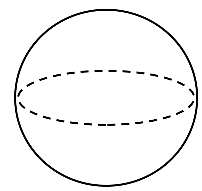
AB Calculus – Chapter 4 REVIEW **WS: REVIEW Related Rates**

Directions: Show all work to complete each problem on your own paper. In Exercises #1-2 assume that x and y are both differentiable functions of t and find the indicated values of dy/dt and dx/dt.

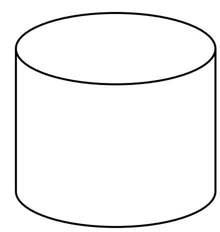
Equation	Find	Given
1. $y = \sqrt{x}$	a. $\frac{dy}{dt}$ when $x = 4$.	$\frac{dx}{dt} = 3$
	b. $\frac{dx}{dt}$ when $x = 25$.	$\frac{dy}{dt} = 2$
2. $y = x^2 - 3x$	a. $\frac{dy}{dt}$ when $x = 3$.	$\frac{dx}{dt} = 2$
	b. $\frac{dx}{dt}$ when $x = 1$.	$\frac{dy}{dt} = 5$

Free Response

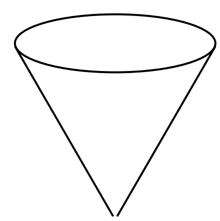
3. A spherical balloon is inflating at a rate of 27π in^3/sec. How fast is the radius of the balloon increasing when the radius is 3 inches? $V = \frac{4}{3}\pi r^3$



4. The voltage, V, in an electrical circuit is related to the current, I, and the resistance, R, by the equation $V = IR$. The current is decreasing at -4 amps/sec as the resistance increases at 20 ohms/sec. How fast is the voltage changing when the voltage is 100 volts and the current is 20 amps?



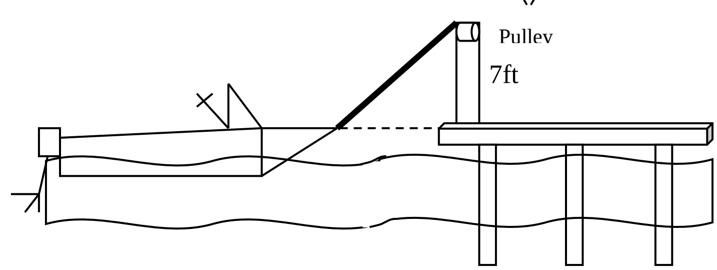
5. A cylindrical tank with a radius of 6 meters is filling with fluid at a rate of 108π m^3/sec. How fast is the height increasing? $V = \pi r^2 h$



6. An inverted conical container has a diameter of 42 in can a depth of 15 inches. If water is flowing out of the vertex of the container at a rate of 35π in^3/sec, how fast is the depth of the water dropping when the height is 5 inches?

$V = \frac{1}{3}\pi r^2 h$

7. A boat is being pulled toward a dock by a rope attached to its bow through a pulley on the dock 7 feet above the bow. If the rope is hauled at a rate of 4 ft/sec, how fast is the boat approaching the dock when 25 feet of rope is out?



8. A 6 foot tall woman is walking at the rate of 4 ft/sec away from a street lamp that is 24 ft tall. How fast is the length of her shadow changing?

9. Oil spilled from a tanker spreads in a circle with circumference increasing at a rate of 40 ft/sec. How fast is the area of the spill increasing when the circumference of the circle is 100π feet? (Hint: Find a formula for the area of a circle in terms of the circumference.)

AB Calculus – Chapter 4 REVIEW **WS: REVIEW Optimization & Modeling**

Directions: Use your own paper to solve each question. Be sure to justify your conclusion using either the 1st Derivative Test or the 2nd Derivative Test. Refer to example #1 on page 16 of your notes if you need a refresher on the 8 steps of the solution process.

10. An open box of maximum volume is to be made from a square piece of material, 3 feet on a side, by cutting equal squares from each corner and turning up the sides. Find the dimensions of the box which give the maximum volume.

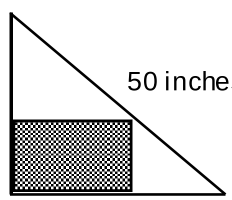
11. Which point(s) on the graph of $y = 4 - x^2$ are closest to the point (0, 2)?

12. A rectangular page is to contain 24 square inches of print. The margins at the top and bottom of the page are each 1.5 inches. The margins on each side are one inch. What should the dimensions of the page be so that the least amount of paper is used?

13. Two posts, one 12 feet high and the other 28 feet high, stand 30 feet apart. They are to be stayed by two wires attached to a single stake running from ground level to the top of each post. Where should the stake be placed to use the least wire?

14. The graphs of $f(x) = \sqrt{x}$, x = 8 and y = 0 bound a region in the first quadrant. Find the dimensions of the rectangle of maximum area that can be inscribed in this region (the sides of the rectangle should be parallel to the axes). Justify using the first derivative number line and the second derivative test.

15. Ms. Fitzhugh wishes to construct a storage box in the corner of her attic. The corner of the attic is triangular shaped, with sides of 30 inches, 40 inches and 50 inches, as shown below. What is the biggest storage box that could be constructed in the attic, if she wants the box to be a rectangle with two sides along the two shorter sides of the attic corner and one corner on the hypotenuse? Justify using the first derivative number line and the second derivative test.



True/False

T F 16. If c is a critical number of f(x), then f(c) is a relative maximum or minimum.
T F 17. If f '(c) does not exist then c is not in the domain of the function f(x).
T F 18. If f is continuous, f '(x)>0 for x<c and f '(x)<0 for x>c, the f(x) has a local maximum of f(c) at x=c.
T F 19. The graph of y=1/x is concave down for x<0, concave up for x>0 and thus has point of inflection at x=0.
T F 20. If x=c is a critical point on f(x) and f "(c) =5 then f(c) is a local minimum.
T F 21. An inflection point on the function f(x) occurs where f "(c)=0.

Answers: 1) a. ¾ b. 20 2) a. 6 b. -5 3) ¼ in/sec 4) 380 volts/sec 5) 3 m/sec 6) -0.714 in/sec 7) -4.166 ft/sec 8) 4/3 ft/sec 9) 2000 ft^2/sec 10) h=1/2 ft, l=2ft, h=2 ft 11) two points: ($\pm 1.5^{1/2}$, 2.5) 12) w=8 in, h= 7 in 13) ½ way between the posts 14) l=16/3 units, h=1.6329 units 15) x=20 ft, y=15 ft, A =300 ft^2
16) F 17) F 18) T 19) F 20) T 21) F