UNIT 3 DERIVATIVE RULES
REVIEW MULTIPLE CHOICE

① $\frac{d}{dx}\left(\frac{1}{x^3} - \frac{1}{x} + x^2\right)\Big|_{x=-1}$

$\frac{d}{dx}\left(x^{-3} - x^{-1} + x^2\right) = -3x^{-4} + x^{-2} + 2x$

$\left(\frac{-3}{x^4} + \frac{1}{x^2} + 2x\right)\Big|_{x=-1} = -3 + 1 - 2 = -4$ B

B

② $f(x) = \frac{1}{2}x^2$ $\qquad 2x - 4y = 3$

$\frac{dy}{dx} = x = \frac{1}{2}$ $\qquad m = \frac{1}{2}$

$x = \frac{1}{2}$ $f\left(\frac{1}{2}\right) = \frac{1}{8}$ $\therefore$ @ $\left(\frac{1}{2}, \frac{1}{8}\right)$ tangent line to $f(x)$ is parallel to $2x - 4y = 3$ B

B

③ $f(x) = x^4 + 2x^2$ $\qquad f'(x) = 1 = 4x^3 + 4x$

$f'(x) = 4x^3 + 4x$ $\qquad$ * Solve using graphing calculator

2nd calc intersect.

$x = 0.2367329$ $\rightarrow$ $f(0.236\ldots) = 0.1152256937$

* For best accuracy use value in calculator do not truncate at intermediate step. using $x = 0.236$ & $y = 0.115$

$\Downarrow$

tangent line to $f(x)$ @ $x = 0.236$

$y = 1\,(x - 0.236) + .115$

$y = 1x - 0.1215072101$

$y = 1x - 0.122$

D D

④ f & g differentiable functions

i) $g(x) > 0$ for all x

ii) $f(0) = 1$

$h(x) = f(x) \cdot g(x)$

$h'(x) = f'(x) \cdot g(x) + f(x) \cdot g'(x)$

$h'(0) = \underbrace{f'(0) \cdot g(0)} + f(0) \cdot g'(0)$

$h'(0) = \quad 0 \quad + (1)\, g'(0)$

$\underbrace{f'(0) = 0}$ or $g(0) = 0$

$h'(0) = f(0) \cdot g'(0)$

$h'(0) = 1 \cdot g'(0)$

$h'(0) = g'(0)$

E

TRY EACH OPTION:

A) If $f(x) = f'(x)$ then $f(x) = e^x$ but $f'(0) = 0$ False

B) If $f(x) = g(x)$ then $h'(x) = 2[f(x)] \cdot f'(x)$ False or $2[g(x)]\, g'(x)$. False

C) If $f(x) = e^x$ then $h'(x) = e^x g(x) + e^x \cdot g'(x) = e^x\left(g(x) + g'(x)\right)$ False

D) If $f(x) = 0$ then $h'(x) = 0$ FALSE

E) If $f(x) = 1$ then $h'(x) = 0 \cdot g(x) + 1\, g'(x)$ & $h'(x) = f(x) \cdot g'(x)$ TRUE

UNIT 3 Review MC

5) $f(x) = \dfrac{2x+3}{3x-2}$

I noticed that $f(x)$ & $f^{-1}(x)$ are equivalent

Not relevant to solution...

inverse $f^{-1}(x)$

$x = \dfrac{2y+3}{3y-2}$

$3xy - 2x = 2y+3$

$y(3x-2) = 2x+3$

$y = \dfrac{2x+3}{3x-2}$

Yep.

$f(1) = 5$

$f'(x) = \dfrac{(3x-2)(2) - (2x+3)(3)}{(3x-2)^2}$

$f'(x) = \dfrac{\cancel{6x} - 4 - \cancel{6x} - 9}{(3x-2)^2} = \dfrac{-13}{(3x-2)^2}\Big|_{x=1} = \dfrac{-13}{1}$

$y = -13(x-1) + 5$

$y = -13x + 13 + 5$

B $\quad 13x + y = 18$

6) $f(x) = \dfrac{e^{2x}}{2x} \quad f'(x)$

$f'(x) = \dfrac{4xe^{2x} - 2e^{2x}}{4x^2} = \dfrac{\cancel{2}e^{2x}(2x-1)}{\cancel{4}_{2}x^2}$

E $\quad f'(x) = \dfrac{e^{2x}(2x-1)}{2x^2}$

7) $f(x) = (x-1)^2 \cdot \sin(x) \qquad f'(x) = 2(x-1)' \cdot \sin x + \cos x\,(x-1)^2$

$f'(x) = (x-1)\left[2\sin x + (x-1)\cos x\right]$

$f'(0) = (-1)\left[\cancel{2(0)} + (-1)(1)\right]$

D $\quad f'(0) = 1$

8) $f(x) = x + \cos x \qquad (0, 1)$

$f'(x) = 1 - \sin x \qquad y = 1(x-0)+1$

$f'(0) = 1 - 0 = 1 \qquad y = 1x + 1$

B

UNIT 3 REVIEW M/C

9

$f(x) = (x^2-2x-1)^{2/3}$

$f'(x) = \frac{2}{3}(x^2-2x-1)^{-1/3}(2x-2)$

$f'(x) = \left.\frac{4(x-1)}{3(x^2-2x-1)^{1/3}}\right|_{x=0} = \frac{-4}{3(-1)} = \frac{4}{3}$

A

10 $\frac{d}{dx}\left(\cos^2(x^3)\right) = \frac{d}{dx}\left[\cos(x^3)\right]^2 = 2\cos^1(x^3)\cdot(-\sin(x^3))\cdot 3x^2$

$= -6x^2\cos(x^3)\cdot\sin(x^3)$

D

OR $= -3x^2\sin(2x^3)$ using $\sin(2\theta) = 2\sin\theta\cos\theta$

13 $y = \arctan(\cos x)$

$\frac{dy}{dx} = \frac{1}{1+(\cos^2 x)}\cdot -\sin x = \frac{-\sin(x)}{1+\cos^2(x)}$

A

14 $y = \cos^2 x - \sin^2 x = \cos(2x)$

$\frac{dy}{dx} = 2\cos x(-\sin x) - 2\sin x\cdot\cos x$ $\boxed{OR}$ $\frac{d}{dx}(\cos(2x))$

$= -4\cos x\sin x$ $\qquad = -\sin(2x)\cdot 2$

$= -2(2\cos x\sin x)$ $\qquad = -2\sin(2x)$

$= -2\sin(2x)$

C

15 $y = 2\cos\left(\frac{x}{2}\right)$

$\frac{dy}{dx} = 2\left(-\sin\left(\frac{x}{2}\right)\cdot\frac{1}{2}\right)$

$\frac{dy}{dx} = -\sin\left(\frac{x}{2}\right)$

2nd derivative:

$\frac{d^2y}{dx^2} = -\cos\left(\frac{x}{2}\right)\cdot\frac{1}{2}$

$= -\frac{1}{2}\cos\left(\frac{x}{2}\right)$

E

UNIT 3 REVIEW MC

(17) $f(x) = \ln(x^2)$ at $x = e^2$

$f(e^2) = \ln(e^4) = 4 \quad (e^2, 4)$

$f'(x) = \frac{1}{x^2} \cdot 2x = \frac{2}{x}\Big|_{x=e^2} = \frac{2}{e^2}$

Slope is all you need STOP.

QUESTION DID NOT ASK FOR LINE:

TANGENT LINE

$y = \frac{2}{e^2}(x - e^2) + 4$

$y = \frac{2}{e^2}x - 2 + 4$

$y = \frac{2}{e^2} + 2$

(B)

UNIT 3 REVIEW FRQ

(1) $y = \frac{2-x}{3x+1}$

$\frac{dy}{dx} = \frac{(-1)(3x+1) - (2-x)(3)}{(3x+1)^2} = \frac{\cancel{-3x} - 1 - 6 + \cancel{3x}}{(3x+1)^2}$

$\frac{dy}{dx} = \frac{-7}{(3x+1)^2}$

(2) $y = \sin^3(1-2x) = [\sin(1-2x)]^3$

3 layers: outer, middle, inner

So 2 chain rules...

$\frac{dy}{dx} = 3[\sin(1-2x)]^2 \cdot [\cos(1-2x)](-2)$

$\frac{dy}{dx} = -6 \cdot \sin^2(1-2x) \cdot \cos(1-2x)$

(3) $y = 2\sin(x) + \cos(2x)$

$\frac{dy}{dx} = 2\cos(x) - 2\sin(2x)$ ✓

$= 2[\cos(x) - \sin(2x)]$ OR

$= 2[\cos x - 2\sin x \cos x]$ OR

$= 2\cos x[1 - 2\sin x]$ OR

UNIT 3 REVIEW FRQ.

(4) $y = x^2 - 4x$ y-intercept $(0,0)$

$\frac{dy}{dx} = 2x - 4 \Big|_{x=0} = -4$ Tangent Line:
$y = -4x$

(5) $s(t) = t^3 - 6t^2 + 12t - 8$

$v(t) = s'(t) = 3t^2 - 12t + 12$
$v(t) = 0$ $3(t^2 - 4t + 4)$
$3(t-2)(t-2) = 0$
$t = 2$
particle is at rest when $v(t) = 0$ @ $t = 2$

(6) $s(t)$ is increasing when $v(t) > 0$
$v(t) > 0$
$3(t-2)^2 > 0$
for all $t \in (-\infty, 2), (2, \infty)$

(7) $a(t) = v'(t) = 6t - 12$
$= 6(t-2) > 0$
$a(t) > 0$ when $t > 2$.

(8) $f(x) = 16\sqrt{x} = 16x^{1/2}$

$f'(x) = 8x^{-1/2}$

$f''(x) = -4x^{-3/2}$

$f'''(x) = 6x^{-5/2} = \frac{6}{x^{5/2}} \Big|_{x=4} = \frac{6}{32} = \frac{3}{16}$

(9) $y = \ln(\sec x + \tan x)$
$\frac{dy}{dx} = \frac{1}{(\sec x + \tan x)} \cdot (\sec x \tan x + \sec^2 x)$
$\frac{dy}{dx} = \frac{\sec x \cancel{(\tan x + \sec x)}}{\cancel{(\sec x + \tan x)}} = \sec(x)$

UNIT 3 REVIEW FRQ.

(11) $\lim_{x \to 0} \frac{\sin x}{2x} = \frac{1}{2} \lim_{x \to 0} \frac{\sin x}{x} = \frac{1}{2}(1) = \frac{1}{2}$

(12) $h(x) = f(x) \cdot g(x)$

$h'(x) = f'(x) \cdot g(x) + f(x) \cdot g'(x)$

$h'(2) = (3)(1) + (5)(-2)$

$= 3 - 10$

$= -7$