Name_____Date_____Period 2 3 4

Applications of Derivatives: **Horizontal Motion**

Class Example: #1) The position $s(t)$ of a particle in motion along a horizontal line at time $t \geq 0$, is given by the equation $s(t) = t^3 - 9t^2 + 15t + 2$. Let $s(t)$ be measured in **meters** and t measured in **seconds**.

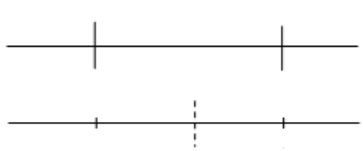
a. Find the velocity $v(t)$ of the particle at any time t.	b. Find the acceleration of the particle at any time t.
c. Find all values of t for which the particle is at rest.	d. Find all values of t for which the acceleration zero.
e. State the $(t, s(t))$ coordinates for the t-values in part (c) and (d).	f-g. Sketch a #line for v(t) & a(t) $v(t) = s'(t)$ $a(t) = s''(t)$
h. If the **velocity of a particle is positive** then the position of the particle is_____. When we are referring to a particle moving along a horizontal line, if the velocity is positive this means the **particle is moving** _____. If the **velocity of a particle is negative** then the position of the particle is_____. When we are referring to a particle moving along a horizontal line, if the velocity is negative this means the **particle is moving** _____.	
i. State the t-intervals for which the particle is moving to the right & give a reason why?	j. State the t-intervals for which the particle is moving to the left & give a reason why?
k. When the velocity and the acceleration are the **same sign**, either both positive or both negative, then the particle is **speeding up** or accelerating. When the velocity and the acceleration are the **opposite signs**, then the particle is **slowing down** or decelerating.	
l. State the t-intervals for which the particle is **speeding up** & give a reason why?	m. State the t-intervals for which the particle is **slowing down** & give a reason why?
n. Draw a horizontal motion diagram _____	
o. Find the **total distance traveled** by the particle on the interval $t \in (0,3)$.	p. Find the **displacement** of the particle on the interval $t \in (0,3)$.

#2) The position $s(t)$ of a particle in motion along a horizontal line at time $t \geq 0$, is given by the equation $s(t) = -t^3 + 12t^2 - 36t + 30$. $S(t)$ is measured in **feet** and t is measured in **seconds**.

a. Find the velocity $v(t)$ of the particle at any time t.	b. Find the acceleration of the particle at any time t.
c. Find all values of t for which the particle is instantaneously at rest.	d. Find all values of t for which the acceleration zero.
e. State the $(t, s(t))$ coordinates for the t-values in part (c) and (d).	f-g. Sketch a #line for v(t) & a(t) $v(t) = s'(t)$ $a(t) = s''(t)$
i. State the t-intervals for which the particle is moving forward & give a reason why?	j. State the t-intervals for which the particle is moving backward & give a reason why?
l. State the t-intervals for which the particle is **speeding up** & give a reason why?	m. State the t-intervals for which the particle is **slowing down** & give a reason why?
n. Draw a horizontal motion diagram _____	
o. Find the **total distance traveled** by the particle on the interval $t \in (1, 4)$.	p. Find the **displacement** of the particle on the interval $t \in (1, 4)$.

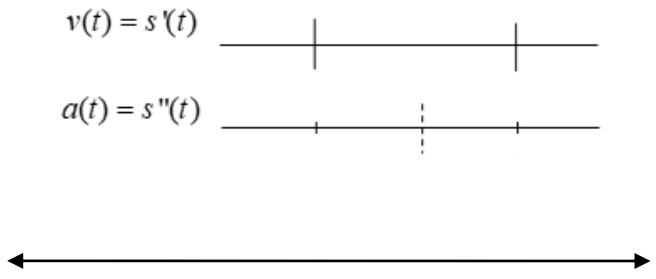
Horizontal Motion Practice Problems:

3. The position of a particle is defined by $x(t)=\frac{8}{3}t^3-11t^2+15t+4$ where $s(t)$ be measured in **meters**, t in **seconds**.

a. Find the velocity $v(t)$ of the particle at any time t.	b. Find the acceleration of the particle at any time t.
c. Find all values of t for which the particle is instantaneously at rest.	d. Find all values of t for which the acceleration zero.
e. State the $(t, s(t))$ coordinates for the t-values in part (c) and (d).	f-g. Sketch a #line for v(t) & a(t) $v(t)=s'(t)$ $a(t)=s''(t)$ 
i. State the t-intervals for which the particle is moving forward & give a reason why?	j. State the t-intervals for which the particle is moving backward & give a reason why?
l. State the t-intervals for which the particle is **speeding up** & give a reason why?	m. State the t-intervals for which the particle is **slowing down** & give a reason why?
n. Draw a horizontal motion diagram _____	
o. Find the **total distance traveled** by the particle on the interval $t\in(0,5)$.	p. Find the **displacement** of the particle on the interval $t\in(0,5)$.

4. A particle moves along a horizontal line in such a way that its position at time t is given by $x(t) = t^3 - 12t^2 + 36t - 10$ where x is measured in feet and t in seconds.

a) Find the velocity and acceleration of the particle.
b) Create a first and second derivative number line to help you justify your answers to the questions below.
c) When is the particle moving forward (to the right)?
d) When is the particle moving backward (to the left)?
e) When is the acceleration positive?
f) When is the particle speeding up?
g) When is the particle slowing down?
h) Draw a motion diagram and label it appropriately.



i) Find the **<u>total distance traveled</u>** and the **<u>displacement</u>** of the particle on the interval $t \in (0,5)$

j) Find the maximum velocity of the particle on the interval $t \in (0,5)$.

k) Find the minimum acceleration of the particle on the interval $t \in (0,5)$.

5. A particle is moving on the x-axis. For $t \geq 0$ the particle's position is given by $x(t) = 2t^3 - 13t^2 + 22t - 2$ meters where t is in seconds. Find the intervals when the particle:

a) is moving right,
b) is moving left,
c) has positive acceleration
d) has negative acceleration,
e) speeding up and
f) slowing down,
g) Find the total distance traveled on $t \in (0,4)$
h) Find the displacement on $t \in (0,4)$

6. The position of a particle along a horizontal number line at time t is given by the function $x(t) = -t^2 + 6t - 8$.

a) What is the largest time interval for which x is an increasing function? In which direction is the motion during this time?
b) At what time(s) does the particle change direction? State why you know.
c) On what time interval is the particle slowing down? State why you know.

7. Two particles are moving along a coordinate line. At the end of t seconds their distances from the origin, in feet, are given by $x_1 = 4t - 3t^2$ and $x_2 = t^2 - 2t$, respectively.

a) When do they have the same velocity?
b) If the speed of a particle is the absolute value of its velocity, then when do the two particles have the same speed?
c) When do they have the same position?