \boxed{\tan x + c}$ (A)

(11) $\int_0^k (2kx - x^2)\,dx = 18$

$\int_0^k (2kx - x^2)\,dx = kx^2 - \frac{1}{3}x^3 \Big|_0^k$

$= \left(k^3 - \frac{1}{3}k^3\right) - (0)$

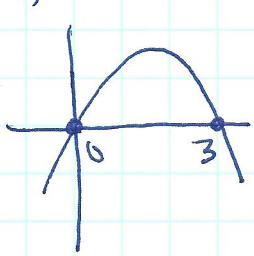
$18 = \frac{2}{3}k^3$

$27 = k^3$ ∴ $\boxed{k=3}$ (C)

(13) $\int_0^1 (3x-2)^2\,dx$

$\int_0^1 (9x^2 - 12x + 4)\,dx = 3x^3 - 6x^2 + 4x \Big|_0^1 = (3-6+4) - 0 = \boxed{1}$ (D)

(15) $y = 3x - x^2$
$= x(3-x)$



$\frac{1}{3-0}\int_0^3 (3x-x^2)\,dx = \frac{1}{3}\left(\frac{3}{2}x^2 - \frac{1}{3}x^3\right)\Big|_0^3$

$\frac{1}{3}\left(\frac{x^2}{6}(9-2x)\right)\Big|_0^3$

$\frac{1}{3}\left(\frac{3}{2}(3) - 0\right) = \boxed{\frac{3}{2}}$ (C)

(16) $a(t) = 6t - 2$ & $v(3) = 25$

$v(t) = \int (6t-2)\,dt$

$v(t) = 3t^2 - 2t + c \longrightarrow v(t) = 3t^2 - 2t + 4$ & $s(1) = 10$

$25 = 27 - 6 + c$

$4 = c$

$s(t) = \int (3t^2 - 2t + 4)\,dt$

$s(t) = t^3 - t^2 + 4t + c$

$10 = 1 - 1 + 4 + c$

$c = 6$

$\boxed{s(t) = t^3 - t^2 + 4t + 6}$ (C)

* (17) $\int \frac{3x^2}{\sqrt{x^3+1}}\,dx$

oops... Save for Ch 7 u-substitution.

we have not learned this yet.

let $u = x^3 + 1$, $du = 3x^2\,dx \rightarrow \int \frac{du}{\sqrt{u}} = \int u^{-1/2}\,du = 2\sqrt{u} + c$

$= \boxed{2\sqrt{x^3+1} + C}$ (A)

(18) $\int (x^2+1)^2\,dx = \int (x^4 + 2x^2 + 1)\,dx = \boxed{\frac{1}{5}x^5 + \frac{2}{3}x^3 + x + c}$ (E)

(20) $\int_1^2 (4x^3 - 6x)\,dx = x^4 - 3x^2 \Big|_1^2 = x^2(x^2-3)\Big|_1^2$

$= (4)(1) - (1)(-2) = 4 + 2 = \boxed{6}$ (C)

* (21) $\frac{1}{2}\int e^{t/2}\,dt = \boxed{e^{t/2} + c}$ C

technically this could be a Ch 7 question too ... but were you able to choose the correct answer?

(23) Avg value of $f(x) = \cos x$ on $[-3, 5]$

$\frac{1}{5+3}\int_{-3}^{5} \cos x\,dx = \frac{1}{8}\sin(x)\Big|_{-3}^{5} = \frac{1}{8}\left(\sin(5) - \sin(-3)\right)$

$= \boxed{\frac{1}{8}\left(\sin(5) + \sin(3)\right)}$ (E)

$f(x) = \sin x$ is odd
$\therefore f(-x) = -f(x)$
$\sin(-x) = -\sin(x)$

CALCULATOR

(24) $\int_1^{500} (13^x - 11^x)\,dx + \int_2^{500} (11^x - 13^x)\,dx$

Calculator overflow error

CAN YOU SIMPLIFY & USE PROPERTIES? YES.

$\int_1^{500} (13^x - 11^x)\,dx - \int_2^{500} (13^x - 11^x)\,dx$

$= \int_1^2 (13^x - 11^x)\,dx = \boxed{14.946}$ (B)

CH 6 REVIEW (continued)

(25) $f(x) = \sin x \cos x$

We will learn this in CH 7 but test each option for now.

$\boxed{\text{I}}$) $F(x) = \dfrac{\sin^2 x}{2}$

$f(x) = \dfrac{\cancel{2} \sin x \cos x}{\cancel{2}}$

Yes

II) $F(x) = \dfrac{\cos^2 x}{2}$

$f(x) = \dfrac{-\cancel{2} \cos x \sin x}{\cancel{2}}$

$f(x) = -\cos x \sin x$

No

$\boxed{\text{III}}$) $F(x) = \dfrac{-\cos(2x)}{4}$

$f(x) = \dfrac{2\sin(2x)}{4}$

$f(x) = \dfrac{\sin(2x)}{2}$

$f(x) = \dfrac{\cancel{2}\sin x \cos x}{\cancel{2}}$

Yes

$\boxed{D}$ I & III

(26) $F(x) = \int \dfrac{(\ln x)^3}{x}\, dx$

$F(9) = F(1) + \int_1^9 \dfrac{(\ln x)^3}{x}\, dx$

$= 0 + 5.82690$

$= 5.826$ or $\boxed{5.827}$ $\boxed{C}$

THANK YOU for SHOWING YOUR WORK