Unit 4 Part 1 Review Sheet
Ch 4 Test (part 1) 6 December 2017

Show all work on a separate sheet of paper
** = Calculator required. Otherwise do not use a calculator

Derivative of an Inverse Function:

1. Given the function $f(x)=x^3-2x+1$ where $g(x)=f^{-1}(x)$, complete the following table & $f'(x)=$ _____

Evaluate	$f(0)=$	$f(2)=$	$f(-3)=$
Solve	$g(\_\_\_)=0$	$g(\_\_\_)=2$	$g(\_\_\_)=-1$
Evaluate	$f'(0)=$	$f'(2)=$	$f'(-1)=$
Evaluate	$g'(1)=$	$g'(5)=$	$g'(-20)=$

** Use the same $f(x)$ to evaluate $g'(116)=$

You will need to use your calculator! Do you know how to use it effectively & efficiently for this type of question?

2. Use the table to find the Derivative of the Inverse Function at each value:
A) $(f^{-1})'(2)=$ B) $(f^{-1})'(3)=$ C) $(f^{-1})'(0)=$ D) $(f^{-1})'(-3)=$

x	1	2	3	4
$f(x)$	-3	0	2	3
$f'(x)$	5	4/3	-8	1/2

Implicit Differentiation:

3. If $x^2+xy+y^3=0$, then, in terms of x and y, $\frac{dy}{dx}=$

(A) $-\frac{2x+y}{x+3y^2}$ (B) $-\frac{x+3y^2}{2x+y}$ (C) $\frac{-2x}{1+3y^2}$ (D) $\frac{-2x}{x+3y^2}$ (E) $-\frac{2x+y}{x+3y^2-1}$

4. If $x^2+xy=10$, then when $x=2$, $\frac{dy}{dx}=$

(A) $-\frac{7}{2}$ (B) –2 (C) $\frac{2}{7}$ (D) $\frac{3}{2}$ (E) $\frac{7}{2}$

5. If $x^2+y^2=25$, what is the value of $\frac{d^2y}{dx^2}$ at the point $(4,3)$?

(A) $-\frac{25}{27}$ (B) $-\frac{7}{27}$ (C) $\frac{7}{27}$ (D) $\frac{3}{4}$ (E) $\frac{25}{27}$

6. Write the equation of the tangent line to the curve $y^3-xy^2=4$ at the point where $y=2$ is

7. Find $\frac{dy}{dx}$ if $\ln(xy)=x+y$

Linear Approximations of a function using Tangent Lines

8. $f(x)=\frac{1}{\sqrt{1-x}}$

i) Write the tangent line equation: $t(x)$ at $c=-3$

ii) Approximate $f(-3.1)$ using $t(-3.1)\approx$

iii) Is $t(-3.1)$ an over/under estimate? Why?

iv) ** Find the difference $f(-3.1)-t(-3.1)$

9. $f(x)=\ln(1-x)$

i) Write the tangent line equation: $t(x)$ at $c=1-e$

ii) Approximate $f(1.1-e)$ using $t(1.1-e)\approx$

iii) Is $t(1.1-e)$ an over/under estimate? Why?

iv) ** Find the difference $f(1.1-e)-t(1.1-e)$

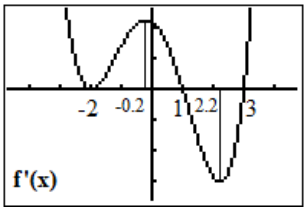
Mean Value Theorem

10. Given $f(x)=\frac{1}{4}x^3+1$ over the interval $[0,2]$, verify the hypotheses of the Mean Value Theorem are satisfied on the given interval and find all values c in that interval that satisfy the conclusion of the theorem.

11. You are driving on a straight highway on which the speed limit is 55mph. At 8:05 AM a police car clocks your velocity at 50 mph and at 8:10 AM a second police car posted 5 miles down the road clocks your velocity at 55 mph. Explain why the police have a right to charge you with a speeding violation. (Anton p306)

1st & 2nd Derivative Tests to identify function behavior

12. Given the graph of the derivative of *f(x)*,



a) State the critical values on $f(x)$.

b) State x-intervals for which $f(x)$ increasing and decreasing. Justify your conclusion.

c) Classify each critical value as a maximum, minimum or neither. Justify your conclusion.

d) State x-intervals for which $f(x)$ is concave up and concave down. Justify your conclusion.

13. If the derivative of a function $f(x)$ is $f'(x)=3(x+2)(x+1)^2(x-3)^3$, find the x-value(s) where a relative maximum occurs. Show the work leading to your answer.

14. Let $f''(x)=-3x^2+6x+1$ and let f have a critical number at $x=1$. Use the Second Derivative Test to determine if there is a relative max or relative min at the critical value.

Optimization

15. Given $f(x)=2x^3-3x^2-12x$, classify all relative and absolute extrema on the interval $[-2,4]$. Use a signed-pattern number line to organize your thoughts. Create a table and make your conclusion.

16. Given $f(x)=\left(\frac{x-4}{x+3}\right)^2$ Complete the information in the charts on your own paper. Graph without your calculator.

a. $f'(x)=$?	d. $f''(x)=$?
b. Critical points	e. Critical points
c. Signed #line	f. Signed #line

State the intervals for which… & include a reason why.	j. Identify inflection point(s)
g. f is increasing & decreasing…	k. vertical asymptote? y-intercept?
h. f is concave up & concave down…	l. $\lim_{x\to\pm\infty} f(x)=$?
i. Make table and classify relative & absolute extrema (max/min)	m. Graph $f(x)$

17. Given $f(x)=x^2e^{-x}$ on the interval $[-1,4]$. Complete the information on your own paper. Graph without a calculator.

a. Show that $f'(x)=e^{-x}(2x-x^2)$	d. Show that $f''(x)=e^{-x}(x^2-4x+2)$
b. Critical points	e. Critical points
c. Signed #line	f. Signed #line

State the intervals for which… & justify conclusion
g. f is increasing/decreasing
h. f is concave up/down

i. Make an x-y table and classify rel & abs max or min. **
j. Identify inflection points **
k. Graph $f(x)$

18. Given $f(x)=2-x^{\frac{2}{3}}$, complete all information in the charts on your own paper. Graph without a calculator.

a. $f'(x)=$?	d. $f''(x)=$?
b. Critical points	e. Critical points
c. Signed #line	f. Signed #line

State the intervals for which… & include a reason why.
g. f is increasing & decreasing
h. f is concave up & concave down

i. Make an x-y table and classify … relative/absolute extrema (max or min)
j. Identify inflection points
k. find zeros ** & Graph $f(x)$

Challenge If $y=x^{1-x}$ we can find the derivative by first taking the natural log of the equation: $\ln(y)=\ln(x^{1-x})$ Use log rules and implicit differentiation to find $\frac{dy}{dx}$. Choose all answers that are true: (A) $-\ln(x)+\frac{(1-x)}{x}$ (B) $\left[\frac{1-x}{x}-\ln(x)\right]x^{1-x}$ (C) $(1-x)x^{-x}-x^{1-x}\ln(x)$ (D) $x^{-x}-x^{1-x}(1+\ln x)$

Show work.