

Derivative of an Inverse Function:

1. Given the function $f(x) = x^3 - 2x + 1$ where $g(x) = f^{-1}(x)$, complete the following table & $f'(x) =$ _____

Evaluate	$f(0) =$	$f(2) =$	$f(-3) =$
Solve	$g(\underline{\quad}) = 0$	$g(\underline{\quad}) = 2$	$g(\underline{\quad}) = -3$
Evaluate	$f'(0) =$	$f'(2) =$	$f'(-3) =$
Evaluate	$g'(1) =$	$g'(5) =$	$g'(-20) =$

** Use the same $f(x)$ to evaluate $g'(116) =$

You will need to use your calculator! Do you know how to use it effectively & efficiently for this type of question?

2. Use the table to find the Derivative of the Inverse Function at each value:

- A) $(f^{-1})'(2) =$ B) $(f^{-1})'(3) =$ C) $(f^{-1})'(0) =$ D) $(f^{-1})'(-3) =$

x	1	2	3	4
$f(x)$	-3	0	2	3
$f'(x)$	5	4/3	-8	1/2

Implicit Differentiation:

3. If $x^2 + xy + y^3 = 0$, then, in terms of x and y , $\frac{dy}{dx} =$

- (A) $-\frac{2x+y}{x+3y^2}$ (B) $-\frac{x+3y^2}{2x+y}$ (C) $\frac{-2x}{1+3y^2}$ (D) $\frac{-2x}{x+3y^2}$ (E) $-\frac{2x+y}{x+3y^2-1}$

4. If $x^2 + xy = 10$, then when $x = 2$, $\frac{dy}{dx} =$

- (A) $-\frac{7}{2}$ (B) -2 (C) $\frac{2}{7}$ (D) $\frac{3}{2}$ (E) $\frac{7}{2}$

5. If $x^2 + y^2 = 25$, what is the value of $\frac{d^2y}{dx^2}$ at the point $(4, 3)$?

- (A) $-\frac{25}{27}$ (B) $-\frac{7}{27}$ (C) $\frac{7}{27}$ (D) $\frac{3}{4}$ (E) $\frac{25}{27}$

6. Write the equation of the tangent line to the curve $y^3 - xy^2 = 4$ at the point where $y = 2$ is

7. Find $\frac{dy}{dx}$ if $\ln(xy) = x + y$

Linear Approximations of a function using Tangent Lines

8. $f(x) = \frac{1}{\sqrt{1-x}}$

- i) Write the tangent line equation: $t(x)$ at $c = -3$
 ii) Approximate $f(-3.1)$ using $t(-3.1) \approx$
 iii) Is $t(-3.1)$ an over/under estimate? Why?
 iv) ** Find the difference $f(-3.1) - t(-3.1)$

9. $f(x) = \ln(1-x)$

- i) Write the tangent line equation: $t(x)$ at $c = 1-e$
 ii) Approximate $f(1.1-e)$ using $t(1.1-e) \approx$
 iii) Is $t(1.1-e)$ an over/under estimate? Why?
 iv) ** Find the difference $f(1.1-e) - t(1.1-e)$

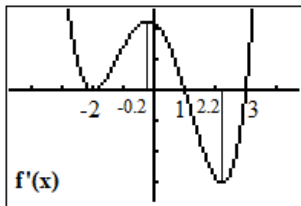
Mean Value Theorem

10. Given $f(x) = \frac{1}{4}x^3 + 1$ over the interval $[0, 2]$, verify the hypotheses of the Mean Value Theorem are satisfied on the given interval and find all values c in that interval that satisfy the conclusion of the theorem.

11. You are driving on a straight highway on which the speed limit is 55mph. At 8:05 AM a police car clocks your velocity at 50 mph and at 8:10 AM a second police car posted 5 miles down the road clocks your velocity at 55 mph. Explain why the police have a right to charge you with a speeding violation. (Anton p306)

1st & 2nd Derivative Tests to identify function behavior

12. Given the graph of the derivative of $f(x)$.



- State the critical values on $f(x)$.
- State x-intervals for which $f(x)$ increasing and decreasing. Justify your conclusion.
- Classify each critical value as a maximum, minimum or neither. Justify your conclusion.
- State x-intervals for which $f(x)$ is concave up and concave down. Justify your conclusion.

13. If the derivative of a function $f(x)$ is $f'(x) = 3(x+2)(x+1)^2(x-3)^3$, find the x-value(s) where a relative maximum occurs. Show the work leading to your answer.

14. Let $f''(x) = -3x^2 + 6x + 1$ and let f have a critical number at $x = 1$. Use the Second Derivative Test to determine if there is a relative max or relative min at the critical value.

Optimization

15. Given $f(x) = 2x^3 - 3x^2 - 12x$, classify all relative and absolute extrema on the interval $[-2, 4]$. Use a signed-pattern number line to organize your thoughts. Create a table and make your conclusion.

16. Given $f(x) = \left(\frac{x-4}{x+3}\right)^2$ Complete the information in the charts on your own paper. Graph without your calculator.

a. $f'(x) = ?$	d. $f''(x) = ?$
b. Critical points	e. Critical points
c. Signed #line	f. Signed #line

State the intervals for which... & include a reason why.	j. Identify inflection point(s)
g. f is increasing & decreasing...	k. vertical asymptote? y-intercept?
h. f is concave up & concave down...	l. $\lim_{x \rightarrow \pm\infty} f(x) = ?$
i. Make table and classify relative & absolute extrema (max/min)	m. Graph $f(x)$

17. Given $f(x) = x^2 e^{-x}$ on the interval $[-1, 4]$. Complete the information on your own paper. Graph without a calculator.

a. Show that $f'(x) = e^{-x}(2x - x^2)$	d. Show that $f''(x) = e^{-x}(x^2 - 4x + 2)$
b. Critical points	e. Critical points
c. Signed #line	f. Signed #line

State the intervals for which... & justify conclusion
g. f is increasing/decreasing
h. f is concave up/down

i. Make an x-y table and classify rel & abs max or min. **
j. Identify inflection points **
k. Graph $f(x)$

18. Given $f(x) = 2 - x^{\frac{2}{3}}$, complete all information in the charts on your own paper. Graph without a calculator.

a. $f'(x) = ?$	d. $f''(x) = ?$
b. Critical points	e. Critical points
c. Signed #line	f. Signed #line

State the intervals for which... & include a reason why.
g. f is increasing & decreasing
h. f is concave up & concave down

i. Make an x-y table and classify ... relative/absolute extrema (max or min)
j. Identify inflection points
k. find zeros ** & Graph $f(x)$

Challenge If $y = x^{1-x}$ we can find the derivative by first taking the natural log of the equation: $\ln(y) = \ln(x^{1-x})$ Use log rules and implicit differentiation to find

$\frac{dy}{dx}$. Choose all answers that are true: (A) $-\ln(x) + \frac{(1-x)}{x}$ (B) $\left[\frac{1-x}{x} - \ln(x)\right]x^{1-x}$ (C) $(1-x)x^{-x} - x^{1-x} \ln(x)$ (D) $x^{-x} - x^{1-x}(1 + \ln x)$

Show work.