Name ANSWER KEY Date ______ Period 2 3 4

Applications of Derivatives: **Horizontal Motion**

Class Example: #1) The position $s(t)$ of a particle in motion along a horizontal line at time $t \geq 0$, is given by the equation $s(t) = t^3 - 9t^2 + 15t + 2$. Let $s(t)$ be measured in **meters** and t measured in **seconds.**

a. Find the velocity $v(t)$ of the particle at any time t.

$v(t) = s'(t) = 3t^2 - 18t + 15$
$= 3(t^2 - 6t + 5)$
$= 3(t-1)(t-5)$

b. Find the acceleration of the particle at any time t.

$a(t) = v'(t) = s''(t) = 6t - 18$
$= 6(t-3)$

c. Find all values of t for which the particle is at rest.

$t = 1, 5$ $\quad$ $t = 0$ INITIAL TIME.

d. Find all values of t for which the acceleration zero.

$t = 3$

e. State the $(t, s(t))$ coordinates for the t-values in part (c) and (d).

t	$s(t) = t^3 - 9t^2 + 15t + 2$	
0	2	
1	9	$1 - 9 + 15 + 2$
3	2	$27 - 81 + 45 + 2$
5	−23	$125 - 225 + 75 + 2$

f-g. Sketch a #line for v(t) & a(t)

$v(t) = s'(t)$: + 0 — 0 + ; → 1 ← 5 →

$a(t) = s''(t)$: — 0 + ; 3

READ!

h. If the **velocity of a particle is positive** then the position of the particle is INCREASING. When we are referring to a particle moving along a horizontal line, if the velocity is positive this means the **particle is moving** to the RIGHT. If the **velocity of a particle is negative** then the position of the particle is DECREASING. When we are referring to a particle moving along a horizontal line, if the velocity is negative this means the **particle is moving** to the LEFT.

i. State the t-intervals for which the particle is moving to the right & give a reason why?

Particle is moving to the right when $v(t) = s'(t) > 0$ on $t \in (0,1), (5, \infty)$

j. State the t-intervals for which the particle is moving to the left & give a reason why?

Particle is moving to the left when $v(t) = s'(t) < 0$ on $t \in (1,5)$.

READ!

k. When the velocity and the acceleration are the **same sign**, either both positive or both negative, then the particle is **speeding up** or accelerating. When the velocity and the acceleration are the **opposite signs**, then the particle is **slowing down** or decelerating.

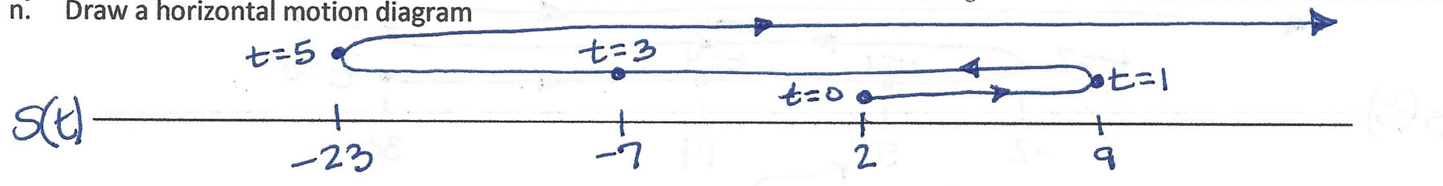
l. State the t-intervals for which the particle is **speeding up** & give a reason why?

when $v(t) > 0$ & $a(t) > 0$ OR when $v(t) < 0$ & $a(t) < 0$ then particle is speeding up on $t \in (1,3)$ $(5, \infty)$.

m. State the t-intervals for which the particle is **slowing down** & give a reason why?

when $v(t) < 0$ & $a(t) > 0$ OR when $v(t) > 0$ & $a(t) < 0$ then the particle is slowing down on $t \in (0,1), (3,5)$.

n. Draw a horizontal motion diagram



o. Find the **total distance traveled** by the particle on the interval $t \in (0,3)$.

$t \in (0,1)$ distance traveled $= 9 - 2 = 7$m
$t \in (1,3)$ distance traveled $= |-7 - 9| = 16$m
∴ total distance traveled on $t \in (0,3)$ is 23 meters.
sec.

p. Find the **displacement** of the particle on the interval $t \in (0,3)$.

displacement = change in position on (0,3)
$= s(3) - s(0) = -7 - 2 = -9$
∴ displacement of the particle on $t \in (0,3)$ is 9 meters to the left.

ANSWER KEY.

#2) The position $s(t)$ of a particle in motion along a horizontal line at time $t \geq 0$, is given by the equation $s(t) = -t^3 + 12t^2 - 36t + 30$. $S(t)$ is measured in **feet** and t is measured in **seconds**.

a. Find the velocity $v(t)$ of the particle at any time t.

$v(t) = -3t^2 + 24t - 36$

$v(t) = -3(t^2 - 8t + 12)$

$-3(t-6)(t-2)$

b. Find the acceleration of the particle at any time t.

$a(t) = -3(2t - 8)$

$= -6(t-4)$

c. Find all values of t for which the particle is instantaneously at rest.

$t = 2, 6$ & $t = 0$ initial

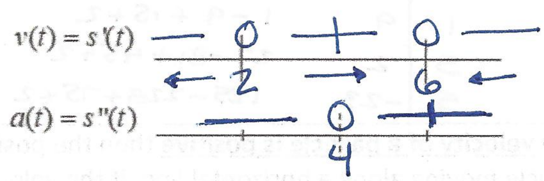
d. Find all values of t for which the acceleration zero.

$t = 4$

e. State the $(t, s(t))$ coordinates for the t-values in part (c) and (d).

t	$s(t) = -t^3 + 12t^2 - 36t + 30$	
0	30	
2	-2	$-8 + 48 - 72 + 30$
4	14	$-64 + 192 - 144 + 30$
6	30	$-216 + 432 - 216 + 30$

f-g. Sketch a #line for v(t) & a(t)



i. State the t-intervals for which the particle is moving forward & give a reason why?

The particle is moving forward on $t \in (2, 6)$ b/c $v(t) = s'(t) > 0$.

j. State the t-intervals for which the particle is moving backward & give a reason why?

The particle is moving backward on $t \in (0, 2), (6, \infty)$ b/c $v(t) = s'(t) < 0$.

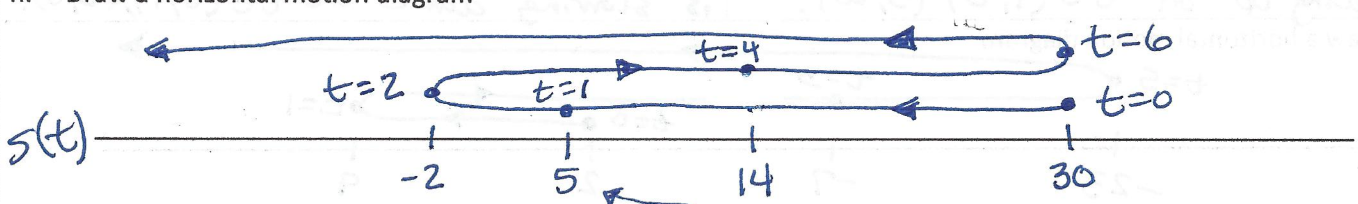
l. State the t-intervals for which the particle is <u>speeding up</u> & give a reason why?

The particle is speeding up when $v(t) > 0$ & $a(t) > 0$ OR $v(t) < 0$ & $a(t) < 0$ which occurs when $t \in (0, 2), (4, 6)$.

m. State the t-intervals for which the particle is <u>slowing down</u> & give a reason why?

The particle is slowing down when $v(t) < 0$ & $a(t) > 0$ OR $v(t) > 0$ & $a(t) < 0$ which occurs when $t \in (2, 4), (6, \infty)$.

n. Draw a horizontal motion diagram



o. Find the <u>total distance traveled</u> by the particle on the interval $t \in (1, 4)$. $s(1) = -1 + 12 - 36 + 30 = 5$

$t \in (1, 2) \rightarrow$ distance $= |-2 - 5| = 7$

$t \in (2, 4) \rightarrow$ distance $= |14 - -2| = 16$

$\therefore$ Total distance traveled on $(1, 4)$ sec.

$= 23$ feet

p. Find the <u>displacement</u> of the particle on the interval $t \in (1, 4)$.

Displacement = change in position

$s(4) - s(1) = 14 - 5 = 9$

$\therefore$ Displacement of the particle on $(1, 4)$ seconds is 9 feet to the right.

2.4.3.5

ANSWER KEY

Horizontal Motion Practice Problems:

3. The position of a particle is defined by $x(t)=\frac{8}{3}t^3-11t^2+15t+4$ where $s(t)$ be measured in **meters**, t in **seconds**.

a. Find the velocity $v(t)$ of the particle at any time t.

$v(t)=s'(t)=8t^2-22t+15$
$=(2t-3)(4t-5)$

b. Find the acceleration of the particle at any time t.

$a(t)=v'(t)=s''(t)=16t-22$
$=2(8t-11)$

c. Find all values of t for which the particle is instantaneously at rest.

$t=\frac{3}{2},\frac{5}{4}$ $\qquad$ $t=0$

INITIAL TIME

d. Find all values of t for which the acceleration zero.

$t=\frac{11}{8}$ $\quad$ 1.375

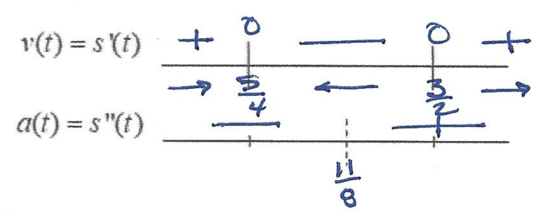
e. State the $(t,s(t))$ coordinates for the t-values in part (c) and (d).

Home Screen

$Y_1(\{0,\frac{5}{4},\frac{11}{8},\frac{3}{2}\})$

t	$s(t)=Y_1$
0	4
$\frac{5}{4}$	10.771
$\frac{11}{8}$	10.760
$\frac{3}{2}$	10.750

f-g. Sketch a #line for v(t) & a(t)



i. State the t-intervals for which the particle is moving forward & give a reason why?

Particle moves forward on
$t\in(0,\frac{5}{4}),(\frac{3}{2},\infty)$
b/c $v(t)=s'(t)>0$.

j. State the t-intervals for which the particle is moving backward & give a reason why?

Particle moves backward
on $t\in(\frac{5}{4},\frac{3}{2})$
b/c $v(t)=s'(t)<0$

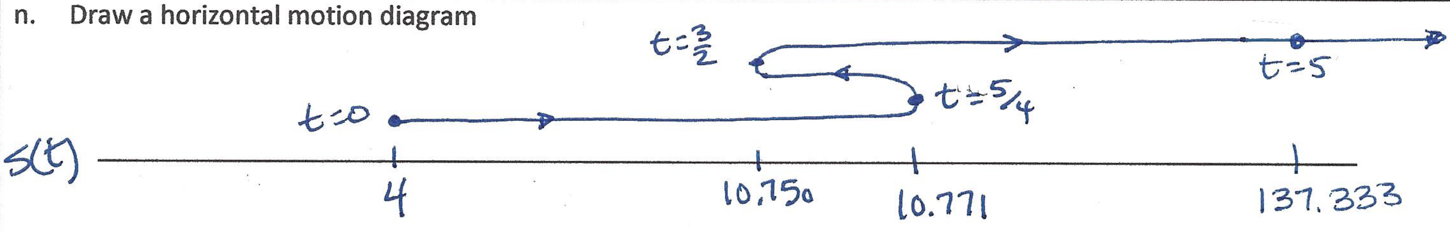
l. State the t-intervals for which the particle is <u>**speeding up**</u> & give a reason why?

Particle speeds up on
$t\in(\frac{5}{4},\frac{11}{8}),(\frac{3}{2},\infty)$
b/c $v(t)$ & $a(t)$ have same signs.

m. State the t-intervals for which the particle is <u>**slowing down**</u> & give a reason why?

Particle slows down on
$t\in(0,\frac{5}{4})(\frac{11}{8},\frac{3}{2})$
b/c $v(t)$ & $a(t)$ have opposite signs.

n. Draw a horizontal motion diagram



o. Find the <u>**total distance traveled**</u> by the particle on the interval $t\in(0,5)$. $s(5)=137.333$

distance $t\in(0,\frac{5}{4})=10.771-4=6.771$
distance $t\in(\frac{5}{4},\frac{3}{2})=|10.750-10.771|=0.021$
distance $t\in(\frac{3}{2},5)=137.333-10.750=126.503$
$\therefore$ Total distance $t\in(0,5)=133.295$ meters

p. Find the <u>**displacement**</u> of the particle on the interval $t\in(0,5)$.

displacement $=\Delta s(t)$
$=s(5)-s(0)$
$=137.333-4$
$=133.333$ meters to the Right

4. A particle moves along a horizontal line in such a way that its position at time t is given by $x(t) = t^3 - 12t^2 + 36t - 10$ where x is measured in feet and t in seconds.

a) Find the velocity and acceleration of the particle.

b) Create a first and second derivative number line to help you justify your answers to the questions below.

$v(t) = s'(t)$

$a(t) = s''(t)$

c) When is the particle moving forward (to the right)?

d) When is the particle moving backward (to the left)?

e) When is the acceleration positive?

f) When is the particle speeding up?

g) When is the particle slowing down?

$s(t)$

h) Draw a motion diagram and label it appropriately.

i) Find the **total distance traveled** and the **displacement** of the particle on the interval $t \in (0,5)$

j) Find the maximum velocity of the particle on the interval $t \in (0,5)$.

k) Find the minimum acceleration of the particle on the interval $t \in (0,5)$.

5. A particle is moving on the x-axis. For $t \geq 0$ the particle's position is given by $x(t) = 2t^3 - 13t^2 + 22t - 2$ meters where t is in seconds. Find the intervals when the particle:

a) is moving right,

b) is moving left,

c) has positive acceleration

d) has negative acceleration,

e) speeding up and

f) slowing down,

g) Find the total distance traveled on $t \in (0,4)$

h) Find the displacement on $t \in (0,4)$

6. The position of a particle along a horizontal number line at time t is given by the function $x(t) = -t^2 + 6t - 8$.

a) What is the largest time interval for which x is an increasing function? In which direction is the motion during this time?

b) At what time(s) does the particle change direction? State why you know.

c) On what time interval is the particle slowing down? State why you know.

7. Two particles are moving along a coordinate line. At the end of t seconds their distances from the origin, in feet, are given by $x_1 = 4t - 3t^2$ and $x_2 = t^2 - 2t$, respectively.

a) When do they have the same velocity?

b) If the speed of a particle is the absolute value of its velocity, then when do the two particles have the same speed?

c) When do they have the same position?

PARTICLE MOTION

#4 $x(t) = t^3 - 12t^2 + 36t - 10$

a) $x'(t) = v(t) = 3t^2 - 24t + 36$
$= 3(t^2 - 8t + 12)$
$= 3(t-6)(t-2)$

$x''(t) = a(t) = 6t - 24$
$= 6(t-4)$

b) $v(t) = 0 \rightarrow t = 2, 6$

$a(t) = 0 \rightarrow t = 4$

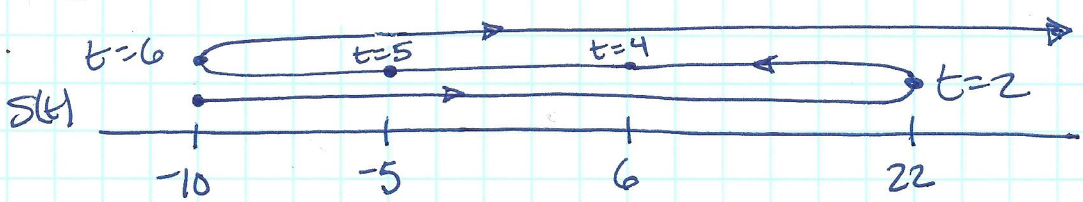
$v(t)$	+	2	—	6	+

$a(t)$	—	4	+

c) Particle moves forward on $t \in (0,2), (6,\infty)$ b/c $v(t) > 0$.
d) Particle moves backward on $t \in (2,6)$ b/c $v(t) < 0$.
e) Acceleration is positive on $t \in (4,\infty)$ b/c $a(t) > 0$.
f) Particle is speeding up on $(2,4)\ (6,\infty)$ b/c $a(t)$ & $v(t)$ have the same signs.
g) Particle is slowing down on $(0,2)\ (4,6)$ b/c $a(t)$ & $v(t)$ have opposite signs.

h)

t	$s(t)$
0	-10
2	22
4	6
6	-10
5	-5



i) Total distance on $(0,2) = 22 - {}^-10 = 32$ feet
$(2,5) = |-5 - 22| = 27$ feet
Total distance $= \boxed{59 \text{ feet}}$

Displacement $= s(5) - s(0)$
$= -5 - -10$
$= 5$ feet to the right.

j) maximum velocity on $(0,5)$

use $a(t)$ # line $a(t)$ — 4 +

Max velocity occurs either endpoint $t=0$ or $t=5$ b/c when $t=4$ velocity is at a minimum. $v(0) = 36$ $v(5) = -9$ (?)

$\therefore V(0) = 36$ max velocity occurs at $t=0$

k) minimum acceleration on $(0,5)$
occurs at $t=0$
when $a(0) = -24$

$a(t)$ is an increasing linear function so left endpoint will be minimum acceleration.

Particle Motion

(5) $x(t) = 2t^3 - 13t^2 + 22t - 2$ meters.

$v(t) = x'(t) = 6t^2 - 26t + 22$
$= 2(3t^2 - 13t + 11)$

$a(t) = x''(t) = 2(6t - 13)$

$t = \frac{13}{6}$

$t = \frac{13 \pm \sqrt{169 - 132}}{6} = \frac{13 \pm \sqrt{37}}{6}$

$t = \frac{13 + \sqrt{37}}{6} = 3.180 \xrightarrow{\text{STO}} B$

$t = \frac{13 - \sqrt{37}}{6} = 1.153 \xrightarrow{\text{STO}} A$

$v(t)$: $+$ | 0 at 1.153 | $-$ | 0 at 3.180 | $+$

$a(t)$: $-$ | 0 at 2.167 | $+$

a) particle is moving right $t \in (0, 1.153)\ (3.180, \infty)$ b/c $v(t) > 0$
b) left $t \in (1.153, 3.180)$ b/c $v(t) < 0$.
c) $a(t) > 0$ $t \in (2.167, \infty)$
d) $a(t) < 0$ $t \in (0, 2.167)$
e) speeding up on $t \in (1.153, 2.167)\ (3.180, \infty)$ b/c $v(t)$ & $a(t)$ have same signs.
slowing down on $t \in (0, 1.153)\ (2.167, 3.180)$ b/c $v(t)$ & $a(t)$ have opposite signs.
g) total distance traveled
$t \in (0, 4)$
d_1 on $t \in (0, 1.153) \Rightarrow (9.149 - -2) = 11.149$ m.
d_2 on $t \in (1.153, 3.180) \Rightarrow |0.814 - 9.149| = 8.336$ m
d_3 on $t \in (3.180, 4) \Rightarrow (6 - 0.814) = 5.186$ m
TOTAL DIST $= 24.671$ m

t	$s(t)$
0	-2
1.153	9.149
3.180	0.814
4	6

h) displacement $= s(4) - s(0)$
$= 6 - -2$
$= 8$ meters to the right.

(6) $x(t) = -t^2 + 6t - 8$

$v(t)$: $+$ | 3 | $-$

$v(t) = -2t + 6$
$-2(t - 3)$

$a(t) = -2$

$a(t)$: $-$

a) $x(t)$ is increasing from $t \in (0, 3)$ b/c $v(t) > 0$
$x(t)$ is moving to the right b/c position increases when $v(t) > 0$.
b) $x(t)$ changes direction at $t = 3$ b/c $v(t) = 0$ & changes signs $+$ to $-$
c) $x(t)$ slows down on $t \in (0, 3)$ b/c $v(t) > 0$ & $a(t) > 0$.

(7) Two particles.

$x_1(t) = 4t - 3t^2$ $\qquad$ $x_2(t) = t^2 - 2t$

a) $v_1(t) = 4 - 6t$ $\qquad$ $v_2(t) = 2t - 2$

$$4 - 6t = 2t - 2$$
$$6 = 8t$$
$$t = \frac{3}{4} \text{ seconds}$$

at time $t = 3/4$ seconds the two particles have the same velocity.

b) speed of a particle is the absolute value of velocity

$|4-6t| = |2t-2| \longrightarrow |2-3t| = |t-1|$

$(4-6t)^2 = (2t-2)^2$ $\qquad$ $(2-3t)^2 = (t-1)^2$

$$4 - 12t + 9t^2 = t^2 - 2t + 1$$
$$8t^2 - 10t + 3 = 0$$
$$(4t-3)(2t-1) = 0$$
$$t = \frac{3}{4} \quad t = \frac{1}{2}$$

Check if $t = \frac{3}{4}$ $\quad v_1\left(\frac{3}{4}\right) = 4 - 6\left(\frac{3}{4}\right) = 4 - \frac{9}{2} = -.5$ ✓

$v_2\left(\frac{3}{4}\right) = 2\left(\frac{3}{4}\right) - 2 = \frac{3}{2} - 2 = -.5$ ✓

if $t = \frac{1}{2}$ $\quad v_1\left(\frac{1}{2}\right) = 4 - 6\left(\frac{1}{2}\right) = 4 - 3 = -1$ ✓

$v_2\left(\frac{1}{2}\right) = 2\left(\frac{1}{2}\right) - 2 = 1 - 2 = -1$ ✓

c) the particles have the same position when $x_1(t) = x_2(t)$

$$4t - 3t^2 = t^2 - 2t$$
$$4t^2 - 6t = 0$$
$$2t(2t-3) = 0$$
$$t = 0 \quad t = \frac{3}{2}$$

If $t = 0$ $\quad x_1(0) = 0$; $x_2(0) = 0$ ✓

If $t = \frac{3}{2}$ $\quad x_1\left(\frac{3}{2}\right) = 4\left(\frac{3}{2}\right) - 3\left(\frac{3}{2}\right)^2 = 6 - \frac{27}{4} = -\frac{3}{4}$ ✓

$x_2\left(\frac{3}{2}\right) = \left(\frac{3}{2}\right)^2 - 2\left(\frac{3}{2}\right) = \frac{9}{4} - 3 = -\frac{3}{4}$ ✓