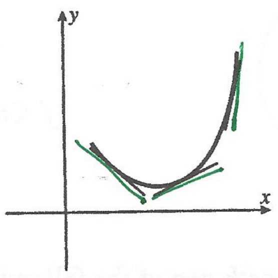
KEY EAGEN

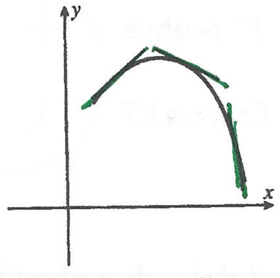
2.5 The Second Derivative Function—Student Notes HH6ed

A function $f(x)$ is *concave upward* on an interval I if $f(x)$ lies above all tangent lines to $f(x)$ in I.

A function $f(x)$ is *concave downward* on an interval I if $f(x)$ lies below all tangent lines to $f(x)$ in I.



concave upward



concave downward

The test for concavity involves the second derivative: If $f(x)$ is twice differentiable on an interval I (meaning $f''(x)$ exists for all x on the interval I) then

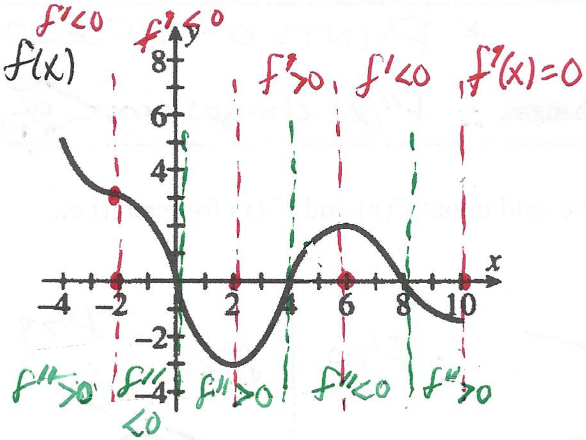
f'' positive

a. If $f''(x) > 0$ for all x on the interval I, then f is *concave upward* on I.

b. If $f''(x) < 0$ for all x on the interval I, then f is *concave downward* on I.

f'' negative

Example 1: Use the graph below to answer true or false to each.



a) $f''(x) > 0$ for $x \in (2, 4)$ TRUE f(x) is concave up on (2,4)

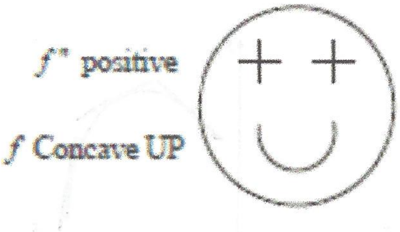
b) $f''(x) < 0$ for $x \in (-4, -2)$ FALSE f(x) is concave up on (-4,-2) so f'' > 0.

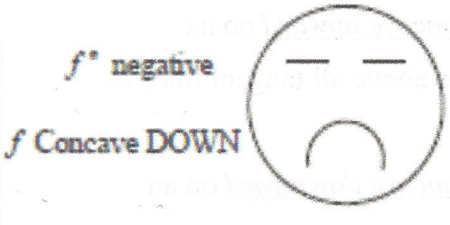
c) $f''(6) = 0$ FALSE

d) $f''(2) > 0$ TRUE f(x) is concave up

e) f is concave upward on $(0, 2)$ TRUE

The concavity test can be remembered with the following pictures … keep in mind these are NOT to be used for justification.





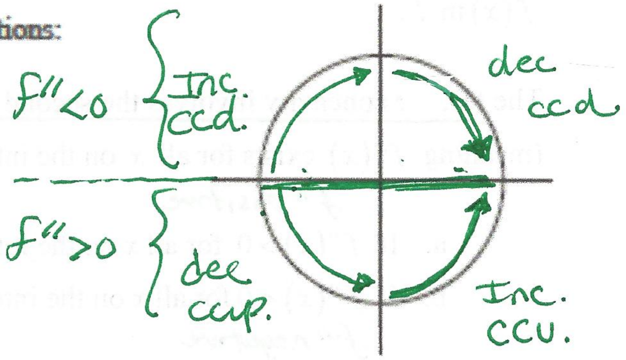
Example: Label each quadrant below with one of the following descriptions:

i) Increasing and Concave Up IV

ii) Increasing and Concave Down II

iii) Decreasing and Concave Up III

iv) Decreasing and Concave Down. I



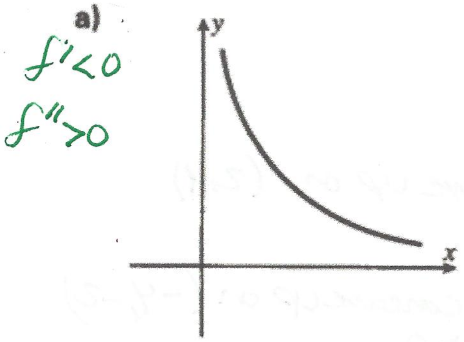
Points of Inflection

A point of inflection is a point on the graph where the concavity changes. ∴ $f''(x)$ changes sign ⊖ to ⊕ or ⊕ to ⊖

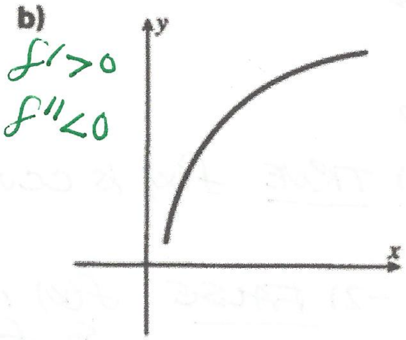
* POINTS OF INFLECTION

Example 2: The graph of a function f is given. What can be said about $f'(x)$ and $f''(x)$ for each (i.e., positive/negative/where)?

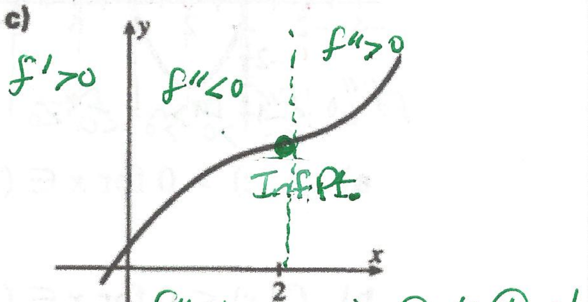
a)



b)

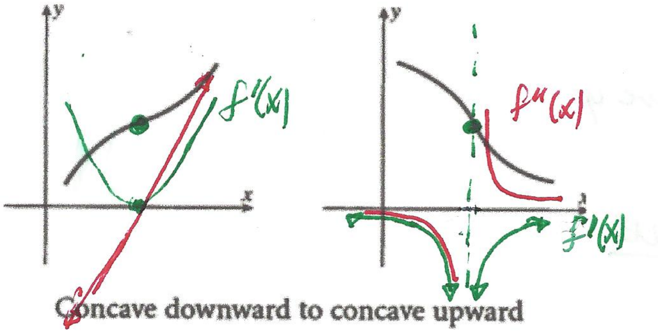


c)

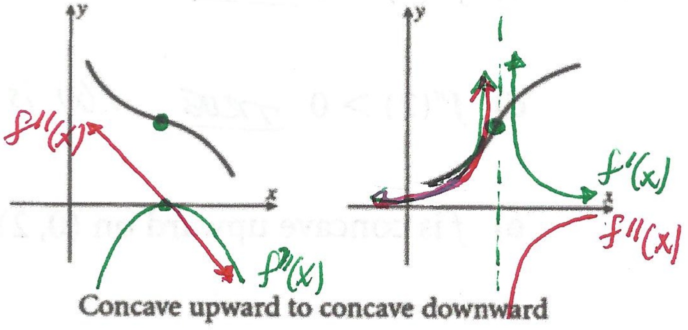


f'' changes sign ⊖ to ⊕ at $x=2$ so $(2, f(2))$ is an Inf. Pt

A *point of inflection* for f is a point on the graph of f where concavity changes from concave downward to concave upward or from concave upward to concave downward.



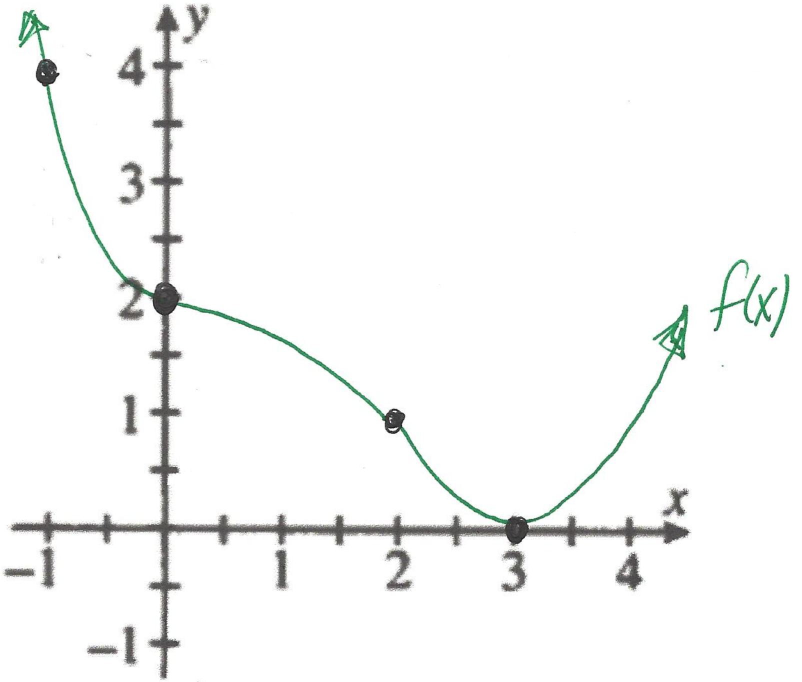
Concave downward to concave upward

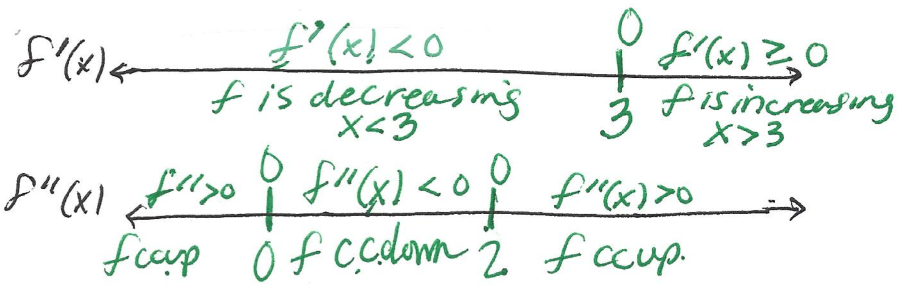


Concave upward to concave downward

<u>Example 3:</u> Sketch a graph of a function having all of the following properties.

$f(-1) = 4, f(0) = 2, f(2) = 1, f(3) = 0$
$f'(x) \leq 0$ for $x < 3$ and
$f'(x) \geq 0$ for $x > 3$.
$f''(x) < 0$ for $0 < x < 2$ and
$f''(x) \geq 0$ elsewhere.





Inflection Points occur
when $f''(x) = 0$ and
$f''(x)$ changes sign ⊖ to ⊕ or
⊕ to ⊖