Ch 6 Building Area Functions part I.

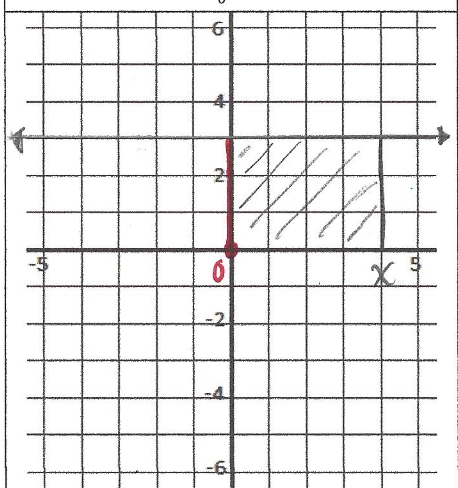
DAY 1 KEY I

- Graph $f(t)$ on the coordinate grid & identify $f(a)$ at the lower limit $x=a$.
- Shade the region indicated by the integral and use geometry to find a formula for the area bounded by the function, the x-axis and the limits of integration.
- Simplify the function to a standard form polynomial: $y=a_0x^n+a_1x^{n-1}+\dots+a_{n-2}x^2+a_{n-1}x^1+a_n$
- Use a colored pencil to shade the area corresponding to x=0.
- Complete the table of values for the area function, $A(x)$.

1a)

$f(t)=3 \ on \ [0,x]$

$A(x)=\int_0^x 3dt$



$A(x)=(x-0)(3)$

$=3x\boxed{+0}$

Table

x	-3	-2	-1	0	1	2	3
A(x)	-9	-6	-3	0	3	6	9

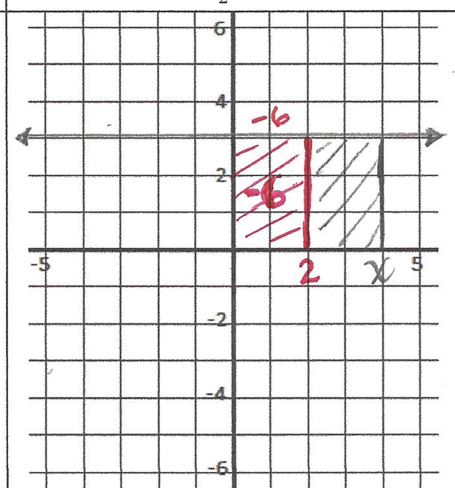
$A(0)=\int_0^0 f(t)dt$

Anti-Derivative

1b)

$f(t)=3 \ on \ [2,x]$

$A(x)=\int_2^x 3dt$



$A(x)=(x-2)(3)$

$=3x\boxed{-6}$

Table

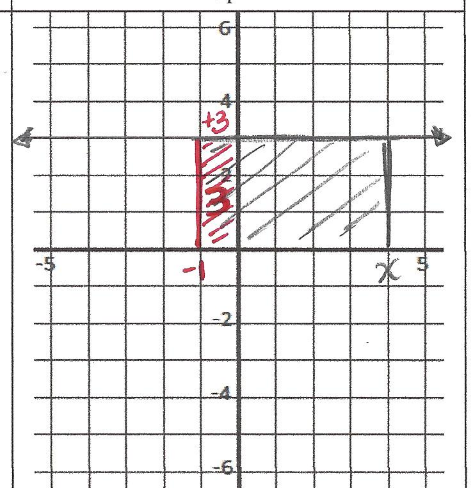
x	-1	0	1	2	3	4	5
A(x)	-9	-6	-3	0	3	6	9

$A(0)=\int_2^0 f(t)dt$

1c)

$f(t)=3 \ on \ [-1,x]$

$A(x)=\int_{-1}^x 3dt$



$A(x)=(x+1)(3)$

$=3x\boxed{+3}$

Table

x	-4	-3	-2	-1	0	1	2
A(x)	-9	-6	-3	0	3	6	9

$A(0)=\int_{-1}^0 f(t)dt.$

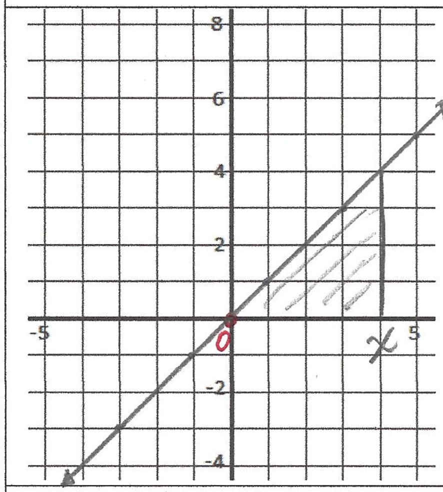
Looking for Patterns & Making Conjectures:

- How is the y-intercept of the $A(x)$ equation and the table related to an area on the graph?
- What is the relationship between the Area function, $A(x)$ and the original function, $f(t)$?
- How are the area functions in (a), (b) & (c) of this set related to each other?

Complete the next three sets of the graphs and tables. After completing each set return to answer these "Making Conjectures:" questions.

2a) $f(t)=t \ \ on\ [0,x]$

$A(x)=\int_0^x t\,dt$



$A(x)=\frac{1}{2}(x-0)(x)$

$=\frac{1}{2}x^2\boxed{+0}$

Table

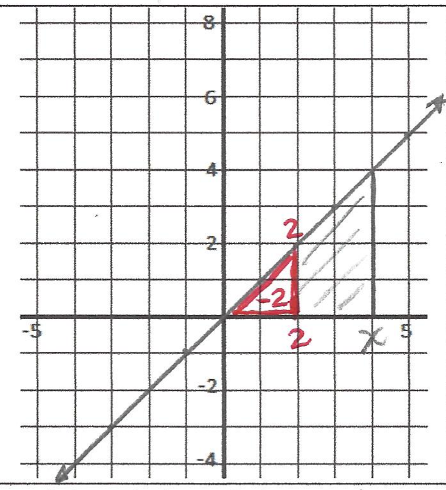
x	-3	-2	-1	0	1	2	3
A(x)	4.5	2	$\frac{1}{2}$	0	$\frac{1}{2}$	2	4.5

$A(0)=\int_0^0 f(t)\,dt$

Anti-Derivative

2b) $f(t)=t \ \ on\ [2,x]$

$A(x)=\int_2^x t\,dt$



$A(x)=\frac{1}{2}(x-2)(2+x)$

$=\frac{1}{2}(x^2-4)$

$=\frac{1}{2}x^2\boxed{-2}$

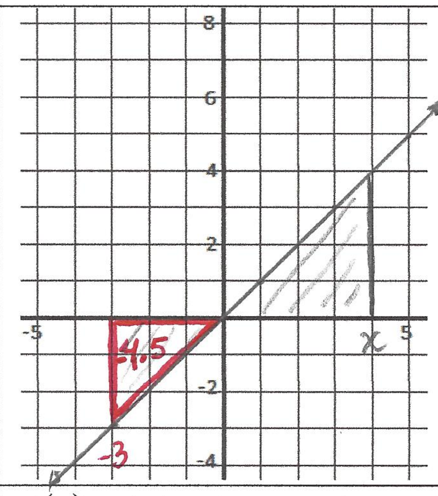
Table

x	-3	-2	-1	0	1	2	3
A(x)	2.5	0	-1.5	-2	-1.5	0	2.5

$A(0)=\int_2^0 f(t)\,dt$

2c) $f(t)=t \ \ on\ [-3,x]$

$A(x)=\int_{-3}^x t\,dt$



$A(x)=\frac{1}{2}(0+3)(-3)+\frac{1}{2}(x-0)(x)$

$=-\frac{9}{2}+\frac{1}{2}x^2$

$=\frac{1}{2}(x^2-9)$

$=\frac{1}{2}(x-3)(x+3)$

$=\frac{1}{2}x^2\boxed{-4.5}$

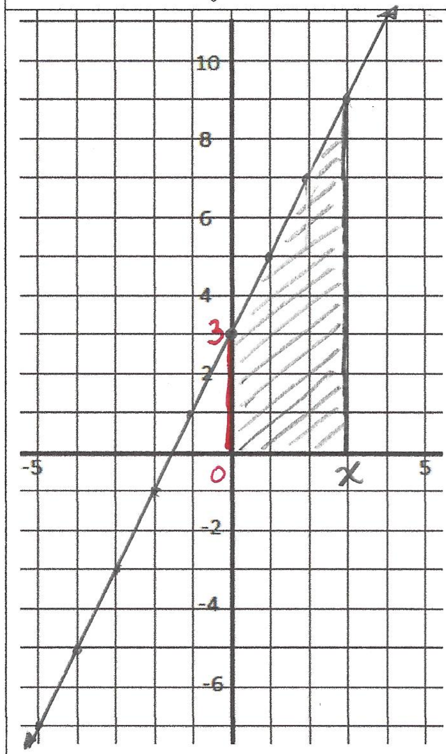
Table

x	-3	-2	-1	0	1	2	3
A(x)	0	-2.5	-4	-4.5	-4	-2.5	0

$A(0)=\int_{-3}^0 f(t)\,dt$

3a) $f(t)=2t+3$ on $[0,x]$

$A(x)=\int_0^x 2t+3\,dt$



$A(x)=\frac{1}{2}(x-0)(3+2x+3)$

$=\frac{1}{2}(x)(2x+6)$

$=(x)(x+3)$

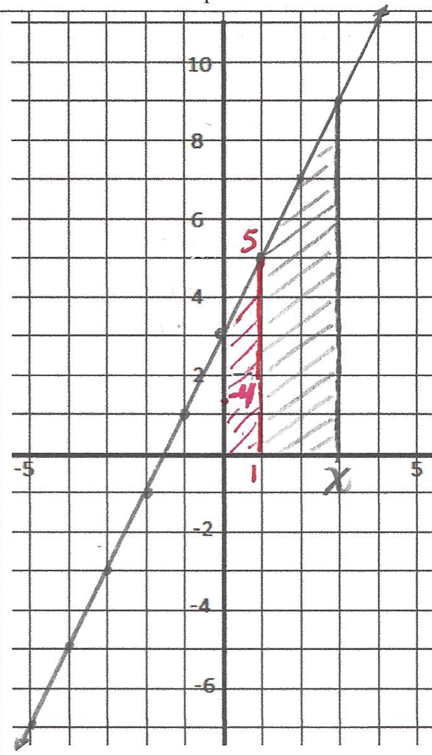
$=x^2+3x\boxed{+0}$

x	-3	-2	-1	0	1	2	3
A(x)	0	-2	-2	0	4	10	18

$A(0)=\int_0^0 f(t)\,dt$

3b) $f(t)=2t+3$ on $[1,x]$

$A(x)=\int_1^x 2t+3\,dt$



$A(x)=\frac{1}{2}(x-1)(5+2x+3)$

$=\frac{1}{2}(x-1)(2x+8)$

$=(x-1)(x+4)$

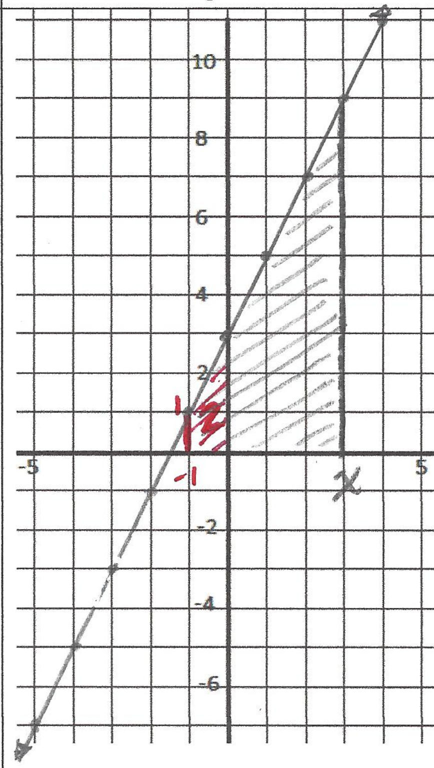
$=x^2+3x\boxed{-4}$

x	-3	-2	-1	0	1	2	3
A(x)	-4	-6	-6	-4	0	6	14

$A(0)=\int_1^0 f(t)\,dt$

3c) $f(t)=2t+3$ on $[-1,x]$

$A(x)=\int_{-1}^x 2t+3\,dt$



$A(x)=\frac{1}{2}(x+1)(1+2x+3)$

$=\frac{1}{2}(x+1)(2x+4)$

$=(x+1)(x+2)$

$=x^2+3x\boxed{+2}$

x	-3	-2	-1	0	1	2	3
A(x)	2	0	0	2	6	12	20

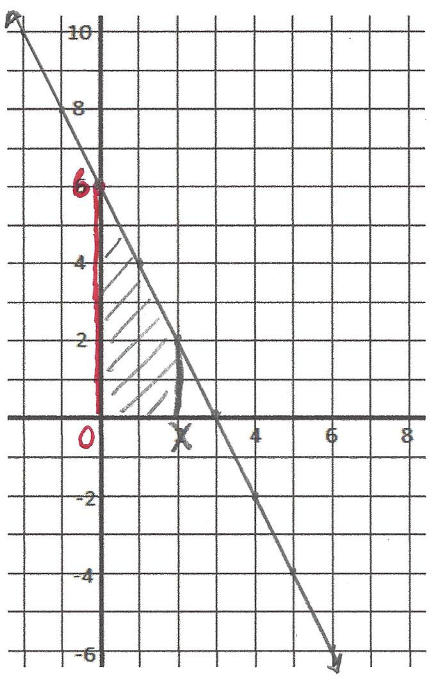
$A(0)=\int_{-1}^0 f(t)\,dt$

Anti-Derivative

DAY 1 KEY I

4a) $f(t)=6-2t \ \ on\ [0,x]$

$A(x)=\int_0^x 6-2t\,dt$



$A(x)=\frac{1}{2}(x-0)(6+6-2x)$

$=\frac{1}{2}(x)(-2x+12)$

$=-(x)(x-6)$

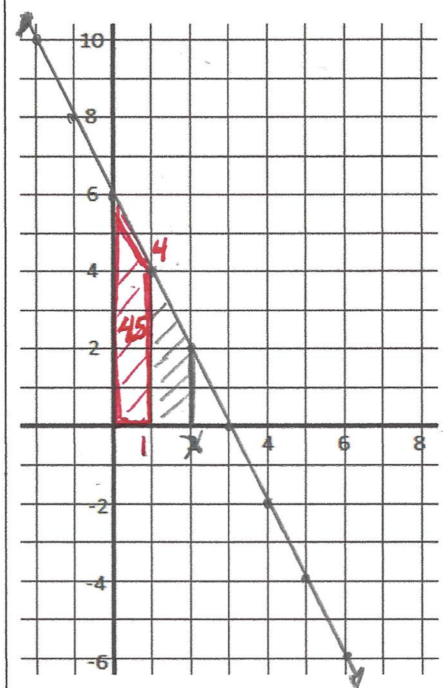
$=-x^2+6x\boxed{+0}$

x	0	1	2	3	4	5	6
A(x)	0	5	8	9	8	5	0

Anti-Derivative

4b) $f(t)=6-2t \ \ on\ [1,x]$

$A(x)=\int_1^x 6-2t\,dt$



$A(x)=\frac{1}{2}(x-1)(4+6-2x)$

$=\frac{1}{2}(x-1)(-2x+10)$

$=-(x-1)(x-5)$

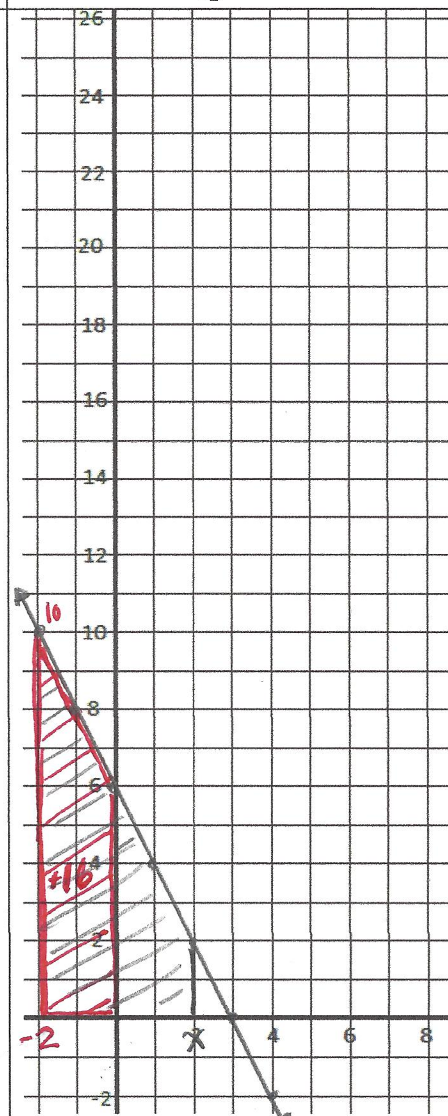
$=-(x^2-6x+5)$

$=-x^2+6x\boxed{-5}$

x	0	1	2	3	4	5	6
A(x)	-5	0	3	4	3	0	-5

4c) $f(t)=6-2t \ \ on\ [-2,x]$

$A(x)=\int_{-2}^x 6-2t\,dt$



$A(x)=\frac{1}{2}(x+2)(10+6-2x)$

$=\frac{1}{2}(x+2)(-2x+16)$

$=-(x+2)(x-8)$

$=-(x^2-6x-16)$

$=-x^2+6x\boxed{+16}$

x	0	1	2	3	4	5	6
A(x)	16	21	24	25	24	21	16

Ch 6 Building Area Functions *part II*

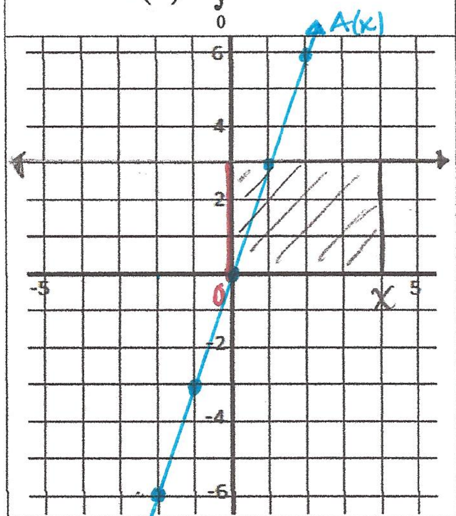
DAY 2 KEY II

- Graph $f(t)$ on the coordinate grid & identify $f(a)$ at the lower limit $x=a$.
- Shade the region indicated by the integral and use geometry to find a formula for the area bounded by the function, the x-axis and the limits of integration.
- Simplify the function to a standard form polynomial: $y=a_0x^n+a_1x^{n-1}+\ldots+a_{n-2}x^2+a_{n-1}x^1+a_n$
- Use a colored pencil to shade the area corresponding to x=0.
- Complete the table of values for the area function, $A(x)$.

1a)

$f(t)=3 \; on \; [0,x]$

$A(x)=\int_0^x 3dt$



$A(x)=(x-0)(3)$

$=3x\boxed{+0}$

Table

x	-3	-2	-1	0	1	2	3
A(x)	-9	-6	-3	0	3	6	9

Graph A(x) $\quad A(0)=\int_0^0 f(t)\,dt$

Anti-Derivative

$\int_0^x 3dt = 3t\Big|_0^x$

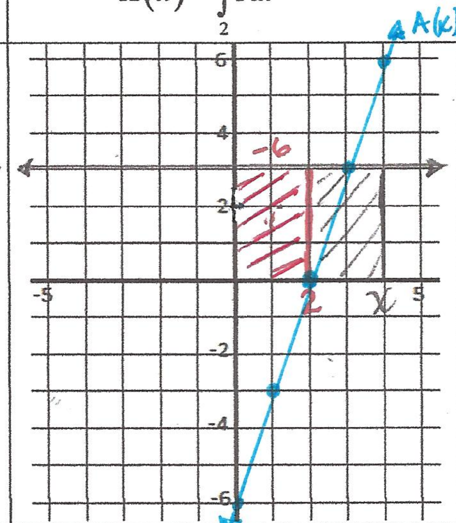
$3(x)-3(0)=3(x-0)$

$A(x)=3x$

1b)

$f(t)=3 \; on \; [2,x]$

$A(x)=\int_2^x 3dt$



$A(x)=(x-2)(3)$

$=3x\boxed{-6}$

Table

x	-1	0	1	2	3	4	5
A(x)	-9	-6	-3	0	3	6	9

$A(0)=\int_2^0 f(t)\,dt$

$\int_2^x 3dt = 3t\Big|_2^x$

$=3(x)-3(2)$

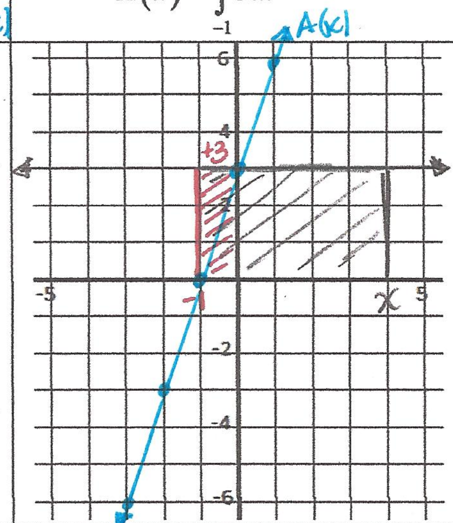
$=3(x-2)$

$A(x)=3x-6$

1c)

$f(t)=3 \; on \; [-1,x]$

$A(x)=\int_{-1}^x 3dt$



$A(x)=(x+1)(3)$

$=3x\boxed{+3}$

Table

x	-4	-3	-2	-1	0	1	2
A(x)	-9	-6	-3	0	3	6	9

$A(0)=\int_{-1}^0 f(t)\,dt$

$\int_{-1}^x 3\,dt = 3t\Big|_{-1}^x$

$=3(x)-3(-1)$

$=3(x--1)=3(x+1)$

$A(x)=3x+3$

notice: area between x = lower limit & x=0 is always the constant value of the Area function.

notice: A(x) degree is always one more than f(t) degree.

DAY 2 KEY II

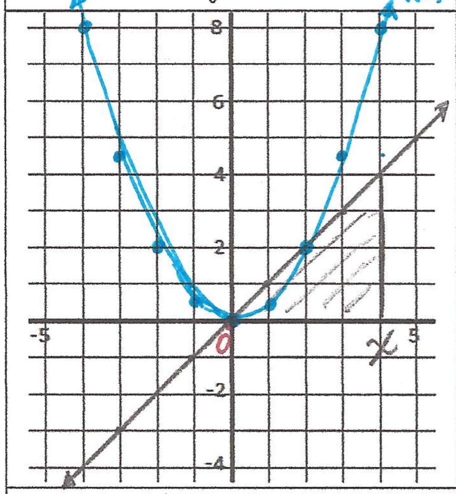
Looking for Patterns & Making Conjectures:

- How is the y-intercept of the $A(x)$ equation and the table related to an area on the graph?
- What is the relationship between the Area function, $A(x)$ and the original function, $f(t)$?
- How are the area functions in (a), (b) & (c) of this set related to each other?

Complete the next three sets of the graphs and tables. After completing each set return to answer these "Making Conjectures:" questions.

2a) $f(t)=t \ on \ [0,x]$

$A(x)=\int_0^x t\,dt$



$A(x)=\frac{1}{2}(x-0)(x)$

$=\frac{1}{2}x^2 \boxed{+0}$

Table

x	-3	-2	-1	0	1	2	3
A(x)	4.5	2	$\frac{1}{2}$	0	$\frac{1}{2}$	2	4.5

A(x) graph $A(0)=\int_0^0 f(t)\,dt$

Anti-Derivative

$A(x)=\int_0^x t\,dt$

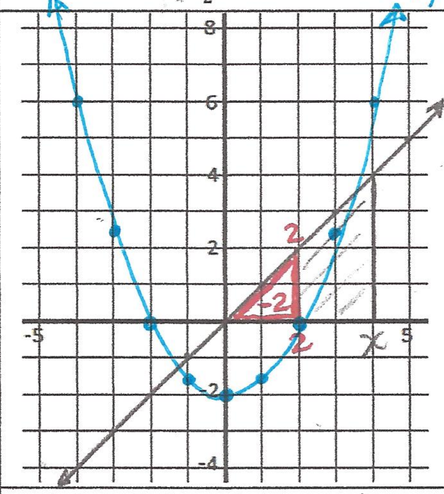
$=\frac{1}{2}t^2 \Big|_0^x$

$=\frac{1}{2}x^2-\frac{1}{2}(0)^2$

$A(x)=\frac{1}{2}x^2$

2b) $f(t)=t \ on \ [2,x]$

$A(x)=\int_2^x t\,dt$



$A(x)=\frac{1}{2}(x-2)(2+x)$

$=\frac{1}{2}(x^2-4)$

$=\frac{1}{2}x^2\boxed{-2}$

Table

x	-3	-2	-1	0	1	2	3
A(x)	2.5	0	-1.5	-2	-1.5	0	2.5

$A(0)=\int_2^0 f(t)\,dt$

$A(x)=\int_2^x t\,dt$

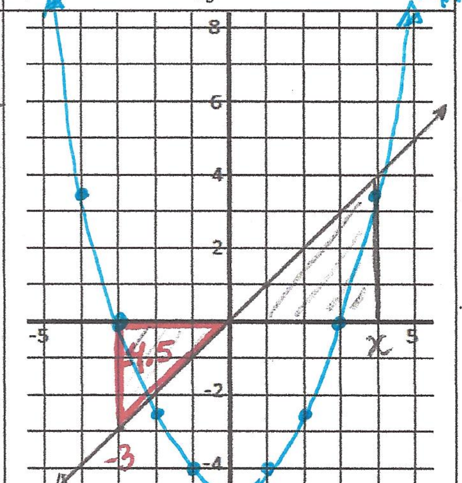
$=\frac{1}{2}t^2\Big|_2^x$

$=\frac{1}{2}(x^2)-\frac{1}{2}(2)^2$

$A(x)=\frac{1}{2}x^2-2$

2c) $f(t)=t \ on \ [-3,x]$

$A(x)=\int_{-3}^x t\,dt$



$A(x)=\frac{1}{2}(0+3)(-3)+\frac{1}{2}(x-0)(x)$

$=-\frac{9}{2}+\frac{1}{2}x^2$

$=\frac{1}{2}(x^2-9)$

$=\frac{1}{2}(x-3)(x+3)$

$=\frac{1}{2}x^2\boxed{-4.5}$

Table

x	-3	-2	-1	0	1	2	3
A(x)	0	-2.5	-4	-4.5	-4	-2.5	0

$A(0)=\int_{-3}^0 f(t)\,dt$

$A(x)=\int_{-3}^x t\,dt$

$=\frac{1}{2}t^2\Big|_{-3}^x$

$=\frac{1}{2}x^2-\frac{1}{2}(-3)^2$

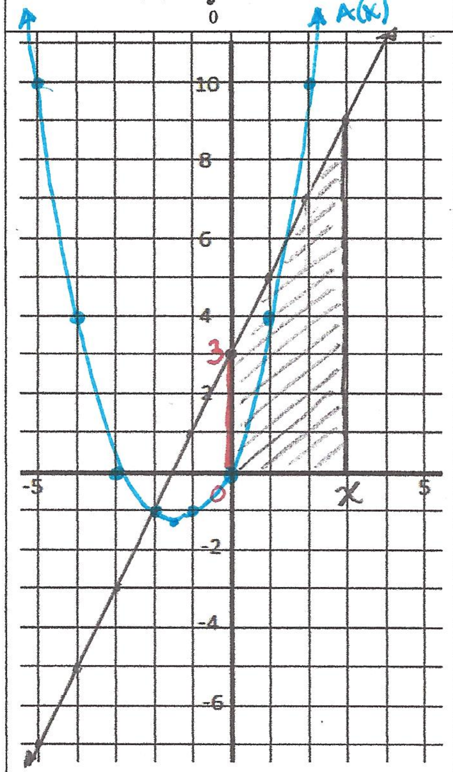
$A(x)=\frac{1}{2}x^2-\frac{9}{2}$

Also notice: the minimum value on A(x) occurs when f(t) changes signs ⊖ to ⊕.

3a)

$f(t)=2t+3 \ on \ [0,x]$

$A(x)=\int_0^x 2t+3dt$



$$A(x)=\tfrac{1}{2}(x-0)(3+2x+3)$$
$$=\tfrac{1}{2}(x)(2x+6)$$
$$=(x)(x+3)$$
$$=x^2+3x\boxed{+0}$$

x	-3	-2	-1	0	1	2	3
A(x)	0	-2	-2	0	4	10	18

$(-1.5, -2.25)$ $A(0)=\int_0^0 f(t)dt$

Anti-Derivative

$A(x)=\int_0^x 2t+3\,dt$

$=t^2+3t\Big|_0^x$

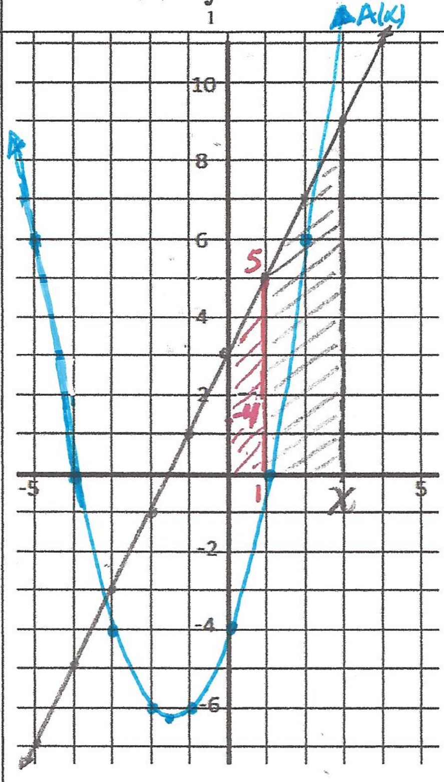
$=(x^2+3x)-(0^2+3(0))$

$=x^2+3x+0$

3b)

$f(t)=2t+3 \ on \ [1,x]$

$A(x)=\int_1^x 2t+3dt$



$$A(x)=\tfrac{1}{2}(x-1)(5+2x+3)$$
$$=\tfrac{1}{2}(x-1)(2x+8)$$
$$=(x-1)(x+4)$$
$$=x^2+3x\boxed{-4}$$

x	-3	-2	-1	0	1	2	3
A(x)	-4	-6	-6	-4	0	6	14

$(-1.5, -6.25)$ $A(0)=\int_1^0 f(t)dt$

$A(x)=\int_1^x 2t+3\,dt$

$=t^2+3t\Big|_1^x$

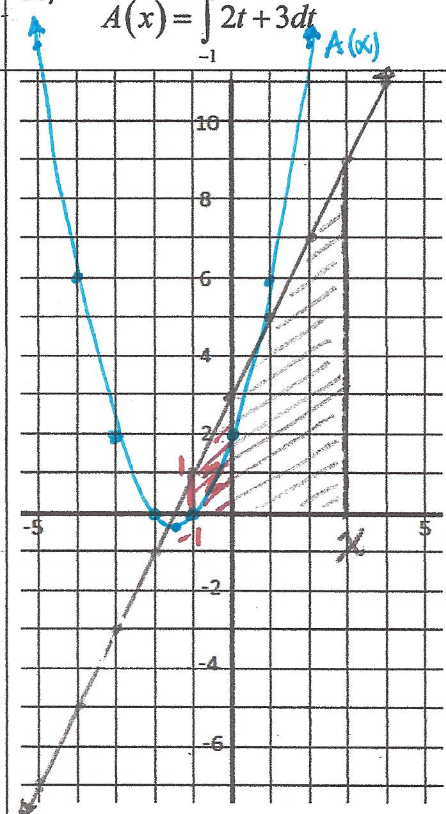
$=(x^2+3x)-(1+3(1))$

$=x^2+3x-4$

3c)

$f(t)=2t+3 \ on \ [-1,x]$

$A(x)=\int_{-1}^x 2t+3dt$



$$A(x)=\tfrac{1}{2}(x+1)(1+2x+3)$$
$$=\tfrac{1}{2}(x+1)(2x+4)$$
$$=(x+1)(x+2)$$
$$=x^2+3x\boxed{+2}$$

x	-3	-2	-1	0	1	2	3
A(x)	2	0	0	2	6	12	20

$(-1.5, -.25)$ $A(0)=\int_{-1}^0 f(t)dt$

$A(x)=\int_{-1}^x 2t+3\,dt$

$=t^2+3t\Big|_{-1}^x$

$=(x^2+3x)-((-1)^2+3(-1))$

$=x^2+3x+2$

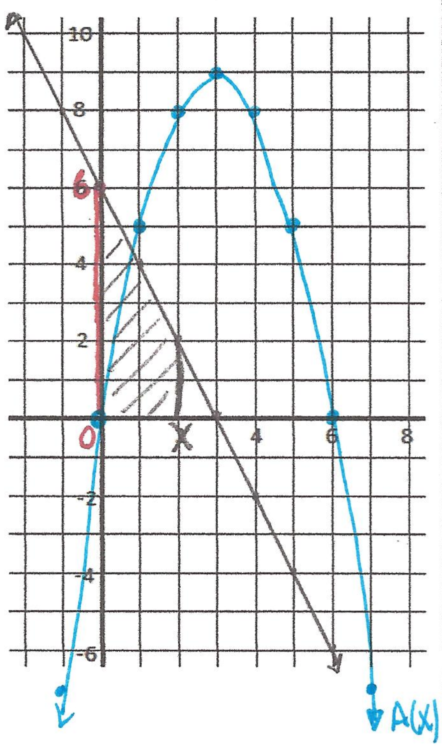
Notice: the minimum value on A(x) occurs when f(t) changes signs $\ominus$ to $\oplus$.

DAY 2 KEY II

4a)

$f(t) = 6 - 2t \text{ on } [0, x]$

$A(x) = \int_0^x 6 - 2t\,dt$



$A(x) = \frac{1}{2}(x-0)(6+6-2x)$

$= \frac{1}{2}(x)(-2x+12)$

$= -(x)(x-6)$

$= -x^2 + 6x \boxed{+0}$

x	0	1	2	3	4	5	6
A(x)	0	5	8	9	8	5	0

Anti-Derivative

$A(x) = \int_0^x 6 - 2t\,dt$

$= 6t - t^2 \Big|_0^x$

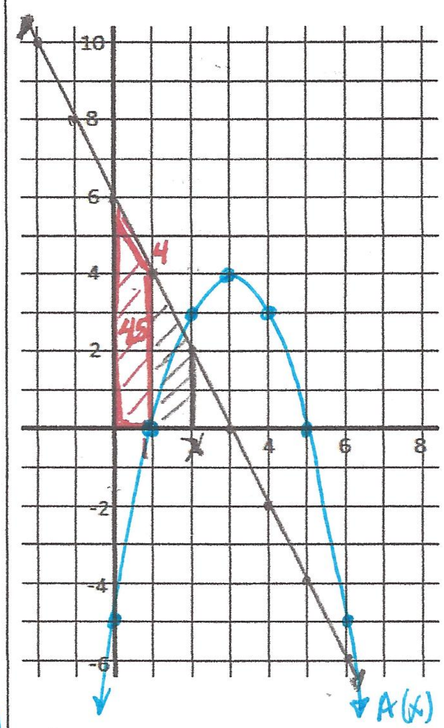
$= (6x - x^2) - (0)$

$A(x) = -x^2 + 6x + 0$

4b)

$f(t) = 6 - 2t \text{ on } [1, x]$

$A(x) = \int_1^x 6 - 2t\,dt$



$A(x) = \frac{1}{2}(x-1)(4+6-2x)$

$= \frac{1}{2}(x-1)(-2x+10)$

$= -(x-1)(x-5)$

$= -(x^2 - 6x + 5)$

$= -x^2 + 6x \boxed{-5}$

x	0	1	2	3	4	5	6
A(x)	-5	0	3	4	3	0	-5

$A(x) = \int_1^x 6 - 2t\,dt$

$= 6t - t^2 \Big|_1^x$

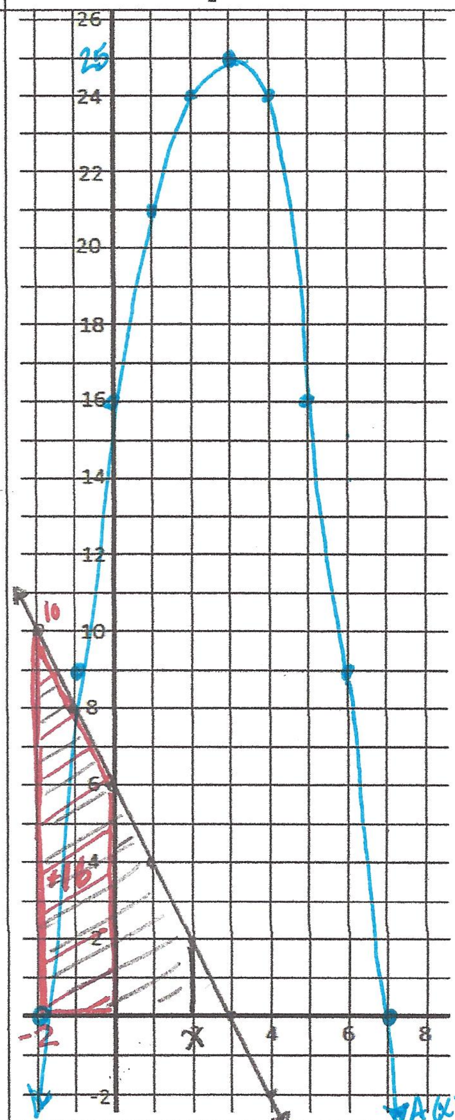
$= (6x - x^2) - (6 - 1)$

$A(x) = -x^2 + 6x - 5$

4c)

$f(t) = 6 - 2t \text{ on } [-2, x]$

$A(x) = \int_{-2}^x 6 - 2t\,dt$



$A(x) = \frac{1}{2}(x+2)(10+6-2x)$

$= \frac{1}{2}(x+2)(-2x+16)$

$= -(x+2)(x-8)$

$= -(x^2 - 6x - 16)$

$= -x^2 + 6x \boxed{+16}$

x	0	1	2	3	4	5	6
A(x)	16	21	24	25	24	21	16

$A(x) = \int_{-2}^x 6 - 2t\,dt$

$= 6t - t^2 \Big|_{-2}^x$

$= (6x - x^2) - (-12 - 4)$

$= -x + 6x + 16$

notice: Maximum A(x) value occurs when f(t) changes signs + to −.

Key III

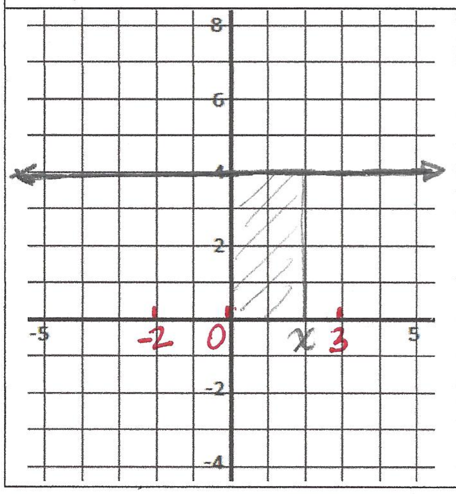
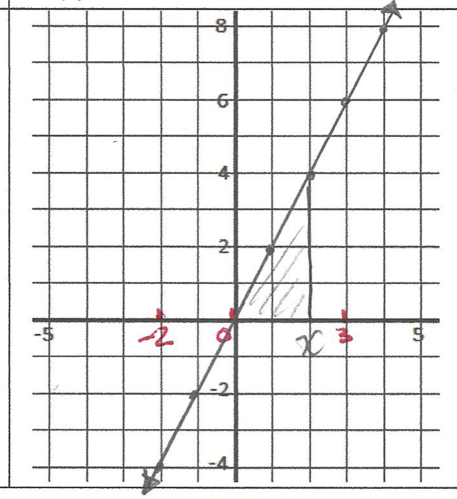
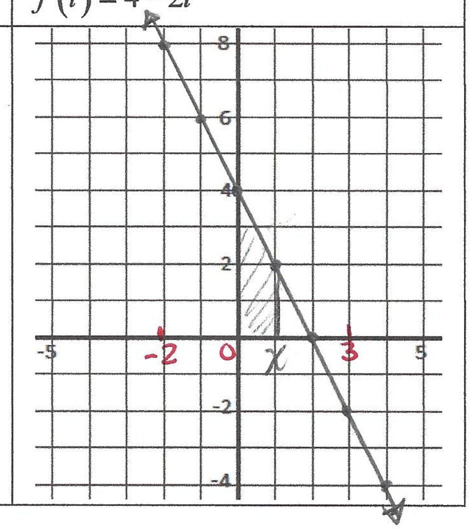
Ch 6 Building Area Functions - part III

We have now seen how to build area functions by:

- Graphing and using geometry to write an area function
- Using a table to record geometric area on the graph and making sure the area function models the data in the table.
- Find the ANTI-DERIVATIVE of $f(t)$, call this $F(x)$ and then evaluate from $x = a\ to\ x = b$ according to the following:

$$A(x)=\int_a^b f(t)=F(t)\Big|_a^b=F(b)-F(a)$$

Graph the three functions, $f(t)$, on the grids provided.

$f(t)=4$	$f(t)=2t$	$f(t)=4-2t$
		

Use the graphs above to determine an area function for each integral below. Note the changes in the lower limits of each integral.

<table>
<tr><td>a) $A(x)=\int_0^x(4)\,dt$</td><td>b) $A(x)=\int_0^x(2t)\,dt$</td><td>c) $A(x)=\int_0^x(4-2t)\,dt$</td></tr>
<tr><td>Build area function from graph
$A(x)=(x-0)(4)$
$=4x-0$</td><td>$A(x)=\frac{1}{2}(x-0)(2x)$
$=x^2-0$</td><td>$A(x)=\frac{1}{2}(x-0)(4+4-2x)$
$=\frac{1}{2}(x)(-2x+8)$
$=-x(x-4)$
$=-x^2+4x-0$</td></tr>
<tr><td>Table a)<table><tr><td>x</td><td>-2</td><td>-1</td><td>0</td><td>1</td><td>2</td></tr><tr><td>A(x)</td><td>-8</td><td>-4</td><td>0</td><td>4</td><td>8</td></tr></table></td><td>Table b)<table><tr><td>x</td><td>-2</td><td>-1</td><td>0</td><td>1</td><td>2</td></tr><tr><td>A(x)</td><td>4</td><td>1</td><td>0</td><td>1</td><td>4</td></tr></table></td><td>Table c)<table><tr><td>x</td><td>-2</td><td>-1</td><td>0</td><td>1</td><td>2</td></tr><tr><td>A(x)</td><td>-12</td><td>-5</td><td>0</td><td>3</td><td>4</td></tr></table>Table a) - Table b) = Table c)</td></tr>
<tr><td>Find Anti-Derivative
$A(x)=\int_0^x 4\,dt$
$=4t\Big|_0^x=4(x)-4(0)$
$=4x-0$</td><td>$A(x)=\int_0^x(2t)\,dt$
$=t^2\Big|_0^x=x^2-0^2$
$=x^2-0$</td><td>$A(x)=\int_0^x(4-2t)\,dt$
$=4t-t^2\Big|_0^x$
$=(4x-x^2)-(4(0)-0^2)$
$=-x^2+4x-0$</td></tr>
</table>

$A(x)_a - A(x)_b = A(x)_c$

5

KEY III

a) $A(x)=\int_{-2}^{x}(4)\,dt$	b) $A(x)=\int_{-2}^{x}(2t)\,dt$	c) $A(x)=\int_{-2}^{x}(4-2t)\,dt$
Build area function from graph $A(x)=(x+2)(4)$ $=4x+8$	Visualize Trapezoid $[-2,x]$ $A(x)=\frac{1}{2}(x+2)(-4+2x)$ $=\frac{1}{2}(x+2)(2x-4)$ $=(x+2)(x-2)$ $=x^2-4$	Visualize Trapezoid $[-2,x]$ $A(x)=\frac{1}{2}(x+2)(8+4-2x)$ $=\frac{1}{2}(x+2)(-2x+12)$ $=-(x+2)(x-6)$ $=-(x^2-4x-12)$ $=-x^2+4x+12$
Table a) x: -4, -2, 0, 1, 3 A(x): -8, 0, 8, 12, 20	Table b) x: -4, -2, 0, 1, 3 A(x): 12, 0, -4, -3, 5	Table c) x: -4, -2, 0, 1, 3 A(x): -20, 0, 12, 15, 15 Table a) − Table b) = Table c)
Find Anti-Derivative $A(x)=\int_{-2}^{x}4\,dt$ $=4t\Big\vert_{-2}^{x}$ $=4x-4(-2)$ $=4x+8$	$A(x)=\int_{-2}^{x}(2t)\,dt$ $=t^2\Big\vert_{-2}^{x}$ $=x^2-(-2)^2$ $=x^2-4$	$A(x)=\int_{-2}^{x}(4-2t)\,dt$ $=4t-t^2\Big\vert_{-2}^{x}$ $=(4(x)-x^2)-(4(-2)-(-2)^2)$ $=4x-x^2-(-8-4)$ $=-x^2+4x+12$

$A(x)_a - A(x)_b = A(x)_c$

a) $A(x)=\int_{3}^{x}(4)\,dt$	b) $A(x)=\int_{3}^{x}(2t)\,dt$	c) $A(x)=\int_{3}^{x}(4-2t)\,dt$
Build area function from graph $A(x)=(x-3)(4)$ $=4x-12$	Visualize Trapezoid $[3,x]$ $A(x)=\frac{1}{2}(x-3)(6+2x)$ $=\frac{1}{2}(x-3)(2x+6)$ $=(x-3)(x+3)$ $=x^2-9$	Visualize Trapezoid $[3,x]$ $A(x)=\frac{1}{2}(x-3)(-2+4-2x)$ $=\frac{1}{2}(x-3)(-2x+2)$ $=-(x-3)(x-1)$ $=-(x^2-4x+3)$ $=-x^2+4x-3$
Table x: -2, 0, 3, 5, 8 A(x): -20, -12, 0, 8, 20	Table x: -2, 0, 3, 5, 8 A(x): -5, -9, 0, 16, 55	Table x: -2, 0, 3, 5, 8 A(x): -15, -3, 0, -8, -25 Table a) − Table b) = Table c)
Find Anti-Derivative $A(x)=\int_{3}^{x}4\,dt$ $=4t\Big\vert_{3}^{x}$ $=4(x)-4(3)$ $=4x-12$	$A(x)=\int_{3}^{x}2t\,dt$ $=t^2\Big\vert_{3}^{x}$ $=x^2-(3)^2$ $=x^2-9$	$A(x)=\int_{3}^{x}(4-2t)\,dt$ $=4t-t^2\Big\vert_{3}^{x}$ $=(4x-x^2)-(4(3)-(3)^2)$ $=4x-x^2-(12-9)$ $=-x^2+4x-3$

$A(x)_a - A(x)_b = A(x)_c$