✯ HW ✓

§4.2 Optimization Practice & HW

Exercises: For each of the following function use derivative techniques to:

I) Find the critical points using derivative techniques and identify them as maxima or minima.

II) Find the absolute maximum and minimum values of each function if it exists.

III) Identify the intervals on which the function is increasing or decreasing.

IV) Find points of inflection and intervals where the functions are concave up or concave down for the

SHOW ALL WORK in HW section of your notebook.

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1. $f(x) = 4x^3 + 3x^2 - 6x + 1$ on the interval [-2, 1]. Use the first derivative test to justify extrema.
2. $f(x) = \sin x - \cos x$ on the interval $[0, \pi]$ Use the second derivative test to justify extrema.
3. $f(x) = \sqrt[5]{x^2}$ on the interval [-1, 32] Use the first derivative test to justify extrema.
4. $f(x) = x - \ln x$ on the interval [0.1, 5] Use the second derivative test to justify extrema.

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5. $f(x) = x + \dfrac{32}{x^2}$ over all real numbers Use the first derivative test to justify extrema.
6. $f(x) = 2x - e^x$ on the interval [-2, 4] Use the second derivative test to justify extrema.
7. $f(x) = 3x\sqrt[3]{x} - 2x$ on the interval [0, 3] Use the first derivative test to justify extrema.
8. $f(x) = \dfrac{x^4 + 1}{x^2}$ Use the second derivative test to justify extrema.

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9. $f(x) = e^x - \ln x^2$ Use the first derivative test to justify extrema.
10. $f(x) = 3\sqrt[3]{x} - 2x$ for the interval [-2, 3] Use the second derivative test to justify extrema.

In exercises 11 and 12, the derivative of the function $y = f(x)$ is given. At what points, if any, does the graph have a relative minimum or relative maximum?

11. $\dfrac{dy}{dx} = (x-1)^2 (x-2)$
12. $\dfrac{dy}{dx} = (x-1)^2 (x-2)(x-4)$

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§4.2 OPTIMIZATION

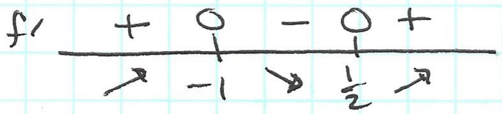
NOTES
H.W.
$\boxed{\text{DAY 62}}$

(p.15)

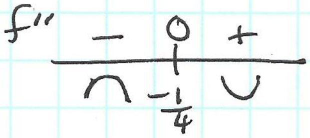
① $f(x) = 4x^3 + 3x^2 - 6x + 1$ on $[-2, 1]$

$f'(x) = 12x^2 + 6x - 6$
$= 6(2x^2 + x - 1)$
$= 6(2x-1)(x+1) = 0$

$f'(x) = 0 \quad x = \frac{1}{2}, -1$



$f''(x) = 6(4x+1)$
$f''(x) = 0 \quad x = -\frac{1}{4}$



$\boxed{\text{TABLE}}$

X	−2	−1	½	1
y	−7	6	$-\frac{3}{4}$	2

$6 = f(-1)$ is a ABS & Rel MAX b/c
f' changes sign ⊕ to ⊖

$-\frac{3}{4} = f(\frac{1}{2})$ is Rel min b/c
f' changes sign ⊖ to ⊕.

$-7 = f(-2)$ is ABS MIN

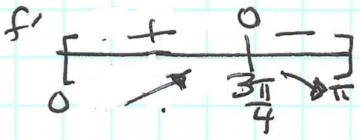
f is increasing on $(-2,-1)$ $(\frac{1}{2}, 1)$ b/c $f' > 0$.
f is decreasing on $(-1, \frac{1}{2})$ b/c $f' < 0$.

$(-\frac{1}{4}, \frac{21}{8}) = (-.25, 6.625)$ is the Inf. Point
b/c $f''(x)$ changes ⊖ to ⊕ @ $x = -\frac{1}{4}$.

$f(x)$ is concave up on $(-\frac{1}{4}, \infty)$ b/c $f'' > 0$
$f(x)$ is concave down $(-\infty, -\frac{1}{4})$ b/c $f'' < 0$.

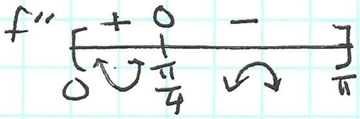
② $f(x) = \sin x - \cos x \quad [0, \pi]$

$f'(x) = \cos x + \sin x$
$f'(x) = 0 \quad \cos x = -\sin x$
$x = \frac{3\pi}{4}$.



$f''(x) = -\sin x + \cos x$
$f''(\frac{3\pi}{4}) = -\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2} = -\sqrt{2}$

$f''(x) = 0 \quad \sin x = \cos x$
$x = \frac{\pi}{4}$



$\boxed{\text{TABLE}}$

X	0	$\frac{3\pi}{4}$	π
Y	−1	$+\sqrt{2}$	+1
	MIN	MAX.	MAX

$f''(\frac{3\pi}{4}) = -(\sqrt{2}) < 0$ & ∴ $f(\frac{3\pi}{4}) = \sqrt{2}$ is
a Rel MAX b/c f is cc down.
by the 2nd Derivative Test.

ABS MIN $= -1$ @ $x = 0$
ABS MAX $= \sqrt{2}$ @ $x = \frac{3\pi}{4}$
REL MIN $= 1$ @ $x = \pi$.

f is increasing on $(0, \frac{3\pi}{4})$ b/c $f' > 0$
f is decreasing on $(\frac{3\pi}{4}, \pi)$ b/c $f' < 0$.

f is concave up $(0, \frac{\pi}{4})$ b/c $f'' > 0$
f is concave down $(\frac{\pi}{4}, \pi)$ b/c $f'' < 0$

Inflection Point @ $(\frac{\pi}{4}, 0)$ b/c
f'' changes sign ⊕ to ⊖.

§4.2 OPTIMIZATION NOTES HW $\boxed{\text{DAY 62}}$

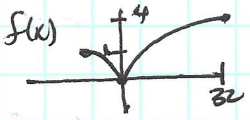
③ $f(x) = \sqrt[5]{x^2} = x^{2/5}$ $[-1, 32]$

$f'(x) = \frac{2}{5}x^{-3/5} = \frac{2}{5x^{3/5}}$

f' $[$ $-$ $\not 0$ $+$ $]$

-1 ↘ 0 ↗ 32

TABLE



x	-1	0	32
y	1	0	4
	MAX REL	MIN ABS	MAX ABS.

$f(-1)=1$ is REL MAX

$f(0)=0$ is ABS MIN

$f(32)=4$ is ABS MAX.

b/c change in sign f' ⊖ to ⊕.

f is increasing on $(0,32)$ b/c $f'>0$

f is decreasing on $(-1,0)$ b/c $f'<0$.

$f''(x) = \frac{-6}{25}x^{-8/5} = \frac{-6}{25x^{8/5}}$ f'' undefined $x=0$.

$f''(x) < 0$ for all x except $x=0$

$\therefore$ $f(x)$ is concave down $(-1,0)(0,32)$.

$f''(x)$ never changes sign so $f(x)$ has no inflection points.

④ $f(x) = x - \ln x$ $[0.1, 5]$

$f'(x) = 1 - \frac{1}{x} = \frac{x-1}{x}$

C.P. $f'(x)=0$ $x=1$

$f'(x)$ und $x=0$

f' $[$ $-$ 0 $+$ $]$

0.1 ↘ 1 ↗ 5

TABLE

x	0	0.1	1	5
y	$\not 0$	$\frac{1}{10}+\ln 10$ 2.403	1	$5-\ln 5$ 3.391

$f(5)=3.391$ is ABS MAX on $[0.1, 5]$.

$f''(1) = 1 > 0$ $\therefore$ f is ccup at C.P $x=1$

$\therefore$ $(1,1)$ is a Rel min. & ABS. min.

by 2nd Deriv Test.

f is increasing on $(1,5)$ b/c $f'>0$

f is decreasing on $(0.1, 1)$ b/c $f'<0$.

$f''(x) = \frac{+1}{x^2}$

$f''(x) = 0$ never

$f''(x)$ und $x=0$

f'' $+$ $\not 0$ $+$

$\cup$ 0 $\cup$

$f(x)$ is concave up for all $[0.1, 5]$, & all $x \neq 0$.

$f(x)$ is never concave down.

$f(x)$ has no inflection points b/c never a sign change on $f''(x)$

§ 4.2 OPTIMIZATION NOTES DAY 63 #5,6,7,8 (p.15)

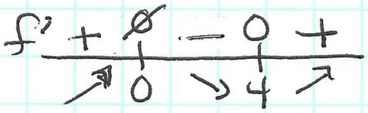
⑤ $f(x) = x + \frac{32}{x^2}$ $\quad x \in \mathbb{R}.$ $\quad x \neq 0$

$f'(x) = 1 - 64x^{-3}$

$1 - \frac{64}{x^3}$

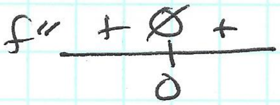
$f'(x) = \frac{x^3 - 64}{x^3} = \frac{(x-4)(x^2+4x+16)}{x^3}$

$$\text{C.P.} \begin{cases} f'(x) = 0 & x = 4 \\ f'(x) \text{ und} & x = 0 \end{cases}$$



$f''(x) = +3(64)(x^{-4})$

$f''(x) = \frac{192}{x^4} > 0$ for all $x \neq 0$



$f(x)$ increasing on $(-\infty, 0)(4, \infty)$
b/c $f' > 0$

$f(x)$ decreasing on $(0, 4)$
b/c $f' < 0$

$f(4) = 6$ Rel min b/c
f' changes sign ⊖ to ⊕.

$f(x)$ is always concave up
$(-\infty, 0)$, $(0, \infty)$
b/c $f''(x) > 0$

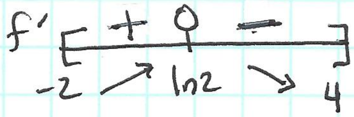
$f(x)$ has no inflection points.

⑥ $f(x) = 2x - e^x$ $\quad [-2, 4]$

$f'(x) = 2 - e^x = 0$

$e^x = 2$

$x = \ln 2$



$f''(x) = -e^x = 0$

never

TABLE	x	-2	$\overset{.693}{\ln 2}$	4
	y	$-4 - \frac{1}{e^2}$ -4.135	$\ln 4 - 2$ $-.6137$	$8 - e^4$ -46.598

2nd DerivTest

$f''(\ln 2) = -e^{\ln 2} = -2 < 0$

$\therefore$ $y \approx -0.6137$ $y = \ln 4 - 2$ is an ABS/REL MAX

b/c $f'' < 0$ at critical point.

$f(-2)$ & $f(4)$ are rel MIN & $f(4) = -46.598$ is ABS MIN.

$f(x)$ is decreasing $(\ln 2, 4)$ b/c $f' < 0$

$f(x)$ is increasing $(-2, \ln 2)$ b/c $f' > 0$

$f''(x) < 0$ always so $f(x)$ is always concave down.

$f(x)$ has no inflection points.

§4.2 OPTIMIZATION NOTES $\boxed{\text{DAY 63}}$ #7, 8

⑦ $f(x) = 3x\sqrt[3]{x} - 2x \qquad [0, 3]$

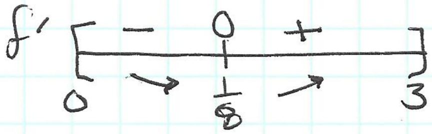
$f(x) = 3x^{4/3} - 2x$

$f'(x) = 4x^{1/3} - 2$

$4\sqrt[3]{x} - 2 = 0$

$\sqrt[3]{x} = \frac{1}{2}$

$x = \frac{1}{8}$



TABLE

x	0	$\frac{1}{8}$	3
y	0	$-\frac{1}{16}$ -0.0625	$9\sqrt[3]{3}-6$ ≈ 6.980
		MIN ABS	MAX. ABS.

f(x) increasing on $(\frac{1}{8}, 3)$ b/c $f' > 0$

f(x) decreasing on $(0, \frac{1}{8})$ b/c $f' < 0$

$f(\frac{1}{8}) = -\frac{1}{16} \approx -0.0625$ is an $\overset{\text{Rel}}{\text{ABS}}$ MIN b/c f′ changes sign ⊖ to ⊕.

$f(3) \approx 6.980$ is ABS MAX.

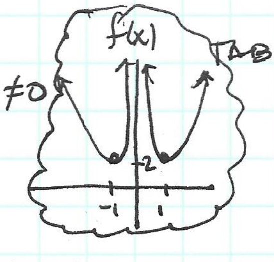
$f(0) = 0$ Rel max.

$f''(x) = \frac{4}{3}x^{-2/3} = \frac{4}{3x^{2/3}} > 0$ for all $x \neq 0$.

f(x) concave up for all $x \in [0, 3]$ b/c $f''(x) > 0$.

f(x) has no points of inflection b/c f″ has no change in sign.

⑧ $f(x) = \frac{x^4+1}{x^2} = x^2 + x^{-2}$ $\quad x \neq 0$



TABLE

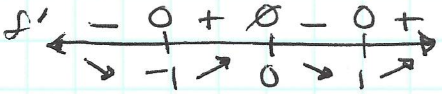
X	−1	0	1
Y	2	und	2

$f'(x) = 2x - \frac{2}{x^3} = \frac{2x^4-2}{x^3}$

$f'(x) = 0 \qquad 2(x^4-1) = 2(x-1)(x+1)(x^2+1) = 0$

@ $x = \pm 1$

$f'(x)$ und @ $x = 0$



2nd Deriv Test @ C.P.

$f''(-1) = 8 > 0$ ∴ $f(-1) = 2$ is a MIN ABS

$f''(+1) = 8 > 0$ $f(1) = 2$ is a min. ABS.

$f(-1) = f(1) = 2$ are both rel & abs minimums on f(x).

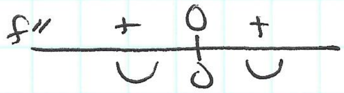
f(x) dec on $x \in (-\infty, -1)\ (0, 1)$ b/c $f' < 0$

f(x) inc on $x \in (-1, 0)(1, \infty)$ b/c $f' > 0$

$f''(x) = 2 + \frac{6}{x^4} = \frac{2x^4+6}{x^4} = \frac{2(x^4+3)}{x^4}$

$f''(x) = 0$ never

$f''(x)$ und $x = 0$



f(x) is concave up on $(-\infty, 0)\ (0, \infty)$ b/c $f'' > 0$.

f(x) has no inflection points.

§4.2 OPTIMIZATION NOTES HW $\boxed{\text{DAY 64}}$ #9-12 p15

⑨ $f(x) = e^x - \ln x^2$

$f(x) = e^x - 2\ln x$

$f'(x) = e^x - \frac{2}{x} = 0$

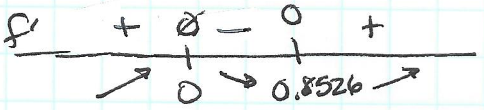
$f'(x) = 0$ TI-84 solve.

$x \approx 0.8526$

$f'(x)$ und $x=0$

$e^x = \frac{2}{x}$

$xe^x = 2$



$\boxed{\text{TABLE}}$

x	0	0.8526
y	∅	2.6646

$f(x)$ increasing on $(-\infty, 0]$ $(0.8526, \infty)$ b/c $f'(x) > 0$

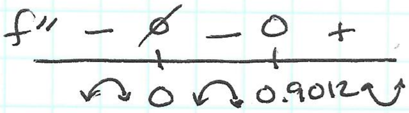
$f(x)$ decreasing on $(0, 0.8526)$ b/c $f'(x) < 0$

$f(0.8526) = 2.6646$ Rel min b/c $f'(x)$ changes ⊖ to ⊕.

$f''(x) = e^x + \frac{2}{x^2} = 0$

$f''(x)$ und $x=0$

$f''(x) = 0$ $x = 0.9012$



$f(x)$ is concave down $(-\infty, 0)$ $(0, 0.9012)$ b/c $f'' < 0$

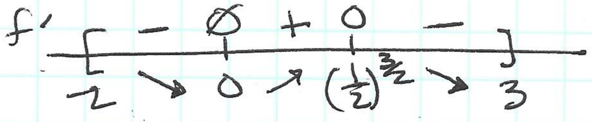
$f(x)$ is concave up $(0.9012, \infty)$ b/c $f'' > 0$

$(0.9012, 2.6706)$ is a point of inflection b/c f'' changes sign ⊖ to ⊕.

⑩ $f(x) = 3\sqrt[3]{x} - 2x$ $[-2, 3]$

$f'(x) = x^{-2/3} - 2 = \frac{1}{x^{2/3}} - 2$

$f'(x) = \frac{1 - 2x^{2/3}}{x^{2/3}}$ $= 0$ @ $x = \left(\frac{1}{2}\right)^{3/2}$ $=$ und @ $x = 0$



$f''(x) = -\frac{2}{3}x^{-5/3} = \frac{-2}{3x^{5/3}} < 0$

for all $x \neq 0$

$\boxed{\text{TABLE}}$

x	−2	0	$\left(\frac{1}{2}\right)^{3/2}$	3
y	0.2202	0	1.4142 $\sqrt{2}$	−1.6733

$f(3)$ ABS MIN, $f(0)$ Rel MIN, $f(-2) \overset{\text{MAX}}{=} .2202$ Rel MAX

$f''\left(\left(\frac{1}{2}\right)^{3/2}\right) < 0$ $\therefore$ $f\left(\left(\frac{1}{2}\right)^{3/2}\right) = 1.414 = \sqrt{2}$ is an ABS (Rel) MAX.

by 2nd Deriv Test. since f is concave down.

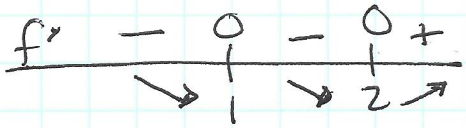
f is increasing $\left(0, \left(\frac{1}{2}\right)^{3/2}\right)$ b/c $f' > 0$

f is decreasing $(-2, 0)$ $\left(\left(\frac{1}{2}\right)^{3/2}, 3\right)$ b/c $f' < 0$.

$f(x)$ is always concave down for $x \in [-2, 0)(0, 3]$. b/c $f''(x) < 0$

§4.2 OPTIMIZATION NOTES HW $\boxed{\text{DAY 64}}$ #9–12

⑪ $\frac{dy}{dx} = (x-1)^2(x-2) = 0$ $x=1$ $x=2$



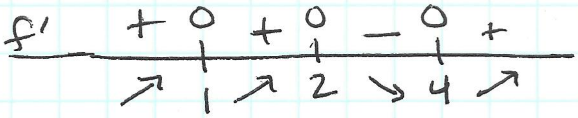
f(x) increasing on (2,∞) b/c f' > 0

f(x) decreasing on (−∞,1)(1,2) b/c f' < 0

f(x) has a rel min = f(2)
b/c f' changes sign
⊖ to ⊕ at x = 2.

f(x) has a terrace point
at (1, f(1)) b/c
f' < 0 on (−∞,1)(1,2).

⑫ $\frac{dy}{dx} = (x-1)^2(x-2)(x-4) = 0$



f(x) increasing on (−∞ 1)(1,2)(4,∞)
b/c f' > 0

f(x) decreasing on (2,4)
b/c f' < 0

f(x) has terrace pt @ (1, f(1))
b/c f' > 0 (−∞,1)(1,2)

f(x) has rel max = f(2)
b/c f' changes sign
⊕ to ⊖ at x = 2

f(x) has rel min = f(4)
b/c f' changes sign
⊖ to ⊕.