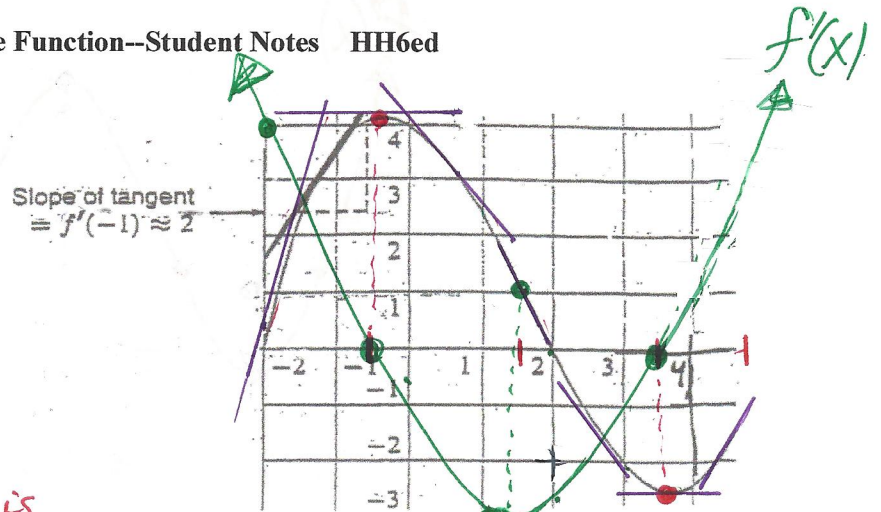


2.3 The Derivative Function--Student Notes HH6ed

The graph at the right is some function  $f(x)$  with a tangent drawn to the curve at the point  $x = -1$ .



1. Complete the table below by indicating, for each of the specified intervals, whether the slope of the function is positive or negative, increasing or decreasing on the interval.

Interval	Slope is Positive/Negative	Slope is Increasing/decreasing
$(-2, -0.5)$	+	decreasing
$(-0.5, 1.5)$	-	decreasing
$(1.5, 3.5)$	-	increasing
$(3.5, 5)$	+	increasing

+ + 0 - - - 0 +  
 +4 +1 0 -1 -2 -1 0 +2  
 decreasing slopes      increasing slopes.

2. Using a straightedge and pencil, lightly sketch the tangents to the function and estimate the slopes of the tangents. Complete the table of values for the derivative function below.

$x$	-2	-1	0	1	2	3	4	5
$f'(x)$	+4	+2	$-\frac{1}{2}$	-3	-2.5	-1	$\frac{1}{2}$	2

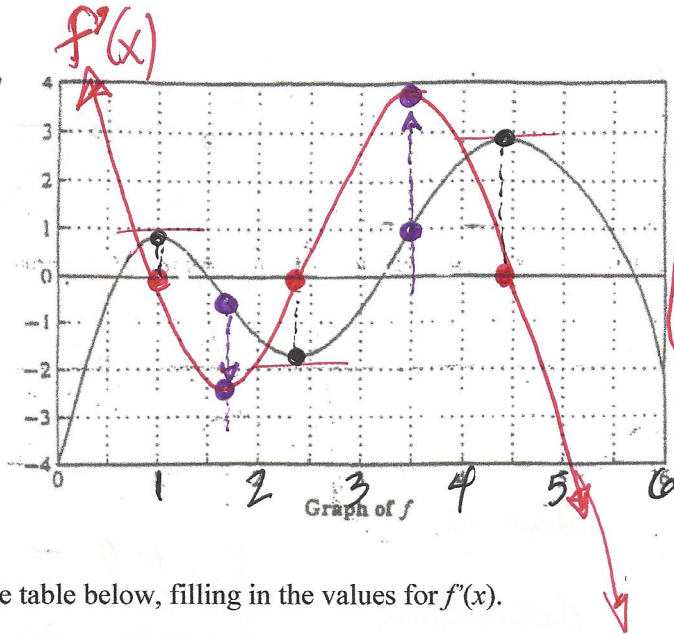
3. Using a colored pencil, sketch the graph of the derivative function by plotting your table values and connecting them with a smooth curve. Do this on the grid of  $f(x)$ .

4. Based on the graphs of  $f(x)$  and its derivative  $f'(x)$ , answer these questions:

- When the derivative function  $f'(x)$  is positive, the graph of  $f(x)$  is increasing
- When the derivative function  $f'(x)$  is negative, the graph of  $f(x)$  is decreasing
- When the derivative function  $f'(x)$  changes sign, the graph of  $f(x)$  is has a local max or local min
- When the derivative function  $f'(x)$  has a turning point, the graph of  $f(x)$  is has an inflection point a change in concavity

The graph of the function  $f$  is shown below.

5.

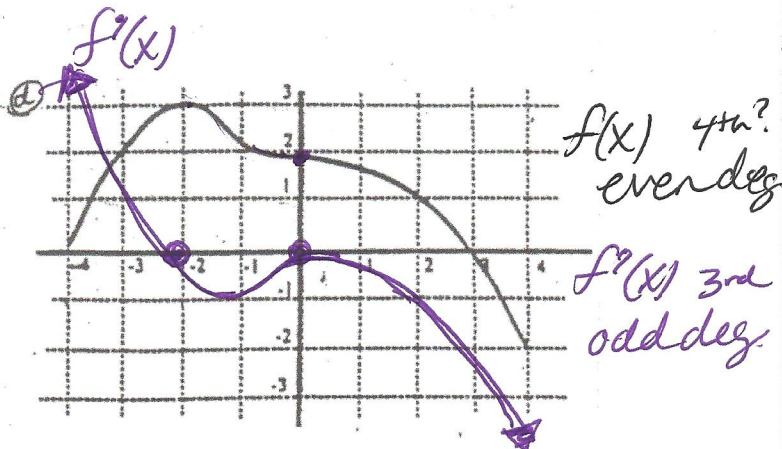
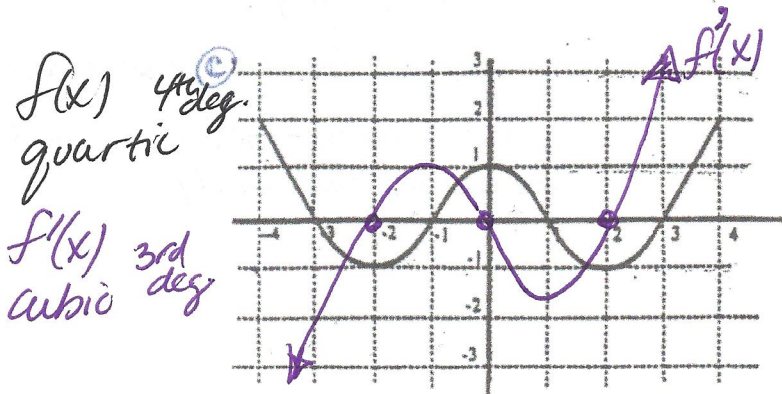
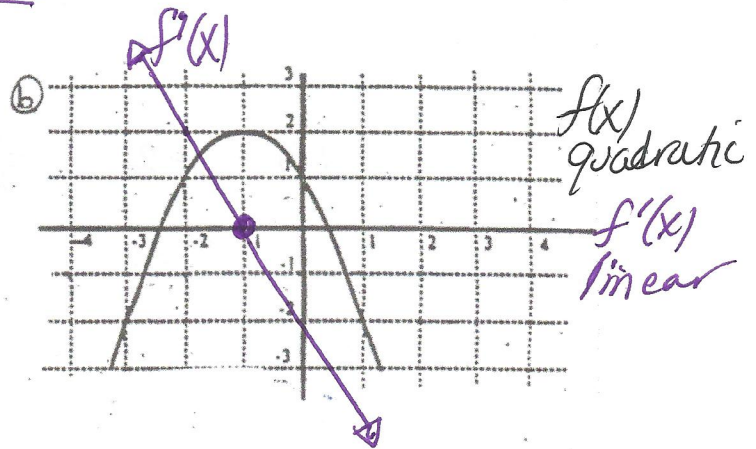
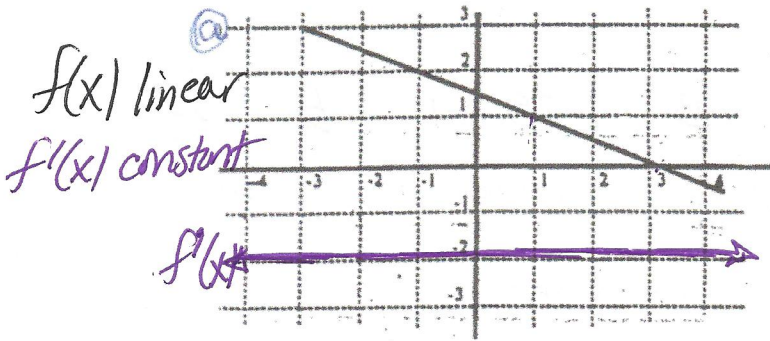


a. Complete the table below, filling in the values for  $f'(x)$ .

$x$	1	2	3	4	5	6
$f'(x)$	0	-2	2.5	2.5	-3	-8

b. Sketch a graph of  $f'(x)$ . Do this on the grid of  $f(x)$ .

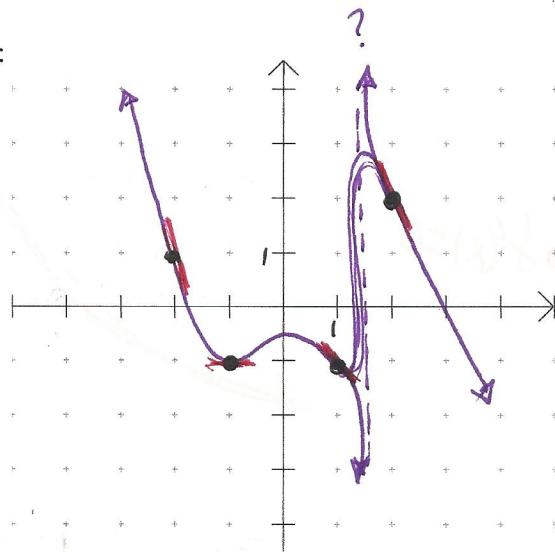
6. For each of the following, sketch a graph of the derivative function on the axes with the function. Use a colored pencil.



Practice:

1. a. Sketch the graph of a function  $f$  that is consistent with these data:

$x$	-2	-1	1	2
$f(x)$	1	-1	-1	2
$f'(x)$	-3	0	-1	-2



b. Write an equation for the tangent line to the function  $f$  at  $x = -2$ .

$$f(-2) = 1$$

$$f'(-2) = -3$$

tangent line

$$y = -3(x + 2) + 1$$

2. The line tangent to a function  $f$  at  $(5, 2)$  passes through the point  $(0, 1)$ . Find  $f(5)$  and  $f'(5)$ .

$$m = \frac{1 - 2}{0 - 5} = \frac{-1}{-5} = \frac{1}{5}$$

$f(5) = 2$  that is given  $(5, 2)$ .

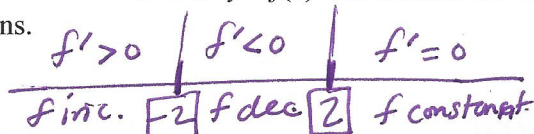
$f'(5) = \frac{1}{5}$  which is the slope.

3. Suppose that  $f'(x) \geq 0$  on the interval  $(2, 7)$ . Explain why  $f(3) \leq f(6)$ .

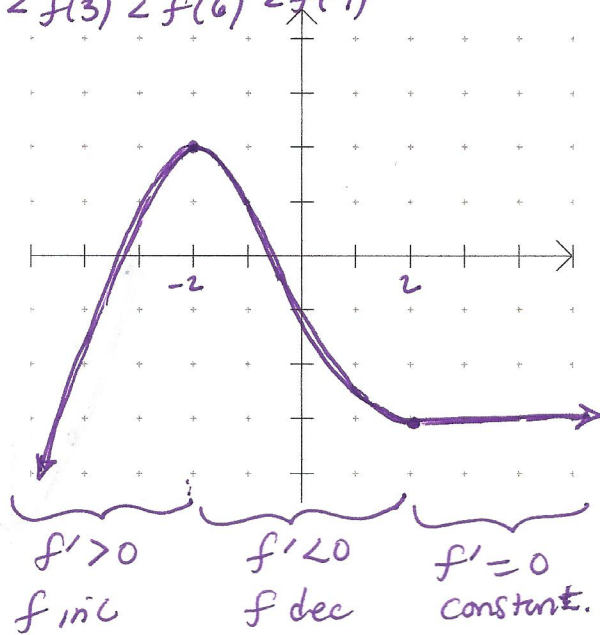
On  $(2, 7)$ ,  $f'(x) \geq 0$  means that the function is increasing b/c slope is positive so  $f(3) \leq f(6)$

$2 < 3 < 6 < 7$   
 $f(2) < f(3) < f(6) < f(7)$

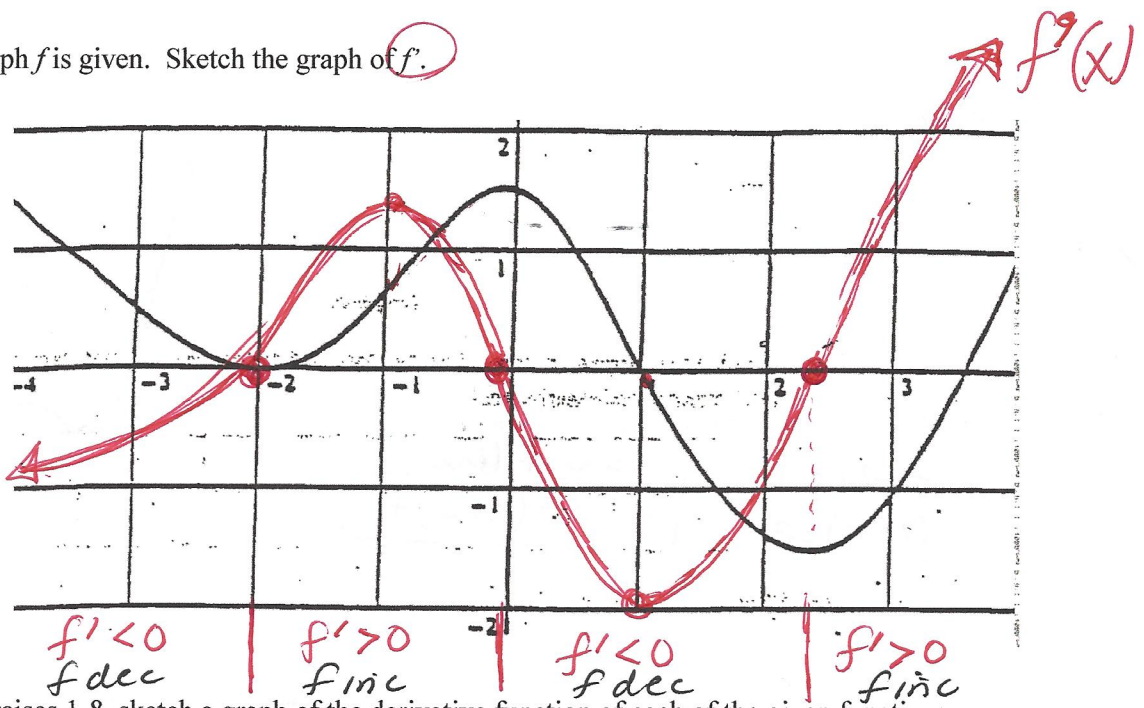
4. Draw the continuous function  $y = f(x)$  that satisfies the following three conditions.



- $f'(x) > 0$  for  $x < -2$
- $f'(x) < 0$  for  $-2 < x < 2$
- $f'(x) = 0$  for  $x > 2$

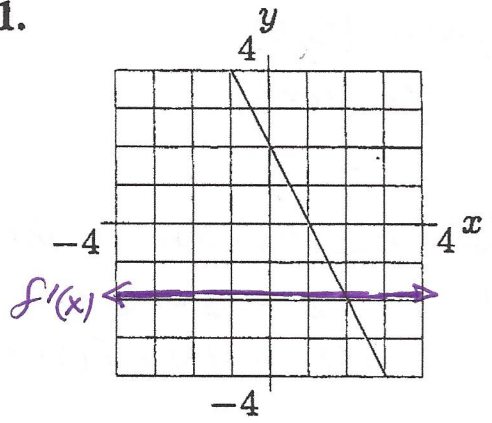


5. The graph  $f$  is given. Sketch the graph of  $f'$ .

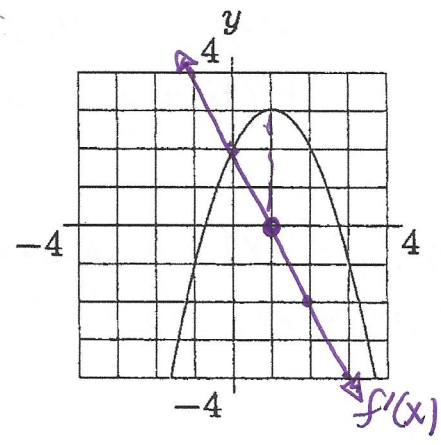


For exercises 1-8, sketch a graph of the derivative function of each of the given functions.

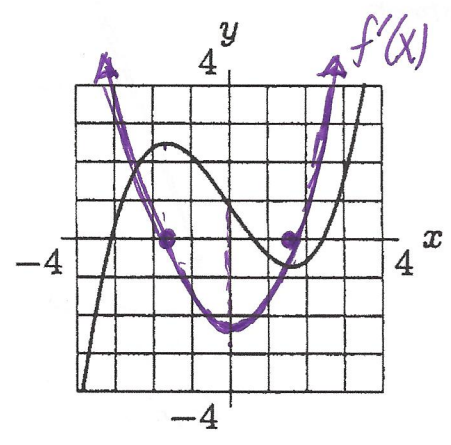
1.



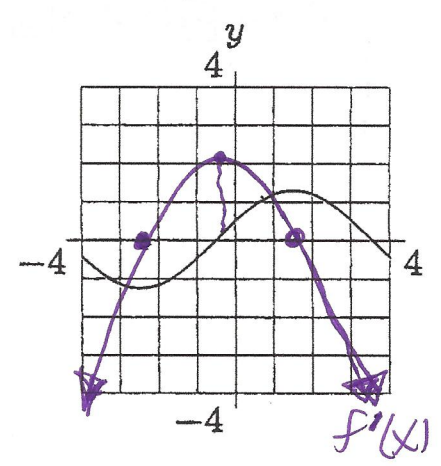
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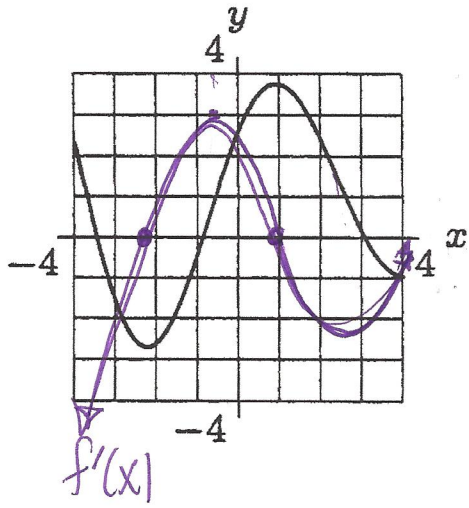
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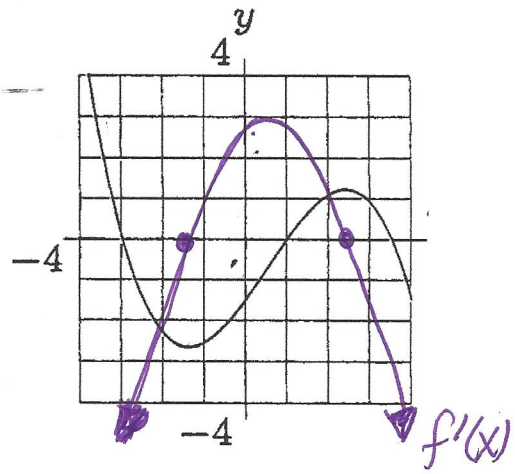
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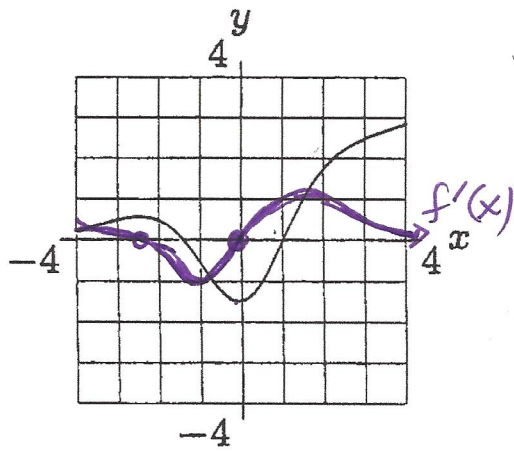
5.



6.



7.



8.

