CH11 (MC) REVIEW.

① Eliminate B, D, E b/c sln curves look like $y = -x^2$
check slopes when $x=1$ when $x=2$

$\boxed{A}$

A) $\frac{dy}{dx} = -1$ A) $\frac{dy}{dx} = -2$

C) $\frac{dy}{dx} = -2$ C) $\frac{dy}{dx} = -4$

too steep. $\longrightarrow \boxed{A}$

② Looks like quiz question... Solution curves look like ellipse rather than circle. Eliminate A, B, C.

Check for * zero slopes at $x=0$

* vertical line, undefined slopes at $y=0$
* check point (1,1) (1,2)

steeper less steep. $\rightarrow \boxed{E}$

$\boxed{E}$

③ no undefined slopes so eliminate D & E.
slopes are + & − eliminate C

check $\frac{dy}{dx} = x - y \Big|_{(1,0)} = 1$ $\boxed{A}$

$\boxed{A}$

$\left.\begin{matrix}(1,0)\\(2,1)\\(3,2)\end{matrix}\right\}$ along the line $y = x-1$ slopes are 1.

④ undefined slopes @ $y=0$ } choose C or E
zero slopes @ $x=0$ }

$\boxed{C}$ positive slopes in QI $\longrightarrow \boxed{C}$

⑤ $\frac{dy}{dx} = \frac{-2}{(x+1)^3}$ $dy = -2\int \frac{1}{(x+1)^3}dx = -2\int (x+1)^{-3}dx$

$y = \frac{\cancel{-2}}{\cancel{-2}}(x+1)^{-2} + c$

$\boxed{B}$ $y = \frac{1}{(x+1)^2} + c$ $\boxed{B}$

⑥ $e^{x-y}\frac{dy}{dx} = -1$

$\frac{e^x}{e^y}dy = -1\,dx$

$\boxed{B}$ $\frac{dy}{e^y} = \frac{-1}{e^x}dx$

$\int e^{-y}dy = \int -e^{-x}dx$

$-e^{-y} = e^{-x} + c$

$-e^{-x} - e^{-y} = C$

$e^{-x} + e^{-y} = (-C) = C \longrightarrow \boxed{B}$

CH 11 REVIEW (MC)

(7)

$$\frac{dy}{dx} = \frac{1}{3}x^{-2/3}$$

$$\int dy = \int \frac{1}{3}x^{-2/3}\,dx$$

C

$$y = x^{1/3} + c \Big|_{(-8,0)} \rightarrow 0 = -2 + c \rightarrow y = x^{1/3} + 2$$

$$c = 2 \qquad y = \sqrt[3]{x} + 2 \quad \boxed{C}$$

(8)

$$x\frac{dy}{dx} = y - 2$$

$$\int \frac{dy}{y-2} = \int \frac{dx}{x}$$

$$\ln|y-2| = \ln|x| + c$$

B

$$y - 2 = x \cdot e^{c}$$

$$y = Cx + 2 \Big|_{(3,8)} \longrightarrow 8 = 3C + 2 \rightarrow y = 2x + 2 \quad \boxed{B}$$

$$6 = 3C$$

$$C = 2$$

(9)

$$y = P_0(2)^{x/3}$$

$$y = (1.2)(2)^{x/3}$$

$$y = (1.2)\,e^{\frac{\ln(2)}{3}t} \qquad k = \frac{1}{3}\ln 2 = .231049$$

B

$$y = (1.2)\,e^{.231049t} \quad \boxed{B}$$

(10)

$$f(x) = \frac{1}{x} \qquad f^{-1}(x) = \frac{1}{x} = h(x)$$

$$y = \frac{1}{x} \longrightarrow \text{inverse } x = \frac{1}{y}$$

$$\therefore y = \frac{1}{x} \qquad h'(x) = \frac{-1}{x^2}$$

B

$$h'(3) = \frac{-1}{9} \rightarrow \boxed{B}$$

(11)

$$P = 2000e^{kt}$$

$$1995 \rightarrow (-3, 1700)$$

$$1998 \rightarrow (0, 2000)$$

$$1700 = 2000e^{-3t}$$

$$\frac{17}{20} = e^{-3t}$$

$$P = 2000e^{-\frac{1}{3}\ln\left(\frac{17}{20}\right)(t)}$$

$$P(7) = 2000e^{-\frac{7}{3}\ln\left(\frac{17}{20}\right)}$$

$$\approx 2922$$

A

$$t = -\frac{1}{3}\ln\left(\frac{17}{20}\right) = \frac{1}{3}\ln\left(\frac{20}{17}\right) \qquad \boxed{A}$$

$\cancel{(2000, \underline{?})}$ 2000 yr $\Rightarrow (7, \underline{?})$

(12)

$$b = \left(\frac{1}{2}\right)^{t/50}$$

$$\left(\frac{1}{2}\right)^{\frac{60}{50}} = \left(\frac{1}{2}\right)^{6/5} \approx 0.435275$$

D

43.527% $\boxed{D}$

CH 11 (MC) REVIEW

(13) $f(x) = 2x^3 - 3x$

$h'(b) = \dfrac{1}{f'(a)}$

$h'(-1) = \dfrac{1}{f'(a)} = ?$

(a, b) on $f(x)$ $\longrightarrow$ (b, a) on inverse $h(x) = f^{-1}(x)$

$(\_, -1)$ solve $\quad (-1, \_)$

$f(x) = -1$

$x = -1.366015$

$\boxed{x = 1} \longrightarrow (1, -1) \therefore (-1, 1)$

on $f(x)$ on $h(x)$

$x = 0.3660254$

C

$f'(x) = 6x^2 - 3$

$f'(1) = 6(1)^2 - 3 = 3 \longrightarrow h'(-1) = \dfrac{1}{f'(1)} = \dfrac{1}{3}$ C

~~(14) $v(t) = (100 - 20t)$ ft/sec~~

~~$20(5-t)$~~

~~$s(t) = 20\int_0^1 (5-t)$~~

(14) $v(t) = (100 - 20t) = 20(5 - t)$ ft/sec

$20\int_0^1 (5 - t)\,dt$

$20\left(5t - \dfrac{t^2}{2}\Big|_0^1\right.$

B

$20\left(5 - \tfrac{1}{2}\right) = 20\left(\tfrac{9}{2}\right) = 90$ ft B

(17) $\dfrac{dP}{dt} = kP$

$P = P_0(2)^{x/50}$

$\dfrac{P_{75}}{P_0} = 2^{\frac{75}{50}}$

$= 2^{\frac{3}{2}}$

$= (\sqrt{2})^3$

$= 2\sqrt{2}$ D

(15) $y\,dy = x\,dx$

$\frac{1}{2}y^2 = \frac{1}{2}x^2 + c$

A

$y^2 = x^2 + c$

$\boxed{y^2 - x^2 = c \text{ or } x^2 - y^2 = -c}$ A

(18) $\dfrac{dP}{dt} = kP$

$P_0\left(\frac{1}{4}\right)^{\frac{x}{2}}$

$P_0\left(\frac{1}{2}\right)^x = e^{kx}$

A

$\frac{1}{2} = e^k$

$\boxed{k = \ln\left(\tfrac{1}{2}\right) \quad k = -\ln(2)}$ A

(16) $x\,dy = y\,dx$

$\dfrac{dy}{y} = \dfrac{dx}{x}$

E

$\ln|y| = \ln|x| + c$

$y = Cx + 0 \quad \therefore$ E

(19) $\dfrac{dy}{dt} = -.11(y - 68)$

$\displaystyle\int \frac{dy}{y - 68} = \int -.11\,dt$

$\ln|y - 68| = -.11t + c$

$y - 68 = Ce^{-.11t}$

$y = Ce^{-.11t} + 68 \Big|_{(0,180)} \longrightarrow y = 112e^{-.11t} + 68$

$180 - 68$

$112 = Ce^{-.11(0)}$

$C = 112$

$\longrightarrow y(.10) = 105.281$

C

(20) $\frac{dT}{dt} = -k(T-10)$ $\boxed{A}$

↓ SOLVE

(21) $\int \frac{dT}{T-10} = \int -k\,dt$

$\ln|T-10| = -kt + c$

$T - 10 = Ce^{-kt}$

$T = Ce^{-kt} + 10$

$(0, 32)$

$C = 22$

$T = 22e^{-kt} + 10$

$(1, 27)$

$\frac{17}{22} = e^{-k}$

$-k = \ln\left(\frac{17}{22}\right)$

$T = 22e^{\ln(17/22)t} + 10$

$T = 22e^{-.257829t} + 10$ $\boxed{C}$

$\boxed{C}$

(22) $\frac{dy}{dx} = \frac{2x}{y}$

$\int y\,dy = \int 2x\,dx$

$\frac{1}{2}y^2 = x^2 + c$

$y^2 = 2x^2 + c$

$y = \pm\sqrt{2x^2 + c}\ \Big|_{(0,4)}$

$4 = \pm\sqrt{0 + c}$

$c = 16$

$y = +\sqrt{2x^2 + 16}$ $\boxed{D}$

$\boxed{D}$

(23) NOT (A), NOT (B)

$\boxed{C}$ & $\boxed{D}$

$\left.\begin{array}{l} g(y)\,dy = f(x)\,dx \\ \frac{dy}{g(y)} = f(x)\,dx \end{array}\right\}$ separable.

(24) $\frac{dy}{dx} = \sqrt{1 + \sin(t^2)}$

$x(2) = 3 + \int_0^2 \sqrt{1 + \sin(t^2)}\,dt$

$\boxed{E}$ $x(2) = 5.333$

(25) $\frac{dy}{dx} = -4y$

$\int \frac{dy}{y} = \int 4\,dx$

$\ln|y| = -4x + c$

$y = Ce^{-4x}\ \Big|_{(0,6)}$

$y = 6e^{-4x}$

$\boxed{E}$

(26) $\int \cos y\,dy = \int x\,dx$

$\sin y = \frac{1}{2}x^2 + c\ \Big|_{(1,0)} \rightarrow 0 = \frac{1}{2} + c \quad c = -\frac{1}{2}$

$y = \arcsin\left(\frac{1}{2}x^2 + c\right)$

$y = \arcsin\left(\frac{1}{2}x^2 - \frac{1}{2}\right) = \arcsin\left(\frac{x^2 - 1}{2}\right)$ $\boxed{A}$

$\boxed{A}$

(27) $\int \frac{dP}{P} = \int k\,dt$

$y = Ce^{kt}$

$\frac{500 = Ce^{2k}}{1500 = Ce^{6k}}$

$\frac{1}{3} = e^{-4k}$

$\rightarrow k = -\frac{1}{4}\ln\left(\frac{1}{3}\right) = \frac{1}{4}\ln(3)$

$y = Ce^{\frac{t\ln 3}{4}}$

$e^{\frac{\ln 3}{4}t} = \frac{1}{2}$

$t = \frac{4\ln\left(\frac{1}{2}\right)}{\ln 3} = \frac{-4\ln(2)}{\ln 3}$

$t = 2.523$ $\boxed{A}$

$\boxed{A}$

(28) $\frac{dy}{dx} = \frac{\sin x}{x}$ $y(1) = 4$ $y(2) = ?$

$y(2) = 4 + \int_1^2 \frac{\sin x}{x}\,dx$

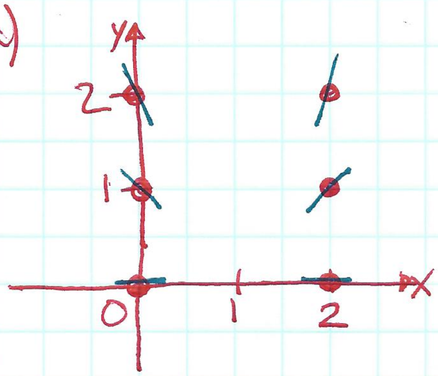
$y(2) = 4.659$

$\boxed{D}$

CH 11 FRQ REVIEW

#1 $\dfrac{dy}{dx} = \dfrac{y^2}{x-1}$

a)



c) $\displaystyle\int \frac{dy}{y^2} = \int \frac{dx}{x-1}$

$-\dfrac{1}{y} = \ln|x-1| + c \Big|_{(2,3)}$

$-\dfrac{1}{3} = \ln(1) + c \quad \therefore c = -\dfrac{1}{3}$

$-\dfrac{1}{y} = \ln|x-1| - \dfrac{1}{3}$

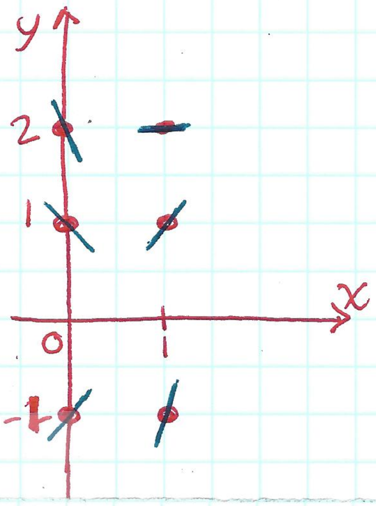
$-\dfrac{1}{y} = \dfrac{3\ln|x-1| - 1}{3} \quad \therefore \boxed{y = \dfrac{-3}{3\ln|x-1| - 1}}$ c)

b) TANGENT LINE at $(2,3)$ $\quad \dfrac{dy}{dx} = \dfrac{9}{1} = 9$

$\boxed{y = 9(x-2) + 3}$

#2 $\dfrac{dy}{dx} = 2x - y$

a)



b) $\dfrac{d^2y}{dx^2}$: $\quad \dfrac{d^2y}{dx^2} = 2 - \dfrac{dy}{dx}$

$= 2 - (2x - y)$

$\dfrac{d^2y}{dx^2} = 2 - 2x + y$

QII $x < 0$ & $y > 0$ $\therefore$ $2 - 2(-) + (+)$

$2 + 2(+) + (+) > 0$

$\dfrac{d^2y}{dx^2} > 0$ for all (x,y) in QII $\therefore$

all solution curves in QII are concave up.

c) $\dfrac{dy}{dx}\Big|_{(2,3)} = 2(2) - 3 = 1$

The point $(2,3)$ is not a critical point.

$\dfrac{d^2y}{dx^2}\Big|_{(2,3)} = 2 - 2(2) + 3 = 1 > 0$

The particular solution is concave up at $(2,3)$ but since $(2,3)$ is not a critical point max or min cannot be determined so $(2,3)$ is NEITHER a max nor min.

d) $\dfrac{dy}{dx} = 2x - y$ is not separable.

Where did this question come from?

Blah