ANSWERS

AB Calculus – Chapter 4 REVIEW **WS: REVIEW Optimization & Modeling**

Directions: Use your own paper to solve each question. Be sure to justify your conclusion using either the 1st Derivative Test or the 2nd Derivative Test. Refer to example #1 on page 16 of your notes if you need a refresher on the 8 steps of the solution process.

10. An open box of maximum volume is to be made from a square piece of material, 3 feet on a side, by cutting equal squares from each corner and turning up the sides. Find the dimensions of the box which give the maximum volume.

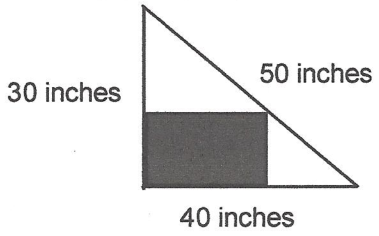
11. Which point(s) on the graph of $y = 4 - x^2$ are closest to the point (0, 2)?

12. A rectangular page is to contain 24 square inches of print. The margins at the top and bottom of the page are each 1.5 inches. The margins on each side are one inch. What should the dimensions of the page be so that the least amount of paper is used?

13. Two posts, one 12 feet high and the other 28 feet high, stand 30 feet apart. They are to be stayed by two wires attached to a single stake running from ground level to the top of each post. Where should the stake be placed to use the least wire?

14. The graphs of $f(x) = \sqrt{x}$, x = 8 and y = 0 bound a region in the first quadrant. Find the dimensions of the rectangle of maximum area that can be inscribed in this region (the sides of the rectangle should be parallel to the axes). Justify using the first derivative number line and the second derivative test.

15. Ms. Fitzhugh wishes to construct a storage box in the corner of her attic. The corner of the attic is triangular shaped, with sides of 30 inches, 40 inches and 50 inches, as shown below. What is the biggest storage box that could be constructed in the attic, if she wants the box to be a rectangle with two sides along the two shorter sides of the attic corner and one corner on the hypotenuse? Justify using the first derivative number line and the second derivative test.



True/False

T F 16. If c is a critical number of f(x), then f(c) is a relative maximum or minimum.

T F 17. If f '(c) does not exist then c is not in the domain of the function f(x).

T F 18. If f is continuous, f '(x)>0 for x<c and f '(x)<0 for x>c, the f(x) has a local maximum of f(c) at x=c.

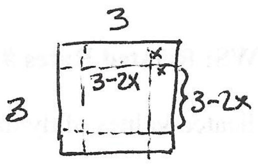
T F 19. The graph of y=1/x is concave down for x<0, concave up for x>0 and thus has point of inflection at x=0.

T F 20. If x=c is a critical point on f(x) and f "(c) =5 then f(c) is a local minimum.

T F 21. An inflection point on the function f(x) occurs where f "(c)=0.

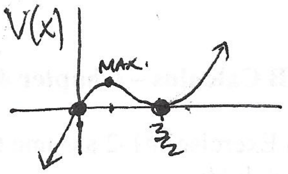
$\boxed{\text{ANSWERS}}$ REVIEW: OPTIMIZATION

⑩.



MAX. Volume $V=(3-2x)(3-2x)(x)$

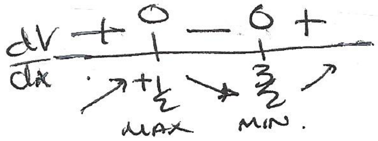
$V=l\cdot w\cdot h$: $V=(3-2x)^2x$



domain $x>0$

x represents height of box.

$$\frac{dV}{dx}=2(3-2x)^1(-2)(x)+(3-2x)^2(1)$$
$$=(3-2x)\left[-4x+3-2x\right]$$
$$=(3-2x)(-6x+3)$$
$$=+3(2x-3)(2x-1)$$
$$x=\tfrac{3}{2}\quad x=\tfrac{1}{2}$$



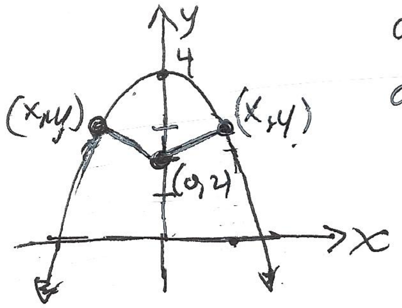
ATQ $\boxed{x=\tfrac{1}{2}}$ maximizes the volume of the open box b/c $\frac{dV}{dx}$ changes signs ⊕ to ⊖. by 1st Derivative test.

$h=\tfrac{1}{2}$ft $l=2$ $w=2$ft

$\boxed{\text{OR}}$ $\frac{d^2V}{dx^2}=24x-24$
$=24(x-1)\Big|_{x=\frac{1}{2}}=-12<0$

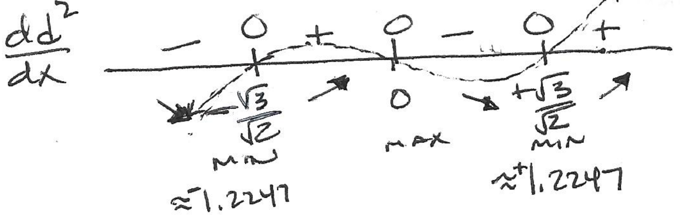
$\therefore$ $\boxed{x=\tfrac{1}{2}}$ maximizes the volume b/c $\frac{d^2V}{dx^2}<0$ & volume is concave down at critical point. by 2nd Deriv Test.

⑪ $y=4-x^2$



MINIMIZE $\underline{d^2}$

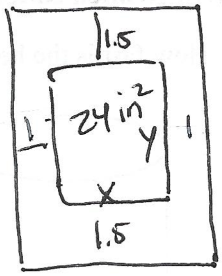
$$d^2=(x-0)^2+(y-2)^2$$
$$d^2=x^2+(4-x^2-2)^2$$
$$d^2=x^2+(2-x^2)^2$$
$$\frac{d(d^2)}{dx}=2x+2(2-x^2)(-2x)$$
$$=2x\left(1-2(2-x^2)\right)$$
$$=2x(2x^2-3)$$
$$=2x(\sqrt{2}x-\sqrt{3})(\sqrt{2}x+\sqrt{3})$$



ATQ: Two possible points on parabola (b/c symmetric graph) $\left(-\frac{\sqrt{3}}{\sqrt{2}},\frac{5}{2}\right)$ $\left(+\frac{\sqrt{3}}{\sqrt{2}},\frac{5}{2}\right)$ that are minimum distance from (0,2) b/c $\frac{dd^2}{dx}$ changes signs from negative to positive at these x-values.

ANSWERS REVIEW OPTIMIZATION

⑫



$(x)(y)=24$

$y=\frac{24}{x}$

$(x+3)(y+2) = \text{Area of Page} \Leftarrow$ Minimize

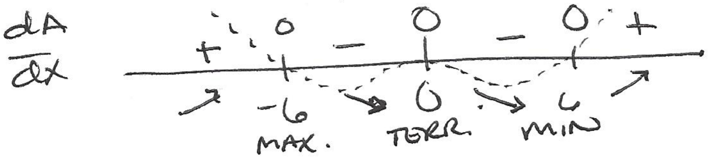
$(x+3)\left(\frac{24}{x}+2\right) = \text{Area}$

$$\frac{dA}{dx} = (x+3)\left(\frac{-24}{x^2}\right) + (1)\left(\frac{24}{x}+2\right)$$

$$\frac{-24(x+3)}{x^2} + \frac{24x+2x^2}{x^2}$$

$$= \frac{\cancel{-24x} - 72 + \cancel{24x} + 2x^2}{x^2}$$

$$= \frac{2x^2-72}{\cancel{x^2}} = \frac{2(x-6)(x+6)}{x^2} = 0 \quad x=\pm 6 \text{ and } x=0$$



ATQ.
Area of page will be minimized when $x=6$ b/c $\frac{dA}{dx}$ changes signs ⊖ to ⊕.

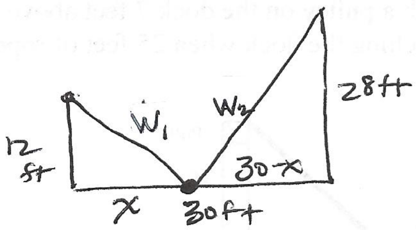
$x=6$ in & $y=4$ in

Page area $= ((6+2)(4+3)$

$=(8)(7)$

$= 56 \text{ in}^2$

⑬



MINIMIZE (W_1+W_2) = total wire

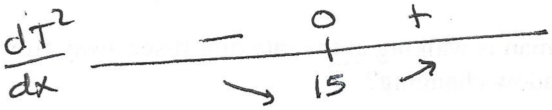
$W_1^2 = x^2+12^2 \qquad W_2^2 = (30-x)^2+28^2$

$T^2 = W_1^2 + W_2^2 = x^2+12^2+(30-x)^2+28^2$

$$\frac{dT^2}{dx} = 2x - 2(30-x)$$

$2x+2x-60 = 4x-60=0$

$x=15$



ATQ: The stake should be place ½ way between each post at $x=15$ ft. to minimize the wire. b/c $\frac{d(T^2)}{dx}$ changes signs ⊖ to ⊕

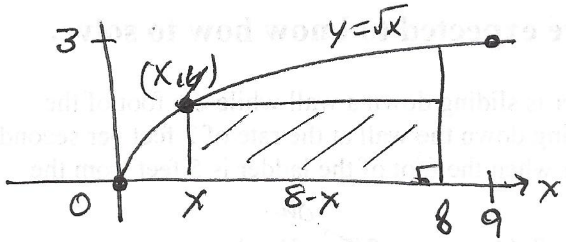
$W_1 = \sqrt{15^2+12^2} = 3\sqrt{41} = 19.2093$

$W_2 = \sqrt{15^2+28^2} = \sqrt{1009} = 31.7647$

50.974 ft of wire.

$\boxed{\text{ANSWERS}}$ REVIEW OPTIMIZATION

⑭



$A = l \cdot h$

$A = (8-x)(\sqrt{x})$

$\frac{dA}{dx} = (-1)\sqrt{x} + (8-x)\left(\frac{1}{2\sqrt{x}}\right)$

$\frac{dA}{dx} = \frac{8-x\ -2x}{2\sqrt{x}} = \frac{8-3x}{2\sqrt{x}} = 0$ und

$x = \frac{8}{3} \quad x = 0$

domain: $0 < x < 8$

$\frac{dA}{dx}$ //// ∅ + 0 −

0 ↗ $\frac{8}{3}$ ↘

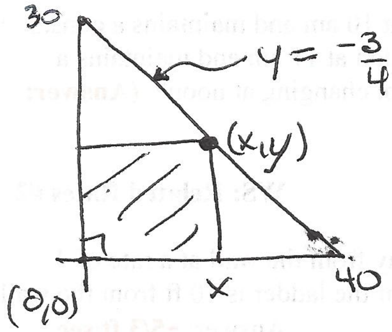
Area of rectangle will be maximized when $x = \frac{8}{3}$ b/c $\frac{dA}{dx}$ changes signs from ⊕ to ⊖

$x = \frac{8}{3}$ length $= 8 - \frac{8}{3} = \frac{16}{3} \approx 5.333$

height $= \sqrt{\frac{8}{3}} \approx \frac{2\sqrt{2}}{\sqrt{3}} = \frac{2\sqrt{6}}{3} \approx 1.6329$

Area ≈ 8.709 unit^2

⑮



$y = -\frac{3}{4}x + 30$

$A = (x)(y) = (x)\left(-\frac{3}{4}x + 30\right)$

$\frac{dA}{dx} = \left(-\frac{3}{4}x + 30\right) + (x)\left(-\frac{3}{4}\right)$

$\frac{dA}{dx} = -\frac{3}{2}x + 30 = 0 \quad x = -30 \cdot -\frac{2}{3} = +20$

$\boxed{x = 20\text{ft} \quad y = 15\text{ ft} \quad A = 300\text{ ft}^2}$

$\frac{dA}{dx}$ + 0 −

↗ 20 ↘

ATQ $\frac{dA}{dx}$ changes signs from ⊕ to ⊖ at $x = 20$ creating a maximum area.

⑯ False — [only if f′ changes signs. May be a terrace point if f′ does not change sign.

⑰ False — [f(x) may have a sharp point or vertical tangent & these points exist when f′ does not.

⑱ True — [f′>0 then f′<0 means f is increasing then decreasing ⟹ MAX.

⑲ False — [There is a change in concavity but since f(0) does not exist there is no inflection point.

⑳ True — [f″=5>0 means f(x) is concave up at a critical point which must be a minimum.

㉑ False — [maybe only if f″ changes signs ⊕ to ⊖ or ⊖ to ⊕ will there be a point of inflection.