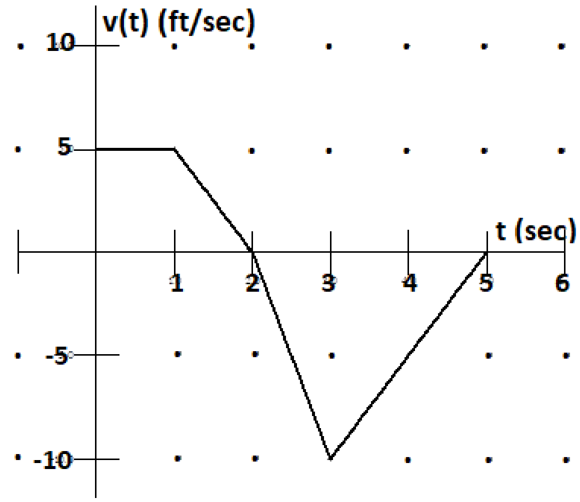
AB CALCULUS: **§5.3 Position vs Distance**
Velocity vs. Speed
Acceleration



All the questions which follow refer to the graph at the right.

1. When is the particle moving at a constant speed?

2. When is the particle moving to the right? Why?

3. When is the particle moving to the left? Why?

4. When is the particle speeding up? Why?

5. When is the particle slowing down? Why?

6. When is the velocity increasing? Why?

7. When is the velocity decreasing? Why?

8. Are your answers to questions 6 & 7 the same as your answers to 4 & 5 respectively? Explain.

9. How fast is the particle moving at time t = 4 and in what direction?

10. What is the particle's velocity at time t = 4?

11. When does the particle change direction?

12. How far does the particle move during the first second and which way?

13. How far to the right does the particle go and when does it arrive there?

14. What is the total distance traveled by the particle?

15. What is the particle's position after 5 secs?

16. Are your answers to questions 14 & 15 the same? Explain.

17. What is the average velocity of the particle over the 5 second time interval?

18. What is the average velocity over the time interval $2< t <3$?

19. What is the average speed over the time interval of #18?

20. What is the particle's acceleration over the 1st second? ______ What does your answer mean?

21. Over what time interval(s) does the particle have positive acceleration? Why?

22. Over what time interval(s) does it have negative acceleration? Why?

23. When is the acceleration undefined on the interval $0 < t < 5$?

24. State an interval of time when the particle has positive acceleration but is slowing down. Explain.

25. State an interval when the acceleration is negative but the particle is speeding up. Explain.

26. What is the particle's acceleration over the time interval (3, 4)?

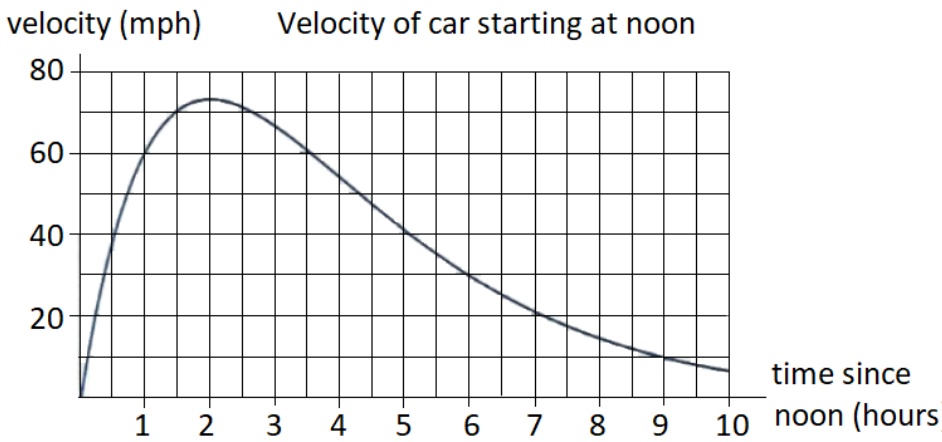
27. What is the particle's average acceleration over the entire 5 second time interval?

28. Over what time interval is the particle's velocity decreasing at a rate of 5 feet/sec every second?

AB Calculus **§5.3**

The Car and Truck Problem

A car starts at noon and travels with the velocity shown in the figure. A truck starts at 1 pm from the same place and travel at a constant velocity of 50 mph.



1. How far away is the car when the truck starts?

2. a. How far has the car gone at 3:00?

   b. What was the car's velocity at 3:00?

3. What was the car's average velocity over the first three hours of its trip?

4. Graph the truck's velocity function.

5. At what point do the two velocity functions intersect? What is the relevance of this point besides the obvious fact that they are traveling at the same speed then?

6. During the period of time when the car is ahead of the truck:

a. When is the distance between them the greatest?

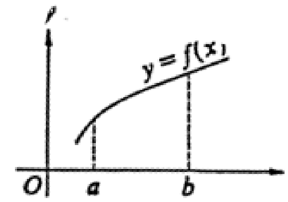
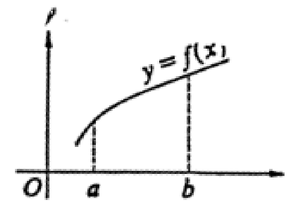
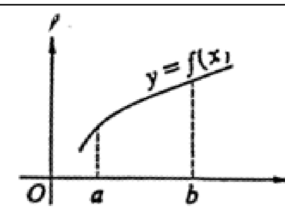
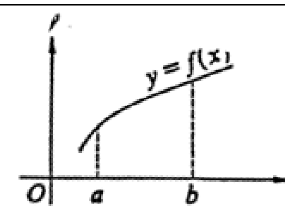
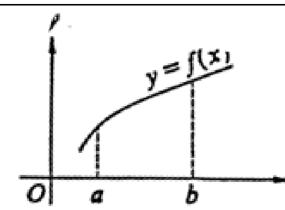
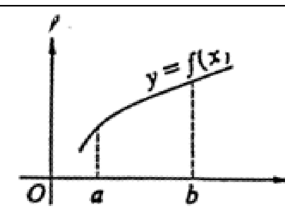
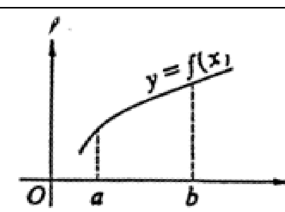
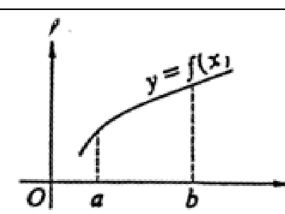
b. Approximate this distance between them

7. a. When does the truck overtake the car?

   b. How far have both traveled then?

AP Calculus AB **Geometric Understanding Fundamental Theorem of Calculus**

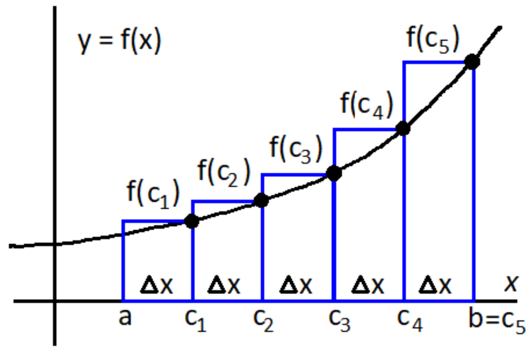
Complete the table below. Let $f = F'$

Expression	What It's Called	Is it a Length, Area, or Slope on f?	Sketch on f
$f(b) - f(a)$			
$\dfrac{f(b) - f(a)}{b - a}$			
$f'(a)$			
$\int_a^b f(t)dt$			
$\dfrac{1}{b-a}\int_a^b f(t)dt$			
$F(b) - F(a)$			
$\dfrac{F(b) - F(a)}{b - a}$			
$F'(a)$			

§5.4 Properties of the Definite Integral

Let's use the following graph to develop an understanding for notation of the Riemann Sums.

Riemann Sum & Definite Integral



$\int_a^b f(x)dx$ can be approximated by:

area = $f(c_1)\Delta x + f(c_2)\Delta x + f(c_3)\Delta x + f(c_4)\Delta x + f(c_5)\Delta x$

$= [f(c_1) + f(c_2) + f(c_3) + f(c_4) + f(c_5)]\,\Delta x$

$= \sum_{i=1}^{5} f(c_i)\,\Delta x$

& if we use ***n*** rectangles instead of 5 we write: $\sum_{i=1}^{n} f(c_i)\Delta x_i$

What will happen if we let n get larger and larger?

Δx will get smaller and smaller.

The width of each rectangle will become smaller and smaller.

The number of rectangles will increase.

We can represent this by using a limit: $\int_a^b f(x)dx = \lim_{n\to\infty}\sum_{i=1}^{n} f(c_i)\Delta x_i$

Let f be a function that is defined on the closed interval $[a,b]$. If Δx is a partition of $[a,b]$ and Δx_i is the width of the *i*th interval, c_i is any point in the subinterval, then the sum $\sum_{i=1}^{n} f(c_i)\Delta x_i$ is called a **Riemann Sum** of f.

Furthermore, if $\lim_{n\to\infty}\sum_{i=1}^{n} f(c_i)\Delta x_i$ exists, we say f is **integrable** on $[a,b]$ and $\int_a^b f(x)dx$ is called the **definite integral** (or Riemann Integral) of f from *a* to *b*. *a* is called the <u>lower limit of integration</u> and *b* is called the <u>upper limit of integration</u>. The width of each rectangle is $\Delta x_i = \dfrac{b-a}{n}$ and in the definite integral is represented by *dx*; the height of each rectangle is $f(c_i)$ and in the definite integral is the function.

General form:	**Examples:**
$\int_a^b f(x)\,dx = \lim_{n\to\infty}\sum_{i=1}^{n} f(c_i)\,\Delta x_i$	$\int_0^5 \sqrt[3]{x+7}\,dx = \lim_{n\to\infty}\sum_{i=1}^{n}\left(\sqrt[3]{\dfrac{5i}{n}+7}\right)\left(\dfrac{5}{n}\right)$ $\int_1^5 \sqrt[3]{x^2+7x}\,dx = \lim_{n\to\infty}\sum_{i=1}^{n}\left(\sqrt[3]{\left(1+\dfrac{4i}{n}\right)^2+7\left(1+\dfrac{4i}{n}\right)}\right)\left(\dfrac{4}{n}\right)$

<u>Example 1:</u> Write the following as definite integrals:

a) $\lim_{n\to\infty}\sum_{i=1}^{n}\left(\dfrac{3i}{n}\right)^2\left(\dfrac{3}{n}\right)$

b) $\lim_{n\to\infty}\sum_{i=1}^{n}\left(\dfrac{2i}{n}\right)^3\left(\dfrac{2}{n}\right)$

c) $\lim_{n\to\infty}\sum_{i=1}^{n}\left[1+\dfrac{2i}{n}+\left(\dfrac{2i}{n}\right)^2\right]\left(\dfrac{2}{n}\right)$

d) $\lim_{n\to\infty}\sum_{i=1}^{n}\left[\sin\left(\dfrac{\pi i}{n}\right)\right]\left(\dfrac{\pi}{n}\right)$

e) $\lim_{n\to\infty}\sum_{i=1}^{n}\left[\sqrt{\dfrac{6i}{n}}\right]\left(\dfrac{2}{n}\right)$

f) $\lim_{n\to\infty}\sum_{i=1}^{n}\left[1+\dfrac{4i}{n}+\left(\dfrac{2i}{n}\right)^2\right]\left(\dfrac{2}{n}\right)$

Properties of Integration:

$\int_a^a f(x)dx = 0$

$\int_a^b cdx = (b-a)\cdot c$ (c is any constant)

$\int_a^b f(x)dx = -\int_b^a f(x)dx$

$\int_a^b k\cdot f(x)dx = k\int_a^b f(x)dx$

$\int_a^b (f(x)+g(x))dx = \int_a^b f(x)dx + \int_a^b g(x)dx$

$\int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx$

<u>Example 2</u>: Suppose $\int_0^1 f(x)dx = 4$ and $\int_0^1 g(x)dx = -2$, find:

a) $\int_0^1 3f(x)dx$

b) $\int_0^1 [f(x)-g(x)]dx$

c) $\int_0^1 [3f(x)+2g(x)]dx$

d) $\int_0^1 [2g(x)-3f(x)]dx$

e) $\int_1^0 [2f(x)]dx$

<u>Example 3:</u> Given $\int_a^b f(x)dx = 5$ *and* $\int_a^b g(x)dx = -4$, use the properties to evaluate the following:

$a.\ \int_b^a f(x)dx$

$b.\ \int_a^b 3f(x)dx$

$c.\ \int_a^b 3dx$

$d.\ \int_a^b (3+f(x))dx$

$e.\ \int_a^b (f(x)+g(x))dx$

$f.\ \int_a^b (f(x)-g(x))dx$

$g.\ \int_a^6 g(x)dx + \int_6^b g(x)dx$

4. If $\int_2^5 (2f(x)+3)dx = 17$, find $\int_2^5 f(x)dx$

5. If *f(x)* is an even function and you know $\int_0^4 f(x)dx = 6$, then what is the value of $\int_{-4}^4 f(x)dx$?

6. If *f(x)* is an odd function and you know $\int_0^4 f(x)dx = 6$, then what is the value of $\int_{-4}^4 f(x)dx$?

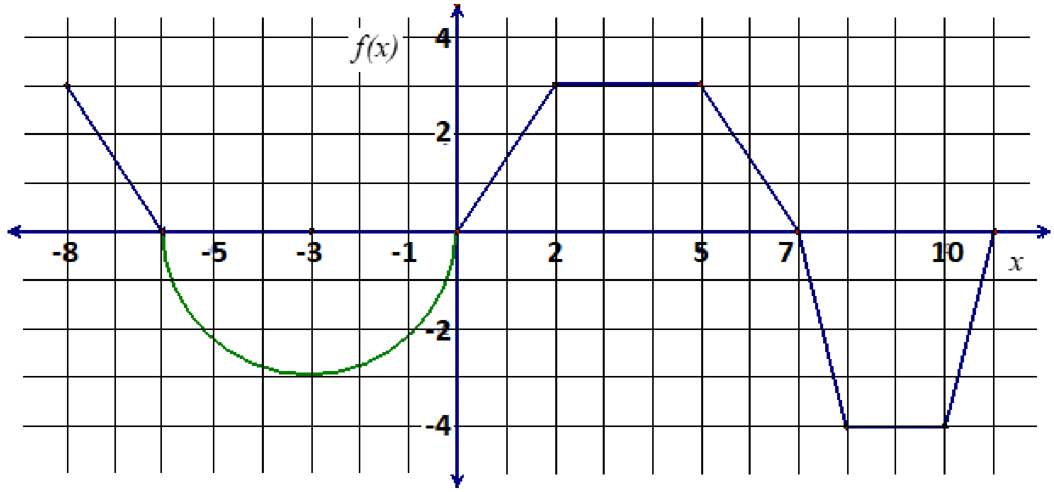
7. Given: $\int_0^7 f(x)dx = 5$, $\int_5^7 f(x)dx = 11$, $\int_7^{10} f(x)dx = 12$ evaluate each of the following integrals:

a. $\int_5^{10} f(x)dx$ *b.* $\int_4^4 f(x)dx$ *c.* $\int_7^0 f(x)dx$

d. $\int_0^5 f(x)dx$ *e.* $\int_5^7 4f(x)dx$ *f.* $\int_{10}^7 \frac{1}{2} f(x)dx$

WS: **<u>§5.4 Properties of the Definite Integral</u>**

<u>Part I:</u> Given the graph of $f\left(x\right)$ on the domain $\left[-8,11\right]$.



1. $\int_{0}^{7} f(x)dx$

2. $\int_{0}^{11} f(x)dx$

3. $\int_{-6}^{0} f(x)dx$

4. $\int_{4}^{4} f(x)dx$

5. $\int_{7}^{0} f(x)dx$

6. $\int_{8}^{4} f(x)dx$

7. $\int_{-8}^{11} f(x)dx$

8. Average value in f over [0, 5]

9. $f'(-6)$

10. $f''(6)$

11. True or False: $\int_{2}^{4} f(x)dx + \int_{4}^{6} f(x)dx = \int_{2}^{6} f(x)dx$

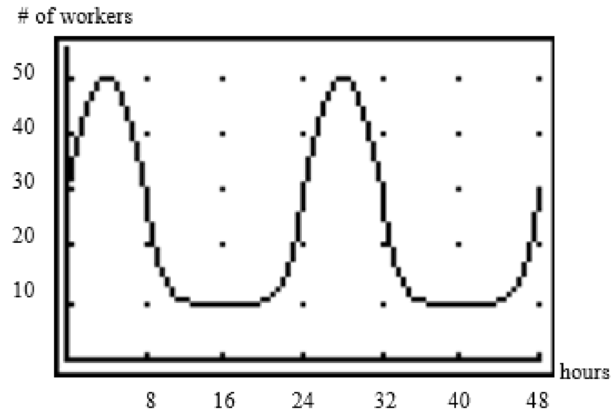
12. True or False: $\int_{2}^{0} f(x)dx + \int_{0}^{6} f(x)dx = \int_{2}^{6} f(x)dx$

13. Over what values of x is f not differentiable?

WS: **<u>§5.4 Properties of the Definite Integral</u>**

Part II. The Worker Problem (Hughes-Hallett, 2nd ed., p. 165, #12 – Revised)

A two-day environmental cleanup operation started at 9 am on the first day. The number of workers fluctuated as shown in figure 3.28. Let $f(t)$ represent the number of workers on the job at time t where t represents the number of hours past 9 am on the first day.



Complete the missing entries in the given table.

QUANTITY	**ESTIMATED VALUE (include units)**	**REAL-WORLD MEANING (as a complete sentence)**
1. $f(8)$		
2. $f(8) - f(4)$		
3. $\dfrac{f(4)-f(0)}{4-0}$		
4.	0 workers/hour	
5. $f'(8)$		
6. $\int_{24}^{32} f(t)\,dt$	(show work)	
7.		The cost of the cleanup operation from 9 am to 5 pm on the 2nd day if workers were paid \$10/hr.
8.		The average rate at which the labor force declined from 5 pm to 1 am on the 1st day.
9.		The average number of workers during the time period 5 pm to 1 am on the 1st day.