AB Review #1 - Non-Calculator

3. If $\begin{cases} f(x) = \dfrac{\sqrt{2x+5}-\sqrt{x+7}}{x-2}, & \text{for } x \neq 2, \\ f(2) = k \end{cases}$ and if f is continuous at $x=2$, then $k=$

(A) 0 (B) $\frac{1}{6}$ (C) $\frac{1}{3}$ (D) 1 (E) $\frac{7}{5}$

4. $\int_0^8 \frac{dx}{\sqrt{1+x}} =$

(A) 1 (B) $\frac{3}{2}$ (C) 2 (D) 4 (E) 6

5. If $3x^2+2xy+y^2=2$, then the value of $\frac{dy}{dx}$ at $x=1$ is

(A) -2 (B) 0 (C) 2 (D) 4 (E) not defined

6. What is $\lim_{h\to 0} \frac{8\left(\frac{1}{2}+h\right)^8 - 8\left(\frac{1}{2}\right)^8}{h}$?

(A) 0 (B) $\frac{1}{2}$ (C) 1 (D) The limit does not exist.

(E) It cannot be determined from the information given.

7. For what value of k will $x+\frac{k}{x}$ have a relative maximum at $x=-2$?

(A) -4 (B) -2 (C) 2 (D) 4 (E) None of these

9. When the area in square units of an expanding circle is increasing twice as fast as its radius in linear units, the radius is

(A) $\frac{1}{4\pi}$ (B) $\frac{1}{4}$ (C) $\frac{1}{\pi}$ (D) 1 (E) π

11. The point on the curve $x^2+2y=0$ that is nearest the point $\left(0,-\frac{1}{2}\right)$ occurs where y is

(A) $\frac{1}{2}$ (B) 0 (C) $-\frac{1}{2}$ (D) -1 (E) none of the above

15. If $f'(x)$ and $g'(x)$ exist and $f'(x) > g'(x)$ for all real x, then the graph of $y = f(x)$ and the graph of $y = g(x)$

(A) intersect exactly once.

(B) intersect no more than once.

(C) do not intersect.

(D) could intersect more than once.

(E) have a common tangent at each point of intersection.

---

17. The graph of $y = 5x^4 - x^5$ has a point of inflection at

(A) $(0,0)$ only (B) $(3,162)$ only (C) $(4,256)$ only

(D) $(0,0)$ and $(3,162)$ (E) $(0,0)$ and $(4,256)$

---

18. If $f(x) = 2 + |x-3|$ for all x, then the value of the derivative $f'(x)$ at $x = 3$ is

(A) -1 (B) 0 (C) 1 (D) 2 (E) nonexistent

Given the parabola $y = x^2 - 2x + 3$:

(a) Find an equation for the line L, which contains the point $(2,3)$ and is perpendicular to the line tangent to the parabola at $(2,3)$.

(b) Find the area of that part of the first quadrant which lies below <u>both</u> the line L and the parabola.

AB Review #2 - Non-Calculator

19. A point moves on the x-axis in such a way that its velocity at time t $(t > 0)$ is given by $v = \frac{\ln t}{t}$. At what value of t does v attain its maximum?

(A) 1 (B) $e^{\frac{1}{2}}$ (C) e (D) $e^{\frac{3}{2}}$

(E) There is no maximum value for v.

---

20. An equation for a tangent to the graph of $y = \arcsin\frac{x}{2}$ at the origin is

(A) $x - 2y = 0$ (B) $x - y = 0$ (C) $x = 0$ (D) $y = 0$ (E) $\pi x - 2y = 0$

---

21. At $x = 0$, which of the following is true of the function f defined by $f(x) = x^2 + e^{-2x}$?

(A) f is increasing.
(B) f is decreasing.
(C) f is discontinuous.
(D) f has a relative minimum.
(E) f has a relative maximum.

---

23. The area of the region bounded by the curve $y = e^{2x}$, the x-axis, the y-axis, and the line $x = 2$ is equal to

(A) $\frac{e^4}{2} - e$ (B) $\frac{e^4}{2} - 1$ (C) $\frac{e^4}{2} - \frac{1}{2}$

(D) $2e^4 - e$ (E) $2e^4 - 2$

---

24. If $\sin x = e^y$, $0 < x < \pi$, what is $\frac{dy}{dx}$ in terms of x ?

(A) $-\tan x$ (B) $-\cot x$ (C) $\cot x$ (D) $\tan x$ (E) $\csc x$

---

30. If a function f is continuous for all x and if f has a relative maximum at $(-1, 4)$ and a relative minimum at $(3, -2)$, which of the following statements must be true?

(A) The graph of f has a point of inflection somewhere between $x = -1$ and $x = 3$.
(B) $f'(-1) = 0$
(C) The graph of f has a horizontal asymptote.
(D) The graph of f has a horizontal tangent line at $x = 3$.
(E) The graph of f intersects both axes.

31. If $f'(x) = -f(x)$ and $f(1) = 1$, then $f(x) =$

(A) $\frac{1}{2}e^{-2x+2}$ (B) e^{-x-1} (C) e^{1-x} (D) e^{-x} (E) $-e^{x}$

33. What is the average value of $3t^3 - t^2$ over the interval $-1 \le t \le 2$?

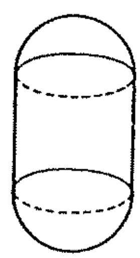
(A) $\frac{11}{4}$ (B) $\frac{7}{2}$ (C) 8 (D) $\frac{33}{4}$ (E) 16

35. At $t = 0$ a particle starts at rest and moves along a line in such a way that at time t its acceleration is $24t^2$ feet per second per second. Through how many feet does the particle move during the first 2 seconds?

(A) 32 (B) 48 (C) 64 (D) 96 (E) 192

36. The approximate value of $y = \sqrt{4 + \sin x}$ at $x = 0.12$, obtained from the tangent to the graph at $x = 0$, is

(A) 2.00 (B) 2.03 (C) 2.06 (D) 2.12 (E) 2.24



The balloon shown is in the shape of a cylinder with hemispherical ends of the same radius as that of the cylinder. The balloon is being inflated at the rate of 261π cubic centimeters per minute. At the instant the radius of the cylinder is 3 centimeters., the volume of the balloon is 144π cubic centimeters and the radius of the cylinder is increasing at the rate of 2 centimeters per minute. (The volume of a cylinder is $\pi r^2 h$ and the volume of a sphere is $\frac{4}{3}\pi r^3$).

(a) At this instant, what is the height of the cylinder?

(b) At this instant, how fast is the height of the cylinder increasing?

AB Review #3 - Non-Calculator

38. $\int \frac{x^2}{e^{x^3}}\,dx =$

(A) $-\frac{1}{3}\ln e^{x^3} + C$

(B) $-\frac{e^{x^3}}{3} + C$

(C) $-\frac{1}{3e^{x^3}} + C$

(D) $\frac{1}{3}\ln e^{x^3} + C$

(E) $\frac{x^3}{3e^{x^3}} + C$

---

43. $\int \sin(2x+3)\,dx =$

(A) $\frac{1}{2}\cos(2x+3) + C$

(B) $\cos(2x+3) + C$

(C) $-\cos(2x+3) + C$

(D) $-\frac{1}{2}\cos(2x+3) + C$

(E) $-\frac{1}{5}\cos(2x+3) + C$

---

45. If $\frac{d}{dx}(f(x)) = g(x)$ and $\frac{d}{dx}(g(x)) = f(x^2)$, then $\frac{d^2}{dx^2}\left(f(x^3)\right) =$

(A) $f\left(x^6\right)$

(B) $g\left(x^3\right)$

(C) $3x^2 g\left(x^3\right)$

(D) $9x^4 f\left(x^6\right) + 6x\,g\left(x^3\right)$

(E) $f\left(x^6\right) + g\left(x^3\right)$

---

3. The Mean Value Theorem guarantees the existence of a special point on the graph of $y = \sqrt{x}$ between $(0,0)$ and $(4,2)$. What are the coordinates of this point?

(A) $(2,1)$
(B) $(1,1)$
(C) $(2,\sqrt{2})$
(D) $\left(\frac{1}{2}, \frac{1}{\sqrt{2}}\right)$
(E) None of the above

---

1. $\int \left(x^3 - 3x\right)dx =$

(A) $3x^2 - 3 + C$

(B) $4x^4 - 6x^2 + C$

(C) $\frac{x^4}{3} - 3x^2 + C$

(D) $\frac{x^4}{4} - 3x + C$

(E) $\frac{x^4}{4} - \frac{3x^2}{2} + C$

3. The slope of the line tangent to the graph of $y = \ln\left(x^2\right)$ at $x = e^2$ is

(A) $\frac{1}{e^2}$ (B) $\frac{2}{e^2}$ (C) $\frac{4}{e^2}$ (D) $\frac{1}{e^4}$ (E) $\frac{4}{e^4}$

---

4. If $f(x) = x + \sin x$, then $f'(x) =$

(A) $1 + \cos x$ (B) $1 - \cos x$ (C) $\cos x$

(D) $\sin x - x\cos x$ (E) $\sin x + x\cos x$

---

6. If $f(x) = \frac{x-1}{x+1}$ for all $x \neq -1$, then $f'(1) =$

(A) -1 (B) $-\frac{1}{2}$ (C) 0 (D) $\frac{1}{2}$ (E) 1

---

8. A particle moves in a straight line with velocity $v(t) = t^2$. How far does the particle move between times $t = 1$ and $t = 2$?

(A) $\frac{1}{3}$ (B) $\frac{7}{3}$ (C) 3 (D) 7 (E) 8

---

9. If $y = \cos^2 3x$, then $\frac{dy}{dx} =$

(A) $-6\sin 3x\cos 3x$ (B) $-2\cos 3x$ (C) $2\cos 3x$

(D) $6\cos 3x$ (E) $2\sin 3x\cos 3x$

Let f be the function defined by $f(x) = \frac{2x-5}{x^2-4}$.

(a) Find the domain of f.

(b) Write an equation for each vertical and each horizontal asymptote for the graph of f.

(c) Find $f'(x)$.

(d) Write an equation for the line tangent to the graph of f at the point $(0, f(0))$.

AB Review #4 – Noncalculator

10. The *derivative* of $f(x) = \frac{x^4}{3} - \frac{x^5}{5}$ attains its maximum value at $x =$

(A) -1 (B) 0 (C) 1 (D) $\frac{4}{3}$ (E) $\frac{5}{3}$

---

11. If the line $3x - 4y = 0$ is tangent in the first quadrant to the curve $y = x^3 + k$, then k is

(A) $\frac{1}{2}$ (B) $\frac{1}{4}$ (C) 0 (D) $-\frac{1}{8}$ (E) $-\frac{1}{2}$

---

12. If $f(x) = 2x^3 + Ax^2 + Bx - 5$ and if $f(2) = 3$ and $f(-2) = -37$, what is the value of $A + B$?

(A) -6 (B) -3 (C) -1 (D) 2

(E) It cannot be determined from the information given.

---

13. The acceleration α of a body moving in a straight line is given in terms of time t by $\alpha = 8 - 6t$. If the velocity of the body is 25 at $t = 1$ and if $s(t)$ is the distance of the body from the origin at time t, what is $s(4) - s(2)$?

(A) 20 (B) 24 (C) 28 (D) 32 (E) 42

---

14. If $f(x) = x^{\frac{1}{3}}(x-2)^{\frac{2}{3}}$ for all x, then the domain of f' is

(A) $\{x \mid x \neq 0\}$ (B) $\{x \mid x > 0\}$ (C) $\{x \mid 0 \leq x \leq 2\}$

(D) $\{x \mid x \neq 0 \text{ and } x \neq 2\}$ (E) $\{x \mid x \text{ is a real number}\}$

---

15. The area of the region bounded by the lines $x = 0$, $x = 2$, and $y = 0$ and the curve $y = e^{\frac{x}{2}}$ is

(A) $\frac{e-1}{2}$ (B) $e - 1$ (C) $2(e-1)$ (D) $2e - 1$ (E) $2e$

---

16. The number of bacteria in a culture is growing at a rate of $3000e^{\frac{2t}{5}}$ per unit of time t. At $t = 0$, the number of bacteria present was 7,500. Find the number present at $t = 5$.

(A) $1{,}200e^2$ (B) $3{,}000e^2$ (C) $7{,}500e^2$ (D) $7{,}500e^5$ (E) $\frac{15{,}000}{7}e^7$

---

17. What is the area of the region completely bounded by the curve $y = -x^2 + x + 6$ and the line $y = 4$?

(A) $\frac{3}{2}$ (B) $\frac{7}{3}$ (C) $\frac{9}{2}$ (D) $\frac{31}{6}$ (E) $\frac{33}{2}$

---

18. $\frac{d}{dx}(\arcsin 2x) =$

(A) $\frac{-1}{2\sqrt{1-4x^2}}$ (B) $\frac{-2}{\sqrt{4x^2-1}}$ (C) $\frac{1}{2\sqrt{1-4x^2}}$

(D) $\frac{2}{\sqrt{1-4x^2}}$ (E) $\frac{2}{\sqrt{4x^2-1}}$

---

20. If F and f are continuous functions such that $F'(x) = f(x)$ for all x, then $\int_a^b f(x)\,dx$ is

(A) $F'(a) - F'(b)$
(B) $F'(b) - F'(a)$
(C) $F(a) - F(b)$
(D) $F(b) - F(a)$
(E) none of the above

Let R be the region enclosed by the graphs of $y = e^{-x}$, $y = e^x$, and $x = \ln 4$.

(a) Find the area of R by setting up and evaluating a definite integral.

(b) Set up, but do not integrate, an integral expression in terms of a single variable for the volume generated when the region R is revolved about the x-axis.

(c) Set up, but do not integrate, an integral expression in terms of a single variable for the volume generated when the region R is revolved about the y-axis.

AB Review #5 - Non-Calculator

21. $\int_0^1 (x+1)e^{x^2+2x}\,dx =$

(A) $\frac{e^3}{2}$ (B) $\frac{e^3-1}{2}$ (C) $\frac{e^4-e}{2}$ (D) e^3-1 (E) e^4-e

---

22. Given the function defined by $f(x)=3x^5-20x^3$, find all values of x for which the graph of f is concave up.

(A) $x>0$
(B) $-\sqrt{2}<x<0$ or $x>\sqrt{2}$
(C) $-2<x<0$ or $x>2$
(D) $x>\sqrt{2}$
(E) $-2<x<2$

---

25. $\int_0^{\pi/4} \tan^2 x\,dx =$

(A) $\frac{\pi}{4}-1$ (B) $1-\frac{\pi}{4}$ (C) $\frac{1}{3}$ (D) $\sqrt{2}-1$ (E) $\frac{\pi}{4}+1$

---

26. The radius r of a sphere is increasing at the uniform rate of 0.3 inches per second. At the instant when the surface area S becomes 100π square inches, what is the rate of increase, in cubic inches per second, in the volume V? $\left(S=4\pi r^2 \text{ and } V=\frac{4}{3}\pi r^3\right)$

(A) 10π (B) 12π (C) 22.5π (D) 25π (E) 30π

---

27. $\int_0^{1/2} \frac{2x}{\sqrt{1-x^2}}\,dx =$

(A) $1-\frac{\sqrt{3}}{2}$ (B) $\frac{1}{2}\ln\frac{3}{4}$ (C) $\frac{\pi}{6}$ (D) $\frac{\pi}{6}-1$ (E) $2-\sqrt{3}$

---

28. A point moves in a straight line so that its distance at time t from a fixed point of the line is $8t-3t^2$. What is the *total* distance covered by the point between $t=1$ and $t=2$?

(A) 1 (B) $\frac{4}{3}$ (C) $\frac{5}{3}$ (D) 2 (E) 5

---

30. $\int_1^2 \frac{x-4}{x^2}\,dx =$

(A) $-\frac{1}{2}$ (B) $\ln 2-2$ (C) $\ln 2$ (D) 2 (E) $\ln 2+2$

32. $\int \frac{5}{1+x^2}\,dx =$

(A) $\frac{-10x}{\left(1+x^2\right)^2}+C$ (B) $\frac{5}{2x}\ln\left(1+x^2\right)+C$ (C) $5x-\frac{5}{x}+C$

(D) $5\arctan x+C$ (E) $5\ln\left(1+x^2\right)+C$

---

34. The average value of $\sqrt{x}$ over the interval $0 \le x \le 2$ is

(A) $\frac{1}{3}\sqrt{2}$ (B) $\frac{1}{2}\sqrt{2}$ (C) $\frac{2}{3}\sqrt{2}$ (D) 1 (E) $\frac{4}{3}\sqrt{2}$

---

35. The region in the first quadrant bounded by the graph of $y = \sec x$, $x = \frac{\pi}{4}$, and the axes is rotated about the x-axis. What is the volume of the solid generated?

(A) $\frac{\pi^2}{4}$ (B) $\pi - 1$ (C) π (D) 2π (E) $\frac{8\pi}{3}$

Let f be the function defined for $\frac{\pi}{6} \le x \le \frac{5\pi}{6}$ by $f(x) = x + \sin^2 x$.

(a) Find all values of x for which $f'(x) = 1$.

(b) Find the x-coordinates of all minimum points of f. Justify your answer.

(c) Find the x-coordinates of all inflection points of f. Justify your answer.

AB Review #6 - Non-Calculator

36. If $y = e^{nx}$, then $\frac{d^n y}{dx^n} =$

(A) $n^n e^{nx}$ (B) $n! e^{nx}$ (C) $n e^{nx}$ (D) $n^n e^x$ (E) $n! e^x$

---

37. If $\frac{dy}{dx} = 4y$ and if $y = 4$ when $x = 0$, then $y =$

(A) $4e^{4x}$ (B) e^{4x} (C) $3 + e^{4x}$ (D) $4 + e^{4x}$ (E) $2x^2 + 4$

---

40. If $\tan(xy) = x$, then $\frac{dy}{dx} =$

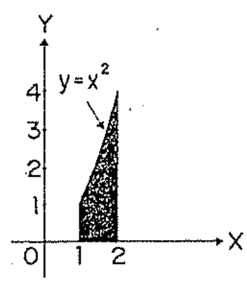
(A) $\frac{1 - y\tan(xy)\sec(xy)}{x\tan(xy)\sec(xy)}$ (B) $\frac{\sec^2(xy) - y}{x}$ (C) $\cos^2(xy)$

(D) $\frac{\cos^2(xy)}{x}$ (E) $\frac{\cos^2(xy) - y}{x}$

---

41. Given $f(x) = \begin{cases} x+1 & \text{for } x < 0, \\ \cos \pi x & \text{for } x \geq 0, \end{cases}$ $\int_{-1}^{1} f(x)\,dx =$

(A) $\frac{1}{2} + \frac{1}{\pi}$ (B) $-\frac{1}{2}$ (C) $\frac{1}{2} - \frac{1}{\pi}$ (D) $\frac{1}{2}$ (E) $-\frac{1}{2} + \pi$



42. Calculate the approximate area of the shaded region in the figure by the trapezoidal rule, using divisions at $x = \frac{4}{3}$ and $x = \frac{5}{3}$.

(A) $\frac{50}{27}$ (B) $\frac{251}{108}$ (C) $\frac{7}{3}$ (D) $\frac{127}{54}$ (E) $\frac{77}{27}$

---

1. If $f(x) = e^{1/x}$, then $f'(x) =$

(A) $-\frac{e^{1/x}}{x^2}$ (B) $-e^{1/x}$ (C) $\frac{e^{1/x}}{x}$ (D) $\frac{e^{1/x}}{x^2}$ (E) $\frac{1}{x} e^{(1/x)-1}$

3. If $f(x) = x + \frac{1}{x}$, then the set of values for which f increases is

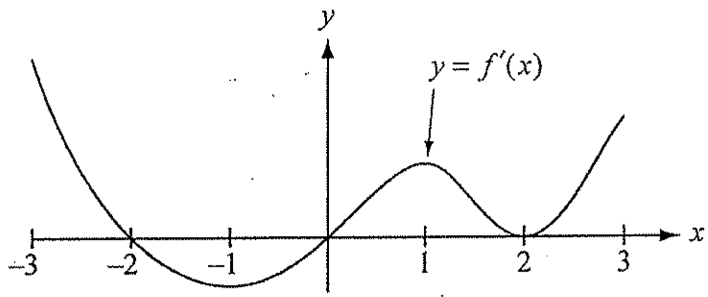
(A) $(-\infty, -1] \cup [1, \infty)$ (B) $[-1, 1]$ (C) $(-\infty, \infty)$

(D) $(0, \infty)$ (E) $(-\infty, 0) \cup (0, \infty)$

---

4. For what non-negative value of b is the line given by $y = -\frac{1}{3}x + b$ normal to the curve $y = x^3$?

(A) 0 (B) 1 (C) $\frac{4}{3}$ (D) $\frac{10}{3}$ (E) $\frac{10\sqrt{3}}{3}$



Note: This is the graph of the <u>derivative</u> of f, <u>not</u> the graph of f.

The figure above shows the graph of f', the derivative of a function f. The domain of the function f is the set of all x such that $-3 \le x \le 3$.

(a) For what values of x, $-3 < x < 3$, does f have a relative maximum? A relative minimum? Justify your answer.

(b) For what values of x is the graph of f concave up? Justify your answer.

(c) Use the information found in parts (a) and (b) and the fact that $f(-3) = 0$ to sketch a possible graph of f on the axes provided below.