REVIEW ANSWERS: EAGEN V1

Review Unit 1: Functions, Graphs & Limits Prepare for TEST date WED. 27-SEPT.

DIRECTIONS: Complete all review questions on a separate paper in your notebook. Show all work.

Non-Calculator (do not use a calculator on these questions)

UE 9/12

1. Are the following functions continuous on the given intervals? Explain why or why not using the definition of continuity.

a. $f(x)=\frac{1}{x-2}$ on $[0,3]$

f(x) is not continuous at $x=2$ b/c

① f(2) dne

② $\lim_{x\to 2} f(x)$ dne

③ ∴ f(x) is discontinuous.

b. $f(x)=\frac{1}{x-2}$ on $[-1,1]$

f(x) is continuous on $[-1,1]$

① f(x) exists for all $x \in [-1,1]$

② $\lim_{x\to c} f(x)$ exists for all $x=c\in[-1,1]$ &

③ $f(c) = \lim_{x\to c} f(x)$ on $[-1,1]$ for $f(x)=\frac{1}{x-2}$

c. $f(x)=\frac{e^x}{e^x-1}$ on $[-1,1]$

f(x) is undefined when $e^x-1=0$

$e^x=1$

$x=0$

which is on $[-1,1]$

∴

① f(0) dne

② $\lim_{x\to 0} f(x)$ dne

③ ∴ f(x) is discontinuous

ED 9/13

2. Use the function $f(x)$ to find

$$f(x)=\begin{cases} x^2-1 & for\ -1\le x<0 \\ 2x & for\ 0<x<1 \\ 1 & for\ x=1 \\ -2x+4 & for\ 1<x<2 \\ 0 & for\ 2<x<3 \end{cases}$$

a. $\lim_{x\to a} f(x)$ for a = -1, 0, 1.

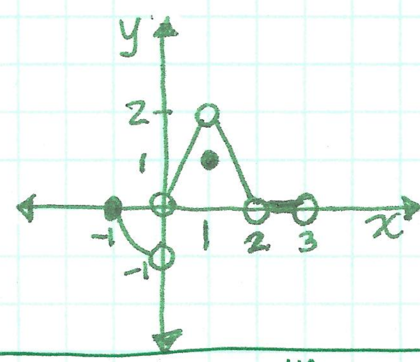
- $\lim_{x\to -1^+} f(x)=0$
- $\lim_{x\to 0^-} f(x)=-1$ $\lim_{x\to 0^+} f(x)=0$ ∴ $\lim_{x\to 0} f(x)$ dne
- $\lim_{x\to 1} f(x)=2$

b. For what value(s) of x does the function have jump or removable discontinuities?

jump → $x=0$, $x=1$

removable → $x=2$, $x=3$

removable b/c one y-value could be defined at $x=2$ & 3 that would remove the discontinuity.



3. Discuss the continuity and asymptotes of $y=\frac{x^2+x-2}{3x^2-4x+1}$.

③ $f(x)=\frac{(x+2)(x-1)}{(3x-1)(x-1)} = \frac{1}{3}+\frac{\frac{7}{3}}{(3x-1)}, x\neq 1$

A) f(x) is discontinuous at $x=1$ where there is a hole $(1, \frac{3}{2})$

- f(x) dne • $\lim_{x\to 1} f(x)=\frac{3}{2}$ ∴ discontinuous at $x=1$

B) f(x) is discontinuous at $x=\frac{1}{3}$ where there is a vertical asymptote

VA: $\lim_{x\to \frac{1}{3}^-} f(x)=-\infty$ $\lim_{x\to \frac{1}{3}^+} f(x)=+\infty$

HA $y=\lim_{x\to\infty}\frac{x^2}{3x^2}=\frac{1}{3}$ ∴ HA $y=\frac{1}{3}$

HA $y=\lim \frac{x^2}{3x^2}=\frac{1}{3}$ • $y=k$ lim

HW
9/14

4. Find k so that $f(x)$ is continuous at $x = 3$: $f(x)=\begin{cases}\dfrac{x^2+2x-15}{x-3} & \text{for } x\neq 3\\ k & \text{for } x=3\end{cases}$

$\xrightarrow{\text{FACTOR}} \dfrac{(x+5)(x-3)}{(x-3)}$ hole $(3, 8)$

$\therefore k=8$ will fill in the y-value of the hole.

$\lim\limits_{x\to 3} \dfrac{x^2+2x-15}{x-3} = \lim\limits_{x\to 3}(x+5) = 8$

5. Let $h(x)=\begin{cases}-2x+3 & \text{for } x<-1\\ x^2+3 & \text{for } x\geq -1\end{cases}$

a. Find $\lim\limits_{x\to -1^-} h(x)$

b. Find $\lim\limits_{x\to -1^+} h(x)$

c. Find $\lim\limits_{x\to -1} h(x)$

d. Is the function continuous at $x = -1$? Explain.

a) $\lim\limits_{x\to -1^-} (-2x+3) = 2+3 = 5$

b) $\lim\limits_{x\to -1^+} (x^2+3) = 1+3 = 4$

c) $\lim\limits_{x\to -1} h(x) =$ dne

d) $f(x)$ is discontinuous at $x=-1$ b/c $\lim\limits_{x\to -1} h(x)$ dne & $h(-1)=4$

FRI
9/15

6. (1986 AB 4) Let $f(x)$ be a function defined as follows $f(x)=\begin{cases}|x-1|+2 & \text{for } x<1\\ ax^2+bx & \text{for } x\geq 1\end{cases}$

a. If $a = 2$ and $b = 3$, is f continuous for all x? Justify your answer.

b. Describe all values of a and b for which f is a continuous function.

a) $f(x)=\begin{cases}|x-1|+2, & x<1\\ 2x^2+3x, & x\geq 1\end{cases}$

$|x-1|+2 \stackrel{?}{=} 2x^2+3x \Big|_{x=1}$

$2 \neq 5$

$\lim\limits_{x\to 1^-} f(x) \neq \lim\limits_{x\to 1^+} f(x)$ & $f(1)=5$

$2 \neq 5$

Since $\lim\limits_{x\to 1} f(x)$ dne $f(x)$ is NOT continuous at $x=1$.

b) If $f(x)$ is continuous then

$|x-1|+2 = ax^2+bx \Big|_{x=1}$

$2 = a+b$

So as long as a & b values satisfy $a+b=2$ $f(x)$ will be continuous at $x=1$

7 ? No #7

MON 9/18

8. Evaluate the limits analytically using limit properties or algebraic strategies. Show all work.

a. $\lim\limits_{x\to3}\dfrac{\sqrt{x^2-5}}{\sqrt[3]{50x-3}}$	b. $\lim\limits_{x\to12}\dfrac{12-x}{\frac{1}{x}-\frac{1}{12}}$	c. $\lim\limits_{x\to0}\dfrac{\sqrt{144+x}-12}{x}$	d. $\lim\limits_{x\to12}\dfrac{x^2+13x+12}{x^2+10x-24}$
e. $\lim\limits_{x\to-\infty}\dfrac{4x-7x^5+18}{5+x-35x^2}$	f. $\lim\limits_{x\to\infty}\dfrac{-6x^5+2x-11}{3x^5+4x^3+9x+1}$	g. $\lim\limits_{x\to\infty}\dfrac{1+3x^2-4x^3}{5x^7-x^2+x+7}$	h. $\lim\limits_{x\to-\infty}\dfrac{\sqrt{64x^2+5x}}{\sqrt[3]{125x^3+8}}$
i. $\lim\limits_{x\to2}\dfrac{\sin(x-2)}{(2-x)}$	j. $\lim\limits_{x\to0}\dfrac{1-\cos(3x)}{(3x)}$	k. $\lim\limits_{x\to7}\dfrac{\lvert x-7\rvert}{7-x}$	l. ~~$\lim\limits_{x\to3}\dfrac{\sqrt{x^2-5}}{\sqrt[3]{50x-3}}$~~

a) $\lim\limits_{x\to3}\dfrac{\sqrt{x^2-5}}{\sqrt[3]{50x-3}} = \dfrac{\sqrt{9-5}}{\sqrt[3]{150-3}} = \dfrac{2}{\sqrt[3]{147}}$

b) $\lim\limits_{x\to12}\dfrac{(12-x)}{\left(\frac{1}{x}-\frac{1}{12}\right)}\cdot\dfrac{(12x)}{(12x)}$

$\lim\limits_{x\to12}\dfrac{\cancel{(12-x)}(12x)}{\cancel{(12-x)}}$

$=144$

c) $\lim\limits_{x\to0}\dfrac{(\sqrt{144+x}-12)}{x}\cdot\dfrac{(\sqrt{144+x}+12)}{(\sqrt{144+x}+12)}$

$\lim\limits_{x\to0}\dfrac{\cancel{144}+x-\cancel{144}}{x(\sqrt{144+x}+12)}$

$\lim\limits_{x\to0}\dfrac{1}{\sqrt{144+x}+12}=\dfrac{1}{24}$

d) $\lim\limits_{x\to12}\dfrac{x^2+13x+12}{x^2+10x-24}$

$=\lim\limits_{x\to12}\dfrac{(x+12)(x+1)}{(x+12)(x-2)}$

$=\dfrac{(24)(13)}{(24)(10)}=\dfrac{13}{10}$

May be a Typo... If $\lim\limits_{x\to-12}f(x)=\dfrac{-11}{-14}$

e) $\lim\limits_{x\to-\infty}\dfrac{-7x^5}{-35x^2}$

$\lim\limits_{x\to-\infty}x^3$

$=-\infty$

OBLIQUE ASYMPTOTE.

f) $\lim\limits_{x\to\infty}\dfrac{-6x^5}{3x^5}$

$=\dfrac{-6}{3}$

$=-2$

HA $\boxed{y=-2}$

g) $\lim\limits_{x\to\infty}\dfrac{-4x^3}{5x^7}$

$=\lim\limits_{x\to\infty}\dfrac{-4}{5x^4}$

$=0$

HA $\boxed{y=0}$

h) $\lim\limits_{x\to-\infty}\dfrac{\sqrt{64x^2}}{\sqrt[3]{125x^3}}$

$=\lim\limits_{x\to-\infty}\dfrac{\lvert 8x\rvert}{5x}$

$\lim\limits_{x\to-\infty}\left(\dfrac{8}{5}\right)\left(\dfrac{\lvert x\rvert}{x}\right)$

$=-\dfrac{8}{5}$

i) $\lim\limits_{x\to2}\dfrac{\sin(x-2)}{-(x-2)}$

$=-1$

j) $\lim\limits_{x\to0}\dfrac{1-\cos(3x)}{3x}=0$

k) $\lim\limits_{x\to7}\dfrac{\lvert x-7\rvert}{7-x}$

$\lim\limits_{x\to7}\dfrac{\lvert x-7\rvert}{(-1)(x-7)}$

$=-1$

l) ~~$\lim\limits_{x\to3}\sqrt{x^2-5}$~~

see (a).

oops repeat of (a)

TUE
9/19

9. Use the Intermediate Value Theorem to determine whether or not the function has a zero on the given interval. Find all zeros on the interval exact to 3-decimal places using a graphing utility.

a. $f(x)=x^3-0.276x^2-0.140157x$ on $x\in[-1,1]$

b. $f(x)=\begin{cases} x^2-2, & x<0 \\ 2-x^2, & x\geq 0 \end{cases}$ on $x\in[-1,1]$

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a) Yes you can use your calculator to evaluate and to Find the zeros.

FIRST I know f(x) has a zero @ x=0.

b/c $f(x)=(x)(x^2-.276x-.140157)$
↑ zero @ x=0.

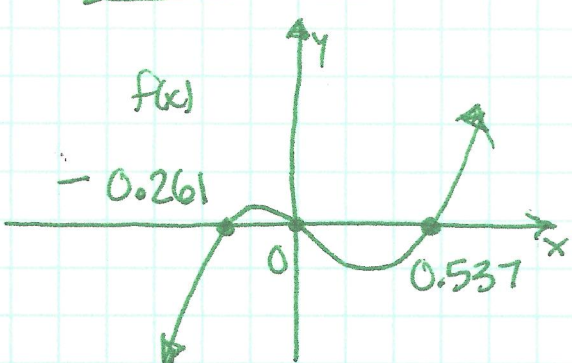
In Zoomdecimal window it looks like only one zero BUT look closer!

change WINDOW: [Xmin Xmax] → [-1,1]
[Ymin, Ymax] → [-.2, +.2]

Use 2nd TRACE CALC
2: zero.

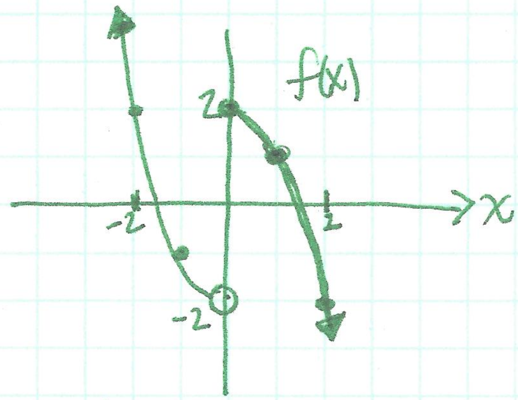
Two more zeros.
Accurate to 3 decimal places.

$\boxed{\begin{array}{l} x=-0.261 \\ x=0.537 \end{array}}$



$\boxed{\text{IVT}}$ $f(-1)=-1.135843<0$
$f(1)=0.58384>0$

Because f(x) changes signs from negative to positive from x=-1 to x=+1 and because f(x) is a continuous polynomial The Intermediate Value Theorem guarantees at least one zero on the interval [-1, 1]



f(x) is not continuous on [-1,1] So the Intermediate Value theorem can not guarantee any zeros on the interval. In fact there are none on [-1,1]

Even though f(-1)=-1 and f(1)=1 and there is a change in signs on the interval, IVT does not apply because f(x) is not continuous.

$f(x)=0, \ x\geq 0$
$2-x^2=0$
$x^2=2$
$\boxed{x=+\sqrt{2}}$

$f(x)=0, \ x<0$
$x^2-2=0$
$x^2=2$
$\boxed{x=-\sqrt{2}}$

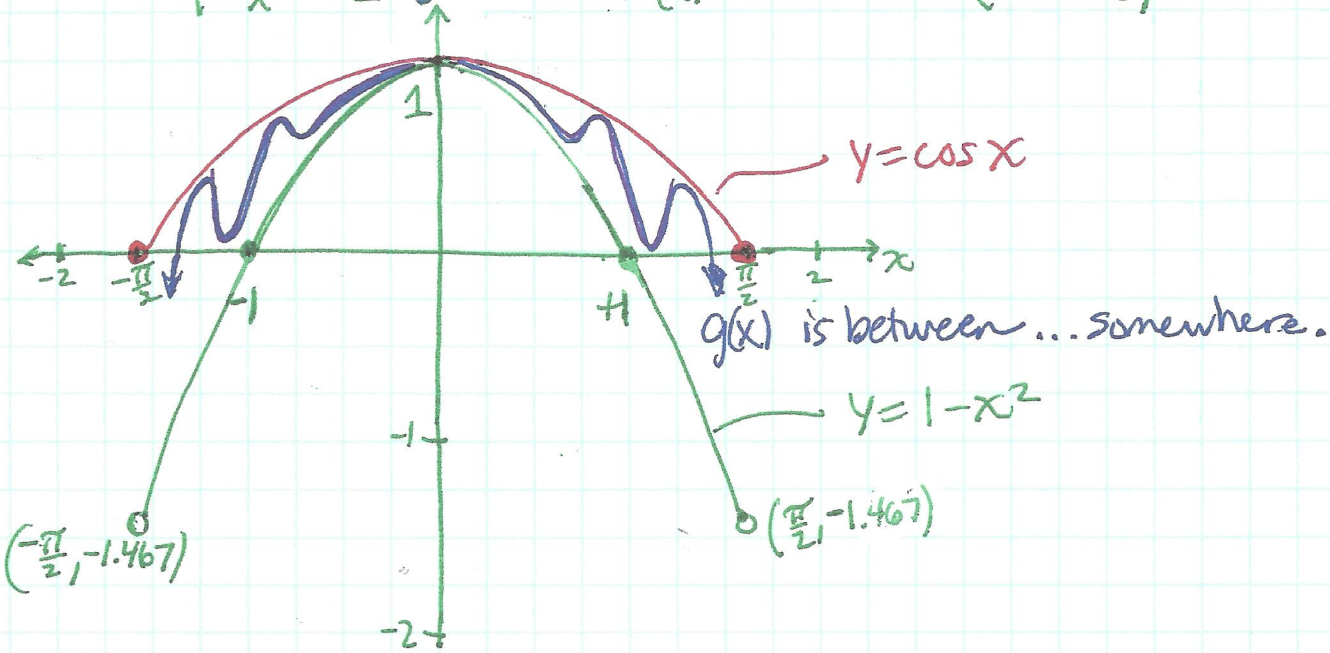
Two zeros exist on f(x) at $x=\pm\sqrt{2}$

NEV 9/20

10. Sketch the graphs of the functions $f(x)=1-x^2$ and $h(x)=\cos(x)$ on the interval $x\in\left(\frac{-\pi}{2},\frac{\pi}{2}\right)$ satisfying the inequality $\begin{matrix} f(x)\le g(x)\le h(x) \\ 1-x^2\le g(x)\le \cos(x)\end{matrix}$. Use the graph to make a conclusion about $\lim_{x\to 0} g(x)$ and explain how you arrived at your conclusion.

$f(x) \le g(x) \le h(x)$

$1-x^2 \le g(x) \le \cos(x)$ on $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$



So $\lim_{x\to 0}\left[f(x) \le g(x) \le h(x)\right]$

$$\lim_{x\to 0}(1-x^2) \le \lim_{x\to 0} g(x) \le \lim_{x\to 0}(\cos x)$$

$$1 \le \lim_{x\to 0} g(x) \le 1$$

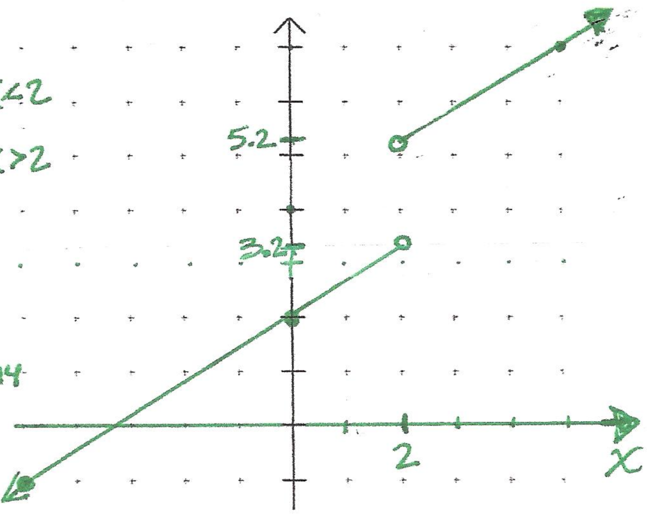
$\therefore \lim_{x\to 0} g(x) = 1$ by the SQUEEZE THEOREM or SANDWICH THM.

Thu 9/21

Calculator **(You may use a calculator on these questions)**

11. For the function $f(x) = 3 + 0.6x + \frac{|x-2|}{x-2}$ $= \begin{cases} 2+0.6x, & x<2 \\ 4+0.6x, & x>2 \end{cases}$

$f(x) = \begin{cases} 3+\frac{3}{5}x - 1, & x<2 \\ 3+\frac{3}{5}x + 1, & x>2 \end{cases}$



a. Find a complete graph of $f(x)$.
b. Where is the function discontinuous? @ $x=2$
c. Evaluate $f(1.9)$, $f(1.99)$ and $f(1.999)$. $f(1.9)=3.14$, $f(1.99)=3.194$, $f(1.999)=3.1994$
d. What is $\lim_{x\to 2^-} f(x) = 3.2$
e. Evaluate $f(2.1)$, $f(2.01)$ and $f(2.001)$. $f(2.1)=5.26$, $f(2.01)=5.206$, $f(2.001)=5.2006$
f. What is $\lim_{x\to 2^+} f(x) = 5.2$

12. Let $f(x) = \frac{|x+1|}{x-2}$.

a. Find $\lim_{x\to -\infty} f(x)$. b. Find $\lim_{x\to \infty} f(x)$. c. Is this function continuous at $x = 2$? Explain.

$$f(x) = \begin{cases} \frac{-(x+1)}{(x-2)}, & x<1 \\ \frac{(x+1)}{x-2}, & x>1 \end{cases}$$

DEFN of $f(x) = |x| = \begin{cases} -x, & x<0 \\ +x, & x>0 \end{cases}$

a) $\lim_{x\to -\infty} \left[\frac{-(x+1)}{(x+2)}\right] = \lim_{x\to -\infty} \frac{-x}{x} = -1$ HA $y=-1$ as $x\to -\infty$

b) $\lim_{x\to \infty} \left[\frac{(x+1)}{(x+2)}\right] = \lim_{x\to \infty} \frac{x}{x} = +1$ HA $y=1$ as $x\to \infty$

c) at $x=2$ $f(x)$ is discontinuous

i) $f(2)$ dne

ii) $\lim_{x\to 2^-} f(x) = 3.2 \neq \lim_{x\to 2^+} f(x) = 5.2$ so $\lim_{x\to 2} f(x)$ dne.

iii) $\therefore$ $f(2) \neq \lim_{x\to 2} f(x)$ so f is discontinuous at $x=2$.

NOTE: In preparation for your test, you should review classwork, homework, quizzes and this review sheet. It is best to study for a test by DOING problems again and not just "looking them over." DOING builds muscle memory so that you are able to answer questions on the test efficiently and accurately. You've got to know your stuff cold so you have enough time to complete the test during one class period.

Good Luck - Good Skill :)