HW: Related Rates DAY 71-72 HW

HW #1: The width of a rectangle is increasing at a rate of 2 cm/sec and its length is increasing at a rate of 3 cm/sec. At what rate is the area of the rectangle increasing when its width is 4 cm and length is 5 cm? At what rate is the length of the diagonal of the rectangle increasing? At what rate is the perimeter of the rectangle increasing?

HW #2: A spherical ball 8 inches in diameter is coated with a layer of ice of uniform thickness. If the ice melts at a rate of 10 in^3/min, how fast is the thickness of the ice decreasing when it is 2 inches thick? How fast is the outer surface area of the ice decreasing at this time?

HW #3: A baseball diamond is a square 90 feet on one side. A runner travels from home plate to first base at 20 ft/sec. How fast is the runner's distance from second base changing when the runner is halfway to first base?

Day 73 in class

HW #4: A rocket rises vertically from a point on the ground that is 100 m from an observer at ground level. The observer notes that the angle of elevation is increasing at a rate of 12 degrees per second when the angle of elevation is 60 degrees. Find the speed of the rocket at that instant. *convert to radians

*convert to radians

HW #5: If $x^2 + y^2 = 25$ and $\frac{dy}{dt} = 6$ cm/sec, find $\frac{dx}{dt}$ when $y = 4$ cm.

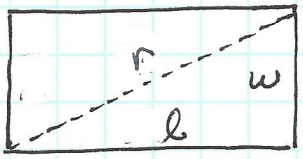
HW #6: Sand is being dumped on a pile in such a way that it always forms a cone whose radius equals its height. If the sand is being dumped at a rate of 10 ft^3/min, at what rate is the height of the pile increasing when there is 1000 ft^3 of sand on the pile?

HW 4.6 RELATED RATES p. 20 # 1-6

HW DAY 71
#1-2-3,6

#1) $\frac{dw}{dt} = 2\frac{cm}{sec}$

A) $\frac{dl}{dt} = 3\frac{cm}{sec}$



$A = l \cdot w$

$\frac{dA}{dt} = \frac{dl}{dt} w + l \cdot \frac{dw}{dt}$

ATQ: When $w = 4cm$ and $l = 5cm$ the area of the rectangle is increasing at a rate of $22\ cm^2/sec$.

$\frac{dA}{dt} = (3)(4) + (5)(2)$

$\frac{dA}{dt} = 22\ \frac{cm^2}{sec}$.

B) $\frac{dr}{dt} = ?$

$4^2 + 5^2 = r^2$

$\sqrt{41} = r$

$l^2 + w^2 = r^2$

$2l\frac{dl}{dt} + 2w\frac{dw}{dt} = 2r\frac{dr}{dt}$

ATQ: When $w = 4cm$ & $l = 5cm$ the length of the diagonal is increasing at a rate of 3.951 cm/sec.

$\frac{(5)(3) + (4)(2)}{\sqrt{41}} = \frac{dr}{dt}$

$\frac{dr}{dt} = \frac{23}{\sqrt{41}} \approx 3.591$ or 3.592

C) $\frac{dP}{dt} = ?$

$P = 2(l + w)$

$\frac{dP}{dt} = 2\left(\frac{dl}{dt} + \frac{dw}{dt}\right)$

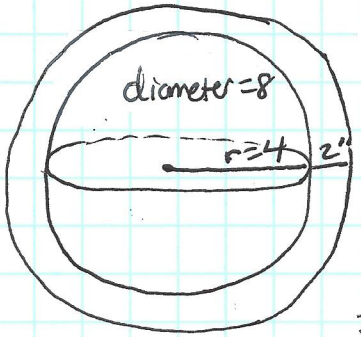
$\frac{dP}{dt} = 2(3 + 2)$

ATQ: When $w = 4cm$ & $l = 5cm$ the Perimeter is increasing at a rate of 10 cm/sec.

$\frac{dP}{dt} = 10\ \frac{cm}{sec}$.

#2)

A)



$\frac{dr}{dt} = ?$ $\frac{dV}{dt} = -10\ \frac{in^3}{min}$ $V = \frac{4\pi}{3} r^3$

When 2" thick $\therefore r = 6$

$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$

$\frac{dr}{dt} = \frac{\frac{dV}{dt}}{4\pi r^2}$

$\frac{dr}{dt} = \frac{-10}{4\pi(6)^2} = \frac{-5}{72\pi}\ \frac{in}{min}$

ATQ: When the ice is 2 inches thick, the thickness of the ice is decreasing at a rate of $\frac{-5}{72\pi}$ in/min

B) $SA = 4\pi r^2$

$\frac{d(SA)}{dt} = 8\pi r \frac{dr}{dt}$

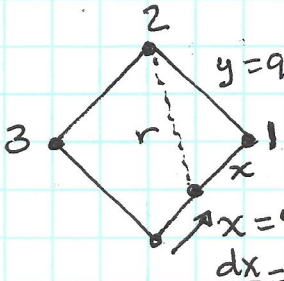
$= 8\pi(6)\left(\frac{-5}{72\pi}\right) = \frac{4 \cdot 2 \cdot 6(-5)\pi}{4 \cdot 3 \cdot 6 \cdot \pi} = \frac{-10}{3}\ \frac{in}{min}$

ATQ: When the ice is 2" thick the surface area is decreasing at a rate of $\frac{10}{3}\ \frac{in}{min}$.

H.W. 4.6c RELATED RATES p.220 #3,4,5

DAY 71 HW #3

#3)



$45^2 + 90^2 = r^2$

$45^2(1+2^2) = r^2$

$45\sqrt{5} = r$

$x^2 + y^2 = r^2$

$x\frac{dx}{dt} + y\frac{dy}{dt} = r\frac{dr}{dt}$

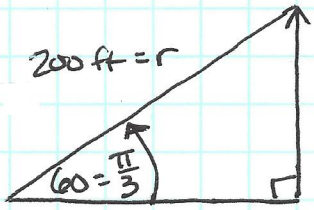
$\frac{dr}{dt} = \left(\frac{1}{r}\right)\left(x\frac{dx}{dt} + y\frac{dy}{dt}\right)$

$\frac{dr}{dt} = \frac{1}{45\sqrt{5}}\left(45\cdot(-20) + 0\right)$

$\frac{dr}{dt} = \frac{-20}{\sqrt{5}} = -4\sqrt{5}\ \frac{ft}{sec}.$

ATQ: When the runner is ½ way to 1st base the distance the runner is from 2nd base is decreasing at a rate of $4\sqrt{5}$ ft/sec.

#4)



$\frac{d\theta}{dt} = \frac{\pi}{15}\ \frac{rad}{sec}.$

$\theta = \frac{\pi}{3}\quad \cos\theta = \frac{1}{2}\ \&\ \sec\theta = \frac{2}{1}$

$\tan\theta = \frac{y}{100}$

$100\tan\theta = y$

$100\sec^2\theta\frac{d\theta}{dt} = \frac{dy}{dt}$

$100\left(\frac{2}{1}\right)^2\left(\frac{\pi}{15}\right) = \frac{dy}{dt}$

$\frac{dy}{dt} = \frac{80\pi}{3}\ \frac{ft}{sec}.$

SAVE for IN CLASS DAY 72 #4 #5

USE RADIANS B/C Radians are a linear unit. If radius is in feet then $\frac{radians}{sec} = \frac{feet}{sec}$. Rocket's height is increasing at a rate of $\frac{80\pi}{3}\ \frac{ft}{sec} \approx 42\ \frac{ft}{sec}$.

#5) CIRCLE w/ radius 5

$x^2 + y^2 = 25$

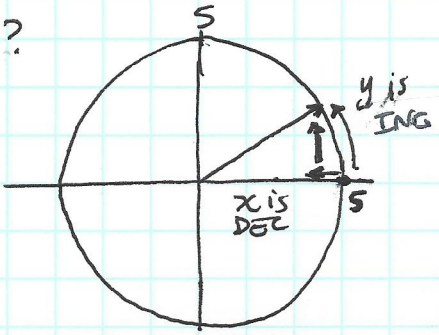
when $y = 4$ cm & $\frac{dy}{dt} = 6\ \frac{cm}{sec} \Rightarrow x = 3$

$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$

$\frac{dx}{dt} = \left(-y\frac{dy}{dt}\right)\left(\frac{1}{x}\right)$

$\frac{dx}{dt} = -4(6)\left(\frac{1}{3}\right) = -8\ \frac{cm}{sec}$

$\frac{dx}{dt} = ?$

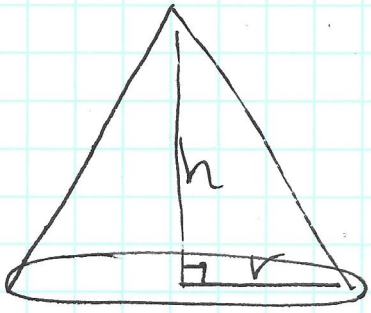


When $y = 4$ cm x is decreasing at a rate of $8\ \frac{cm}{sec}$

HW 4.6 RELATED RATES p. 20 #6

DAY 11 HW #6

#6



$V = \frac{1}{3}\pi r^2 h$ & $h = r$ & $\frac{dV}{dt} = 10 \frac{ft^3}{min}$

$V = \frac{\pi}{3} h^3$

$\frac{dh}{dt} = \frac{dr}{dt}$

$\frac{dV}{dt} = \pi h^2 \frac{dh}{dt}$

$V = 1000\ ft^3$

$\frac{\pi}{3} h^3 = 1000$

$h^3 = \frac{3000}{\pi}$

$h = \sqrt[3]{\frac{3000}{\pi}}\ ft.$

$\frac{dh}{dt} = \frac{\frac{dV}{dt}}{(\pi h^2)}$

$\frac{dh}{dt} = \frac{+10}{\pi \left(\sqrt[3]{\frac{3000}{\pi}}\right)^2} \frac{ft^3/min}{ft^2}$

$\pi \left(\frac{1}{\pi^{2/3}}\right) = \sqrt[3]{\pi}$

$\frac{dh}{dt} = 0.0328 \frac{ft}{min}$

$\frac{10}{\sqrt[3]{\pi}\ \sqrt[3]{3000^2}} \frac{ft}{min}$

ATQ: When there is $1000\ ft^3$ of sand on the pile, the height is increasing at a rate of $0.0328\ ft/min$.