Name____Date_______________Period________

LIMIT PROPERTIES & Algebra Techniques for Finding Limits Activity: Finding Limits Analytically

Evaluate each of the following limits.

1. $\lim_{x\to 3} 5 =$ ______ 2. $\lim_{x\to 3} x =$ ______ 3. $\lim_{x\to 0}(-6) =$ ______ 4. $\lim_{x\to -0.5} x =$ ______

Use your results to complete the following:

5. For the constant function: $f(x) = A,\ \lim_{x\to c} f(x) = \lim_{x\to c} A =$ ______ , where c is any number.

6. For the identity function: $f(x) = x,\ \lim_{x\to c} f(x) = \lim_{x\to c} x =$ ______ , where c is any number.

Evaluate each of the following limits.

7. $\lim_{x\to 3} x =$ ______ 8. $\lim_{x\to 3} 6 =$ ______ 9. $\lim_{x\to 3}(x+6) =$ ______

10. $\lim_{x\to -5} 7 =$ ______ 11. $\lim_{x\to -5} x =$ ______ 12. $\lim_{x\to -5}(7-x) =$ ______

13. $\lim_{x\to 3}(6x) =$ ______ 14. $\lim_{x\to -5}(7x) =$ ______ 15. $\lim_{x\to 3}\left(\frac{x+6}{6x}\right) =$ ______

16. $\lim_{x\to -5}\left[(7x)\right]^2 =$ ______ 17. $\left[\lim_{x\to -5}(7x)\right]^2 =$ ______

18. $\lim_{x\to 3}\sqrt{x+6} =$ ______ 19. $\sqrt{\lim_{x\to 3} x+6} =$ ______

20. $\lim_{x\to 2}\left[3x^4 - x^3 + 2x^2 + x - 1\right] =$ ______

Use your results from above to complete the following:

21. $\lim_{x\to c}\left[f(x)+g(x)\right] =$ ____________________________

In words, the limit of the sum of two functions equals ______________________________________.

22. $\lim_{x\to c}\left[f(x)-g(x)\right] =$ ____________________________

In words, the limit of the difference of two functions equals __________________________________.

23. $\lim_{x\to c}\left[f(x)\bullet g(x)\right] =$ ____________________________

In words, the limit of the product of two functions equals ____________________________________.

24. If $\lim_{x\to c} f(x)$ is known and if $n \geq 2$ is a positive integer, then

$\lim_{x\to c}\left[f(x)\right]^n =$ _____________________ and $\lim_{x\to c}\sqrt[n]{f(x)} =$ ________________________.

In words, the limit of the power a function equals ____.

25. $\lim_{x\to c}\left[\frac{f(x)}{g(x)}\right] =$ ____________________

In words, the limit of the quotient of two functions equals ___________________________________.

26. If P is a polynomial function then, $\lim_{x\to c} P(x) =$ ____________________ .

Finding Limits Analytically: <u>Techniques:</u>

1. **<u>Substitution Theorem:</u>** For a polynomial or a rational function, $\lim_{x \to c} f(x) = f(c)$, if the denominator of the rational function is not zero. If substitution yields $\frac{0}{0}$, the following may be needed in order to find the limit.
2. **<u>Factor Numerator and/or Denominator</u>**
3. **<u>Rationalize the Numerator or Denominator</u>**
4. **<u>Combine Fractions</u>**

<u>Examples:</u>

1. $\lim_{x \to 3} \frac{x^4 + 3x^3 - 13x^2 - 27x + 36}{x^2 + 3x - 4} =$

2. $\lim_{x \to 2} \frac{\frac{1}{x} - \frac{1}{2}}{x - 2} =$

3. $\lim_{x \to 3} \frac{x^3 - 27}{x - 3} =$

4. $\lim_{x \to 0} \frac{\sqrt{x+3} - \sqrt{3}}{x} =$

5. $\lim_{x \to 49} \frac{x - 49}{\sqrt{x} - 7} =$

<u>Exercises</u>

Find each of the limits analytically.

1. $\lim_{x \to 0} \frac{2 - \sqrt{4+x}}{x}$

2. $\lim_{x \to 2} \frac{1 - \frac{2}{x}}{x^2 - 4}$

For Exercises 3-8, find the limits if $\lim_{x \to c} f(x) = 2$ **and** $\lim_{x \to c} g(x) = -3$

3. $\lim_{x \to c} \sqrt{2f(x) - 4g(x)}$	4. $\lim_{x \to c} \left[f(x) + 1 \right]^3$	5. $\lim_{x \to c} \frac{2f(x) + 3g(x)}{g(x) - f(x)}$
6. $\lim_{x \to c} \left[f(x) \bullet 2g(x) \right]$	7. $\lim_{x \to c} \left[f(x) \bullet (g(x) + 3) \right]$	8. $\lim_{x \to c} \frac{f(x)^2}{1 - g(x)}$

AB Calculus LIMITS No calculator!

Find the limits if they exist. Show work in the space provided. Write your final answer in the right column…HERE!

1	$\lim_{x\to 3}\sqrt{9-x^2}$	
2 *	$\lim_{x\to -2}\left(\frac{x^3+8}{x+2}\right)$	
3	$\lim_{x\to 0}\left(\frac{\frac{1}{x+3}-\frac{1}{3}}{x}\right)$	
4 *	$\lim_{x\to 27}\left(\frac{\sqrt[3]{x}-3}{x-27}\right)$	
5	$\lim_{x\to 0}\left(\frac{1}{x^3}\right)$	
6	$\lim_{x\to 10}\frac{\sqrt{x}-10}{x-100}$	
7 *	$\lim_{x\to \infty}\left(\frac{4x+3}{\sqrt{16x^2-2}}\right)$ $\lim_{x\to -\infty}\left(\frac{4x+3}{\sqrt{16x^2-2}}\right)$	
8	$\lim_{x\to 3}\left(\frac{2x^2-3x-9}{x-3}\right)$	
9	$\lim_{x\to 3}\left(\frac{2x^2-3x-8}{x-3}\right)$	
10	$\lim_{x\to 3}\left(\frac{2x^2-3x-9}{x-5}\right)$	
11	$\lim_{x\to 1}\left(\frac{1-\sqrt{4x^2-3}}{x-1}\right)$	
12	$\lim_{x\to -5}\left(\frac{\lvert x+1\rvert}{x+1}\right)$	
13	$\lim_{x\to -1}\left(\frac{\lvert x+1\rvert}{x+1}\right)$	

#14-16 Find all vertical and horizontal asymptotes. Vertical Horizontal/Oblique

14. $f(x) = \dfrac{5x^2 - 3x + 2}{36 - 21x^2 + 3x^4}$ ____________________

15. $f(x) = \dfrac{2x + 1}{\sqrt{9x^2 - 1}}$ ____________________

16. $f(x) = \dfrac{x^3 + 5x + 1}{x^2 - 5x + 6}$ ____________________

#17-28: Use this information to find limits.

$\lim\limits_{x\to 3} f(x) = 3,\ \lim\limits_{x\to 3} g(x) = 4,\ \lim\limits_{x\to 3} h(x) = 0,\ \lim\limits_{x\to 3^+} j(x) = 5,\ \lim\limits_{x\to 3^-} j(x) = 0,\ \lim\limits_{x\to 3^+} k(x) = 0,\ \lim\limits_{x\to 3^-} k(x) = 5$

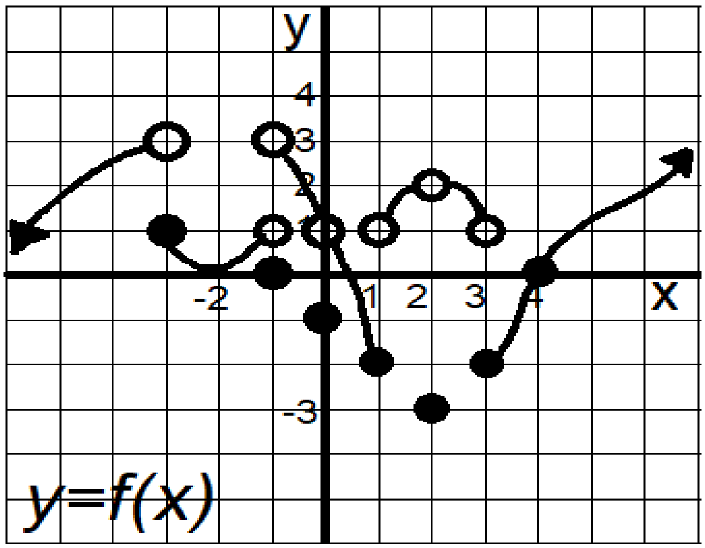
17. $\lim\limits_{x\to 3}\left[f(x) + g(x)\right]$	18. $\lim\limits_{x\to 3}\left[f(x) \bullet h(x)\right]$	19. $\lim\limits_{x\to 3}\left[\dfrac{h(x)}{f(x)}\right]$	20. $\lim\limits_{x\to 3}\left[\dfrac{g(x)}{h(x)}\right]$
21. $\lim\limits_{x\to 3}\left[j(x)\right]$	22. $\lim\limits_{x\to 3}\left[k(x)\right]$	23. $\lim\limits_{x\to 3}\left[j(x) + k(x)\right]$	24. $\lim\limits_{x\to 3}\left[j(x) \bullet k(x)\right]$
25. $\lim\limits_{x\to 3}\left[j(x) + h(x)\right]$	26. $\lim\limits_{x\to 3}\left[j(x) \bullet h(x)\right]$	27. $\lim\limits_{x\to 3}\left[f(g(x) - 1)\right]$	28. $\lim\limits_{x\to 9}\left[j(\sqrt{x})\right]$

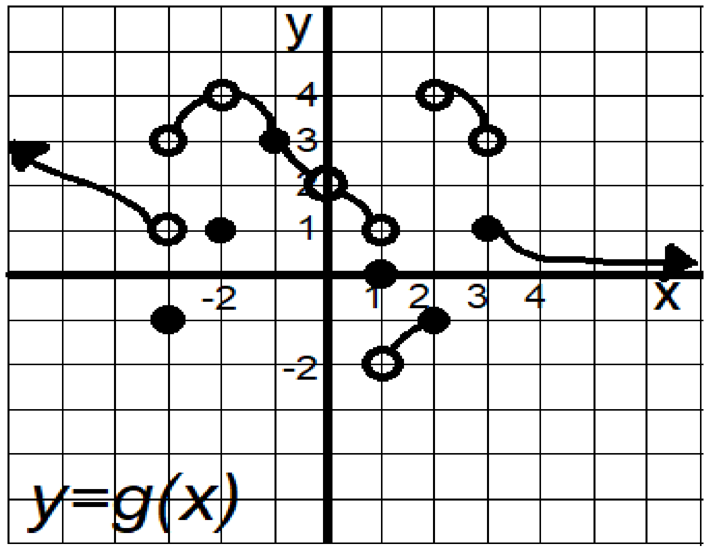
29) Draw pictures and write sentences to show two different ways the limit of a function will not exist.

More Work with Limits

Note that the limit properties (limit of a sum, difference, product, quotient of two functions) only apply if **the limits of both of the functions exist** at the specified point.

Practice: Given the functions *f(x)* and *g(x)* below, evaluate the limits or explain why they don't exist. NOTE: In the event you think the limit does not exist, be sure to check to see if the left-hand-limit equals the right-hand-limit to confirm your conclusion.





1. $\lim_{x\to 0^-}\left[f(x)+g(x)\right]$	2. $\lim_{x\to 0^+}\left[f(x)+g(x)\right]$	3. $\lim_{x\to 0}\left[f(x)+g(x)\right]$	4. $\lim_{x\to 0}\left[f(x)\cdot g(x)\right]$
5. $\lim_{x\to 3}\left[f(x)+g(x)\right]$	6. $\lim_{x\to 3}\left[f(x)\cdot g(x)\right]$	7. $\lim_{x\to 3} f(g(x))$	8. $\lim_{x\to 0} f(g(x))$
9. $\lim_{x\to -3}\left[f(x)\cdot g(x)\right]$	8. $\lim_{x\to 2} g(f(x))$	11. $\lim_{x\to -1} f(g(x))$	12. $\lim_{x\to 4} g(f(x))$

Compute Limits Analytically using algebraic strategies, showing appropriate work and notation. Strategies:
1) direct substitution
2) factor
3) rationalize using conjugate factor
4) simplify complex fractions.

<u>Examples</u>: Evaluate each of the following.

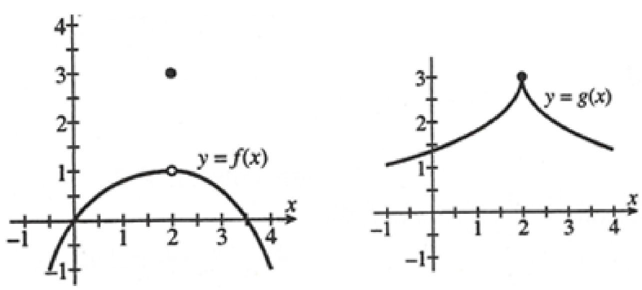
13. $\lim_{x\to 6}\left(4x^2-10x+3\right)$

14. $\lim_{x\to 1}\left(\frac{x^2-6x+5}{x-1}\right)$

15. $\lim_{x\to 5}\sqrt[3]{\frac{4x+44}{6x-29}}$

16. $\lim_{x\to 3} f(x)$ where $f(x)=\begin{cases} x^2 & x\ge 3 \\ 6x-4 & x<3 \end{cases}$

17. *given the graphs* $\lim_{x\to 2}\left[f(x)+g(x)\right]$



18. $\lim_{x\to 0}\left(\frac{\sqrt{2+x}-\sqrt{2}}{x}\right)$

19. $\lim_{x\to 1}\left(\frac{\frac{1}{x}-x}{\frac{1}{x}-1}\right)$

20. $\lim_{x\to 9}\frac{\sqrt{x}-3}{x-9}$

21. $\lim_{x\to 5}\frac{5-x}{\left(\frac{1}{5}-\frac{1}{x}\right)}$