§3.1 Derivatives of Powers and Polynomials -- Student Notes

The Power Rule If n is any real number, then: $\frac{d}{dx}\left(x^n\right) = nx^{n-1}$

1. Differentiate: a) $f(x) = \frac{1}{x^2}$ b) $y = \sqrt[5]{x^3}$

2. Find an equation of the tangent line to the curve $y = x\sqrt{x}$ at the point $(1, 1)$. Illustrate by graphing the curve and its tangent line.

The Constant Multiple Rule If c is a constant and f is a differentiable function, then

$$\frac{d}{dx}\left[cf(x)\right] = c\frac{d}{dx}f(x)$$

The Sum/Difference Rule If f and g are both differentiable, then:

$$\frac{d}{dx}\left[f(x) \pm g(x)\right] = \frac{d}{dx}f(x) \pm \frac{d}{dx}g(x)$$

3. $\frac{d}{dx}\left(x^8 - 8x^5 + 2x^4 + 10x^2 - 6x + 5\right)$

4. Find the points on the curve $y = x^4 - 10x^2 + 2$ where the tangent line is horizontal.

Section 3.1 Practice A. Find the derivatives of each function. For #8-16 first re-write f(x) as the sum or difference of x-expressions raised to a real power. Do not leave negative exponents in your final answer.

1. $y = 0$

2. $y = -4x + \pi$

3. $y = 1.2x^2 - ex$

4. $y = \frac{2}{3}x$

5. $y = 5x^2 - 4x + 9\pi^3$

6. $y = x^\pi + x^e + e^\pi$

6. $y = 2x^{\frac{1}{4}}$

7. $y = \sqrt{x}$

8-16: Re-write f(x) before differentiating!

8. $y = \sqrt[3]{x} + \sqrt[3]{x^2} + \sqrt[3]{x^4}$

9. $y = \sqrt[4]{x} + \sqrt[4]{x^3} + \sqrt[4]{x^5}$

10. $y = \frac{1}{x}$

11. $y = \frac{1}{x^2} + \frac{1}{x^3}$

12. $y = \frac{1}{\sqrt{x}}$

13. $y = \frac{1}{\sqrt[3]{x}}$

14. $y = 4t^2 - \frac{5}{\sqrt{t}} + \frac{1}{t^3}$

15. $y = \frac{5t^4 - 3t^3 - 8t^2 + t}{t^3}$

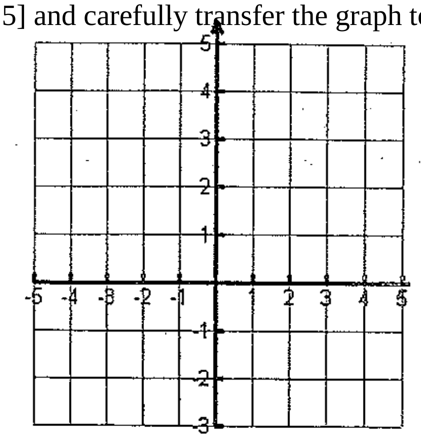
16. $y = \sqrt{x}\left(4x^3 - 5x^2 + 7\right)$

17. Find $\frac{dy}{dx}$ if $y = \frac{x^3}{a} + bx^2 - cx$

18. Find $\frac{dw}{dq}$ if $w = 3ab^2q^3$

§3.2 The Exponential Function – Student Notes

1. Use your calculator to graph $f(x) = 2^x$ with a window of [-5, 5] x [-3, 5] and carefully transfer the graph to the grid at the right.



a. Find the domain of $f(x)$.____________________

b. Find the range of $f(x)$.______________________

c. Where is $f(x)$ increasing?____________________

d. Where is $f(x)$ decreasing?____________________

e. Describe the concavity of $f(x)$.____

f. On the same axes sketch $f'(x)$ using a different color. You can use your calculator by typing Y2=nderiv (Y1,X, X). *(nderiv is under "MATH"-"8" Y1 is under "VARS" "Y-VARS"-"FUNC")*

g. Find the domain of $f'(x)$: _________________ range:________________________

h. Where is $f'(x)$ increasing?__________________ decreasing?____________________

i. Describe the concavity of $f'(x)$.____

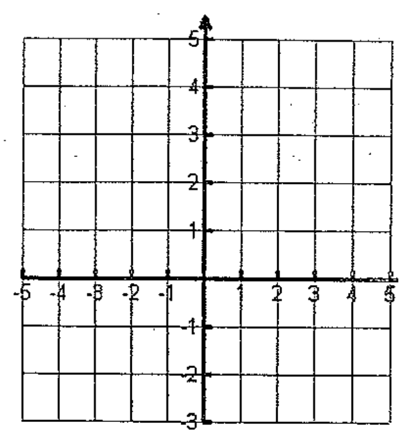
j. What are the *y*-intercepts of each?____

k. How do the two graphs differ? ____

l. Estimate, to the best of your ability, the equation of $f'(x)$. (Try various numerical values until your graph of $f(x)$ matches the graph of $f'(x)$.)

$f'(x)$ = ___________________________________

2. Use your calculator to graph $f(x) = 3^x$ with a window of [-5, 5] x [-3, 5] and carefully transfer the graph to the grid at the right.
On the same axes sketch $f'(x)$ using a different color.
Can use the calculator again to find $f'(x)$



Estimate, to the best of your ability, the equation of $f'(x)$.

$f'(x)$ = ____________________________

3. You should have discovered that for exponential functions, the *derivative is proportional to the original function; i.e.,* $f'(x) = kf(x)$. Note that the constant of proportionality for $f(x) = 2^x$ is less than one and that the

constant of proportionality for $f(x) = 3^x$ is greater than one. Therefore, if $f(x) = a^x$, then for some value of *a* between 2 and 3, the constant of proportionality equals one. That means if $f(x) = a^x$, then for some *a*, $f(x) = f'(x)$. What value of *a* has the property that $f(x) = f'(x)$?

$f(x)$ = _____

But, in general, if $f(x) = a^x$, then $f'(x)$ = ____

Exponential Practice Find $f'(x)$ for each of the following functions.

1. $f(x) = 4^x$
2. $f(x) = e^x$
3. $f(x) = 6^x$
4. $f(x) = 8^x$
5. $f(x) = x^4$
6. $f(x) = \pi^x$
7. $f(x) = 7^x$
8. $f(x) = 9^x$
9. $f(x) = x^e$
10. $f(x) = 2 \cdot 3^x$
11. $f(x) = 2^x + x^2$
12. $f(x) = 4^x - 3^x$
13. $f(x) = e^x + 2x^3$
14. $f(x) = 2^{x+3}$
15. $f(x) = e^{x+\pi}$

§3.3 Product and Quotient Rules -- Student Notes

The Product Rule If *f* and *g* are both differentiable, then:

$$\frac{d}{dx}\left[f(x)g(x)\right] = f'(x)g(x) + f(x)g'(x)$$

Example 1: $h(x) = \left(x^3+1\right)\left(2^x\right)$

$$h'(x) = (\qquad)\left(2^x\right) + \left(x^3+1\right)(\qquad\qquad)$$

$$= \left(2^x\right)[\qquad\qquad\qquad\qquad]$$

1. If $h(x) = xe^x$, find $h'(x)$.

2. Use two different methods to differentiate the function: $h(t) = \sqrt{t}\,(1-t)$.

The Quotient Rule If *f* and *g* are both differentiable, then:

$$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{f'(x)g(x) - f(x)g'(x)}{\left[g(x)\right]^2}$$

Example 2: $h(x) = \dfrac{x^2+1}{x^3-5}$

$$h'(x) = \frac{(\qquad)\left(x^3-5\right) - (x^2+1)(\qquad)}{(\qquad)^2}$$

$$= \frac{(\qquad\qquad) - (\qquad\qquad)}{\left(x^3-5\right)^2}$$

$$= \frac{\qquad\qquad\qquad}{\left(x^3-5\right)^2}$$

3. Let $y = \dfrac{x^2+x-2}{x^3+6}$, then find $\dfrac{dy}{dx}$.

4. Find an equation of the tangent line to the curve $y = \frac{e^x}{x}$ at the point $(1, e)$.

5. Given $f(x) = \frac{3x^2 + 2\sqrt{x}}{x}$, find $\frac{d}{dx}[f(x)]$.

Hint: re-write f(x) before taking the derivative.

Practice. Find each of the following derivatives:

6. $\frac{d}{dx}\left[\left(x^3 - 2x + 1\right)\left(x^4 + x - 3\right)\right]$

7. $\frac{d}{dx}\left(\frac{x^2 + 1}{x^2 - 1}\right)$

8. $\frac{d}{dx}\left(\frac{2x + 1}{3^x}\right)$

9. $\frac{d}{dx}\left(2^x \cdot e^x\right)$

10. $\frac{d}{dx}\left(\sqrt{x}\, e^x\right)$

11. $\frac{d}{dx}\left(\frac{e^x}{1 - 5x}\right)$

§3.4 The Chain Rule – Student Notes

The Chain Rule. If f and g are both differentiable and $F = f \circ g$ is the composite function defined by $F(x) = f\left(g(x)\right)$, then F is differentiable and F' is given by the product

$$F'(x) = f'\left(g(x)\right)g'(x)$$

1. Find $F'(x)$ if $F(x) = \sqrt{5x^2 + 3}$

2. Differentiate $y = \dfrac{1}{\left(1-3x^2\right)^3}$

3. Differentiate $y = \left(x^5 - 1\right)^{1000}$

4. Find $f'(x)$ if $f(x) = \dfrac{1}{\sqrt[3]{x^2 + x + 1}}$

5. Differentiate $g(t) = \left(\dfrac{t-7}{2t+1}\right)^5$

6. Differentiate $y = \left(2x+1\right)^5\left(x^3 - x + 1\right)^4$

For #7-10: Write the equation of the tangent line at the x-value where you evaluated the derivative. Is this tangent line above or below the curve of the function? How do you know?

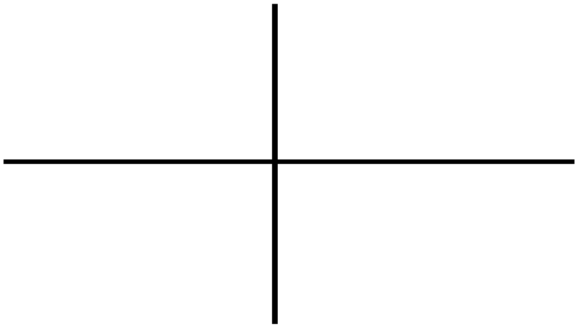
7. Differentiate $y = e^{x^2}$ and evaluate $f'(-4)$

8. If $f(x) = 3^{2x-1}$, then $f'(2)$ = ?

9. If $y = e^{\left(2x^3-3x+4\right)}$, then $y'(-1)$ = ?

10. If $f(x) = 5^{2x}$, then $f'(3)$ = ?

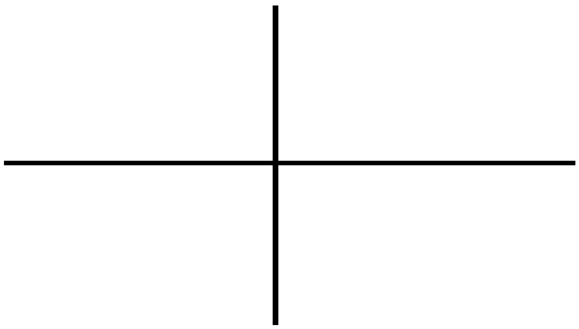
§3.5 The Trigonometric Functions and Their Derivative -- Student Notes



1. Quickly sketch a labeled graph of $y = \sin x$ over $[-2\pi, 2\pi]$

2. Sketch a derivative of $y = \sin x$ in a different color.

3. What do you think is $\dfrac{d(\sin x)}{dx} = ?$



1. Quickly sketch a labeled graph of $y = \cos x$ over $[-2\pi, 2\pi]$

2. Sketch a derivative of $y = \cos x$ in a different color.

3. What do you think is $\dfrac{d(\cos x)}{dx} = ?$

4. Use the quotient rule to find $\dfrac{d(\tan x)}{dx}$

5. Use the quotient rule to find $\dfrac{d(\sec x)}{dx}$

Differentiate:

a) $y = 2\sin(3\theta)$

b) $y = \cos^2 x$

c) $y = \cos(x^2)$

d) $y = e^{-\sin t}$

e) $y = 2\tan(3t)$

f) $y = \tan(1-\theta)$

g) $y = \cos(x)\sin(x)$

h) $y = e^x \sin x$

i) $y = \sin(3x) + \cos(2x)$

§3.1 - §3.5 Applying the Derivative Rules using Tables

The purpose of this worksheet is to abstract the concept of the derivative rules by causing you to apply them to functions that you do not know. Two functions, $f(x)$ and $g(x)$, have the values and first derivatives shown in the table. Use this information to find the following.

x	$f(x)$	$g(x)$	$f'(x)$	$g'(x)$
-4	2	-2	-1	1
-3	1	-1	-2	2
-2	-2	1	0	3
-1	-1	4	2	1
0	0	5	1	0
1	2	3	0	-2
2	3	2	1	-1
3	3	1	-1	-3
4	1	-1	-2	-4

1. $h(x) = f(x) - g(x)$
   Find $h'(2)$

2. $h(x) = f(x) + 3g(x)$
   Find $h'(0)$

3. $h(x) = 2f(x) - 4g(x)$
   Find $h'(-3)$

4. $h(x) = 2f(x) - 1$
   Find $h'(3)$

5. $h(x) = 3g(x) - x^2$
   Find $h'(1)$

6. $h(x) = x\ f(x)$
   Find $h'(-1)$

7. $h(x) = [\ f(x)\ ]^2$
   Find $h'(-3)$

8. $h(x) = f(x)g(x)$
   Find $h'(2)$

9. $h(x) = x^2 f(x)g(x)$
   Find $h'(-1)$

10. $h(x) = f(x)/g(x)$
    Find $h'(-2)$

11. $h(x) = f(3x)$
    Find $h'(-1)$

12. $h(x) = g\left(x^2\right)$
    Find $h'(-2)$

13. $h(x) = f\left(x^3 - x\right)$
    Find $h'(1)$

14. $h(x) = f(g(x))$
    Find $h'(4)$

15. $h(x) = g(f(x))$
    Find $h'(-3)$

16. $h(x) = [\ f(x)\ ]^3\ g(-2x)$
    Find $h'(2)$

17. $h(x) = x^2 / f(x)$
    Find $h'(-1)$

18. $h(x) = f(\ln\ x)/g(2x+1)$
    Find $h'(1)$

	ON YOUR PAPER	***ON***

CALCULATOR

1 Given the function $f(x)$, **show the calculation** necessary to find the:

a) average rate of change, average velocity,
or slope of the secant on the interval $x \in [4.3, 5.6]$ ....>

b) instantaneous rate of change, instantaneous velocity,
or slope of the tangent line at $x = 4.95$ ....>

2 The height of a projectile propelled from a platform 120 feet in the air with an initial velocity of 96 ft/sec is given by the function $h(t) = -\frac{1}{2}a_0t^2 + v_0t + h_0$. Note: Earth's gravitational constant is 32 ft/sec^2.

Write the equation for $h(t)$ =____________________________ and **show the calculation** necessary to find the:

a) average rate of change, average velocity, or slope of the secant on each of the intervals

Interval	$t \in [0,1]$	$t \in [1,2]$	$t \in [2,3]$	$t \in [3.012, 5.789]$	$t \in [4.218, 6.357]$
Algebraic Expression in terms of h(t)					

Since function is defined, write the expression for the slope of secant and evaluate the expression on the calculator. Record 3-decimal accuracy.

Evaluation (3-decimal accuracy)					

b) instantaneous rate of change, instantaneous velocity,
or slope of the tangent line at $t = 3.724$ seconds..........>

c) Examine the values for the first three intervals what do they tell you about the behavior of the function. You should be able to conclude two specific ideas.

3 Given the table of values

$x_{(sec)}$	0	1	2	3	4	5	6	7
$f(x)_{(meters)}$	120	200	248	264	248	200	120	8

show the calculation necessary to find the:

ON PAPER: must pull values from the table and use in calculation

a) average rate of change, average velocity,
or slope of the secant on the interval $x \in [4, 6]$ ....>

b) instantaneous rate of change, instantaneous velocity,
or slope of the tangent line at $x = 5$ ....>

Using appropriate **MATHEMATICAL NOTATION** to write what is required to justify Continuity & Differentiability.

4 Definition of Continuity in 3 parts.

5 Definition of Differentibility.