ANSWERS

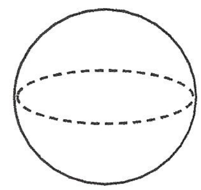
AB Calculus – Chapter 4 REVIEW **WS: REVIEW Related Rates**

Complete on your own paper. In Exercises #1-2 assume that x and y are both differentiable functions of t and find the indicated values of dy/dt and dx/dt.

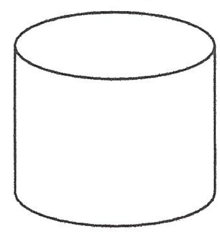
Equation	**Find**	**Given**
1. $y = \sqrt{x}$	a. $\dfrac{dy}{dt}$ when $x = 4$.	$\dfrac{dx}{dt} = 3$
	b. $\dfrac{dx}{dt}$ when $x = 25$.	$\dfrac{dy}{dt} = 2$
2. $y = x^2 - 3x$	a. $\dfrac{dy}{dt}$ when $x = 3$.	$\dfrac{dx}{dt} = 2$
	b. $\dfrac{dx}{dt}$ when $x = 1$.	$\dfrac{dy}{dt} = 5$

Free Response

3. A spherical balloon is inflating at a rate of 27π in^3/sec. How fast is the radius of the balloon increasing when the radius is 3 inches? $V = \frac{4}{3}\pi r^3$

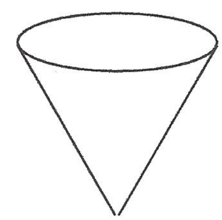


4. The voltage, V, in an electrical circuit is related to the current, I, and the resistance, R, by the equation $V = IR$. The current is decreasing at -4 amps/sec as the resistance increases at 20 ohms/sec. How fast is the voltage changing when the voltage is 100 volts and the current is 20 amps?

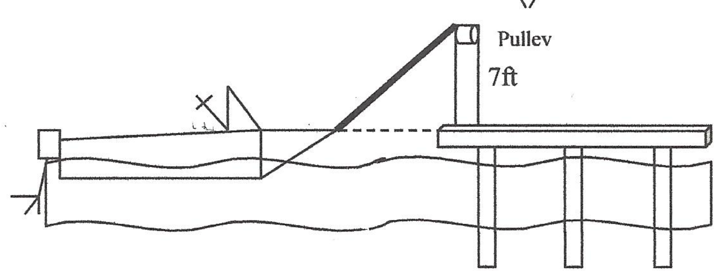


5. A cylindrical tank with a radius of 6 meters is filling with fluid at a rate of 108π m^3/sec. How fast is the height increasing? $V = \pi r^2 h$

6. An inverted conical container has a diameter of 42 in can a depth of 15 inches. If water is flowing out of the vertex of the container at a rate of 35π in^3/sec, how fast is the depth of the water dropping when the height is 5 inches? $V = \frac{1}{3}\pi r^2 h$



7. A boat is being pulled toward a dock by a rope attached to its bow through a pulley on the dock 7 feet above the bow. If the rope is hauled at a rate of 4 ft/sec, how fast is the boat approaching the dock when 25 feet of rope is out?



8. A 6 foot tall woman is walking at the rate of 4 ft/sec away from a street lamp that is 24 ft tall. How fast is the length of her shadow changing?

9. Oil spilled from a tanker spreads in a circle with circumference increasing at a rate of 40 ft/sec. How fast is the area of the spill increasing when the circumference of the circle is 100π feet? (Hint: Find a formula for the area of a circle in terms of the circumference.)

AB CALCULUS – CHAPTER 4 REVIEW $\boxed{\text{ANSWERS}}$

WS. REVIEW RELATED RATES

In Exercises #1-2 assume that x and y are both differentiable functions of t and find the indicated values of dy/dt and dx/dt.

Equation	**Find**	**Given**
1. $y=\sqrt{x}$	a. $\frac{dy}{dt}$ when $x=4$.	$\frac{dx}{dt}=3$
	b. $\frac{dx}{dt}$ when $x=25$.	$\frac{dy}{dt}=2$
2. $y=x^2-3x$	a. $\frac{dy}{dt}$ when $x=3$.	$\frac{dx}{dt}=2$
	b. $\frac{dx}{dt}$ when $x=1$.	$\frac{dy}{dt}=5$

$\frac{dy}{dt}=\frac{1}{2\sqrt{x}}\frac{dx}{dt}$

$2\sqrt{x}\,\frac{dy}{dt}=\frac{dx}{dt}$

1a. $\frac{dy}{dt}=\frac{1}{2\sqrt{x}}\frac{dx}{dt}$

$\frac{dy}{dt}=\frac{1}{2\sqrt{4}}\cdot 3=\boxed{\frac{3}{4}}$

1b. $\frac{dy}{dt}=\frac{1}{2\sqrt{x}}\cdot\frac{dx}{dt}$

$2=\frac{1}{2\sqrt{25}}\cdot\frac{dx}{dt}$

$\frac{dx}{dt}=2\sqrt{x}\cdot\frac{dy}{dt}$

$\frac{dx}{dt}=2\sqrt{25}\cdot 2$

$=4\cdot 5=\boxed{20}$

2. $\frac{dy}{dt}=2x\frac{dx}{dt}-3\frac{dx}{dt}$

$\frac{dy}{dt}=\frac{dx}{dt}(2x-3)$

$\frac{dx}{dt}=\frac{1}{(2x-3)}\cdot\frac{dy}{dt}$

2a. $\frac{dy}{dx}=2(3)(2)-3(2)$

$=12-6$

$=\boxed{6}$

2b. $\frac{dx}{dt}=\frac{1}{(2(1)-3)}\cdot 5$

$\frac{dx}{dt}=\frac{5}{-1}=\boxed{-5}$

Free Response

3. A spherical balloon is inflating at a rate of 27π in^3/sec. How fast is the radius of the balloon increasing when the radius is 3 inches?

$\frac{dV}{dt}=27\pi\ \frac{in^3}{sec}$

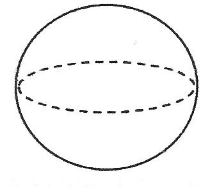
$\frac{dr}{dt}=?\qquad r=3\text{ in}$

$\frac{dV}{dt}=4\pi r^2\frac{dr}{dt}$

$27\pi=4\pi(3)^2\cdot\frac{dr}{dt}$

$\boxed{\frac{1}{4}=\frac{dr}{dt}}$

$V=\frac{4}{3}\pi r^3$



4. The voltage, V, in an electrical circuit is related to the current, I, and the resistance, R, by the equation $V=IR$. The current is decreasing at -4 amps/sec as the resistance increases at 20 ohms/sec. How fast is the voltage changing when the voltage is 100 volts and the current is 20 amps?

$V=IR$

$\frac{dI}{dt}=-4\ \frac{amps}{sec}$

$\frac{dR}{dt}=+20\ \frac{ohms}{sec.}$

$\frac{dV}{dt}=\boxed{?}$

$V=100$ volts.
$I=20$ amps.

$\frac{dV}{dt}=I\frac{dR}{dt}+R\frac{dI}{dt}$

$=20(20)+5(-4)$

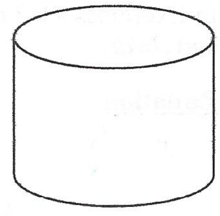
$=400-20=\boxed{380\ \frac{\text{Volts}}{\text{sec.}}}$

$V=IR$

$100=20(R)\ \therefore R=5$

5. A cylindrical tank with a radius of 6 meters is filling with fluid at a rate of 108π m^3/sec. How fast is the height increasing?

$V = \pi r^2 h$



$r = 6$

$\frac{dV}{dt} = 108\pi \frac{m^3}{sec}$

$\frac{dh}{dt} = \boxed{?}$ $\quad \frac{dr}{dt} = 0$

$\frac{dV}{dt} = \pi\left(2rh\frac{dr}{dt} + r^2\frac{dh}{dt}\right)$

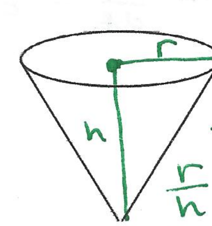
$108\pi = \pi\left(\cancel{2(6)(h)(0)} + 6^2\frac{dh}{dt}\right)$

$108 = 36\frac{dh}{dt}$

$\frac{dh}{dt} = \frac{108}{36} = 3 \frac{m}{sec}$

6. An inverted conical container has a diameter of 42 in can a depth of 15 inches. If water is flowing out of the vertex of the container at a rate of 35π in^3/sec, how fast is the depth of the water dropping when the height is 5 inches?

$V = \frac{1}{3}\pi r^2 h$



CONE [diam = 42 in. $\Rightarrow$ r = 21 in; height = 15 inches]

$\frac{dV}{dt} = -35\pi \frac{in^3}{sec.}$ $\quad \frac{dr}{dt} = \frac{7}{5}\frac{dh}{dt}$

$\frac{dh}{dt} = \boxed{?}$ $\quad \underline{h = 5 \text{ in.}}$

$\frac{dV}{dt} = \frac{\pi}{3}\left(2rh\frac{dr}{dt} + r^2\frac{dh}{dt}\right)$

$-35\pi = \frac{\pi}{3}\left(2(7)(5)\frac{7}{5}\boxed{\frac{dh}{dt}} + 7^2\boxed{\frac{dh}{dt}}\right)$

$-105 = (98 + 49)\frac{dh}{dt}$

$\frac{-105}{147} = \frac{dh}{dt} \approx 0.714 \frac{in}{sec}$

$\frac{r}{h} = \frac{21}{15}$

$\frac{r}{h} = \frac{7}{5}$

$r = \frac{7}{5}h$

$\frac{dr}{dt} = \frac{7}{5}\frac{dh}{dt}$

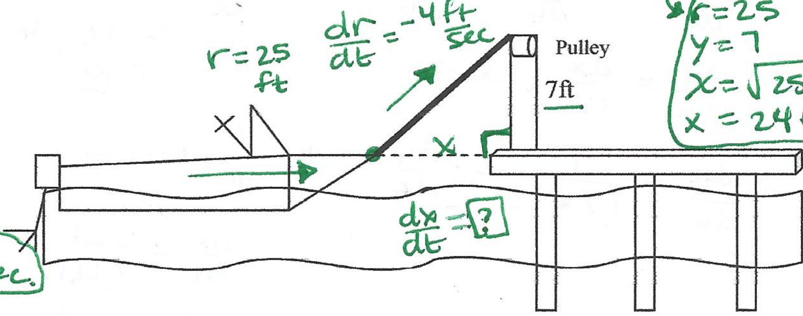
7. A boat is being pulled toward a dock by a rope attached to its bow through a pulley on the dock 7 feet above the bow. If the rope is hauled at a rate of 4 ft/sec, how fast is the boat approaching the dock when 25 feet of rope is out?

$x^2 + 7^2 = r^2$

$2x\frac{dx}{dt} + 0 = 2r\frac{dr}{dt}$

$x\frac{dx}{dt} = r\frac{dr}{dt} \Rightarrow \frac{dx}{dt} = \frac{r}{x}\frac{dr}{dt}$

$\frac{dx}{dt} = \frac{25(-4)}{24} = -4.166$ ft/sec.



$r = 25$

$y = 7$

$x = \sqrt{25^2 - 7^2}$

$x = 24$ ft

8. A 6 foot tall woman is walking at the rate of 4 ft/sec away from a street lamp that is 24 ft tall. How fast is the length of her shadow changing?

$\frac{x}{6} = \frac{x+y}{24}$

$4x = x + y$

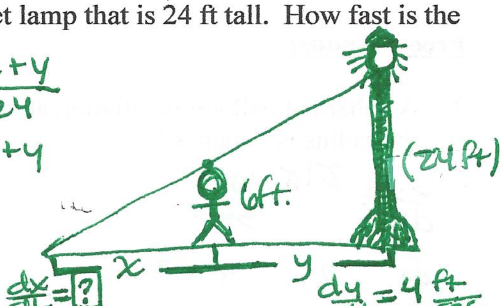
$3x = y$

$3\frac{dx}{dt} = \frac{dy}{dt}$

$3\frac{dx}{dt} = \frac{dy}{dt}$

$\frac{dx}{dt} = \frac{1}{3}\frac{dy}{dt}$

$\frac{dx}{dt} = +\frac{4}{3}$ ft/sec.



9. Oil spilled from a tanker spreads in a circle with circumference increasing at a rate of 40 ft/sec. How fast is the area of the spill increasing when the circumference of the circle is 100π feet? (Hint: Find a formula for the area of a circle in terms of the circumference.)

$\frac{dC}{dt} = 40 \frac{ft}{sec}$

$\frac{dA}{dt} = \boxed{?}$

when $C = 100\pi$ ft. $\therefore r = 50$ ft.

$C = 2\pi r$

$r = \frac{C}{2\pi}$

$A = \pi\left(\frac{C}{2\pi}\right)^2$

$A = \frac{C^2}{4\pi} = \frac{1}{4\pi}C^2$

$A = \pi r^2$

$A = \frac{1}{4\pi}(C)^2$

$\frac{dA}{dt} = \frac{1}{4\pi}2C\frac{dC}{dt}$

$\frac{dA}{dt} = \frac{1}{2\pi}C\frac{dC}{dt}$

$\frac{dA}{dt} = \frac{1}{2\pi}(100\pi)(40) = 2000 \frac{ft^2}{sec}$

$\frac{dA}{dt} =$

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