I

**RELATED RATES (5 MC & 1 FRQ)** **AP REVIEW**

**Multiple Choice**

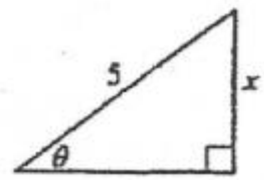
Identify the choice that best completes the statement or answers the question.

____ 1. A railroad track and a road cross at right angles. An observer stands on the road 70 meters south of the crossing and watches an eastbound train traveling at 60 meters per second. At how many meters per second is the train moving away from the observer 4 seconds after it passes through the intersection?

a. 57.60
b. 57.88
c. 59.20
d. 60.00
e. 67.40

____ 2. In the triangle shown above, if $\theta$ increases at a constant rate of 3 radians per minute, at what rate is $x$ increasing in units per minute when $x$ equals 3 units?

a. 3
b. $\frac{15}{4}$
c. 4
d. 9
e. 1




____ 3. The radius of a circle is decreasing at a constant rate of 0.1 centimeter per second. In terms of the circumference C, what is the rate of change of the area of the circle, in square centimeters per second?

a. $-(0.2)\pi C$
b. $-(0.1)C$
c. $-\frac{(0.1)C}{2\pi}$
d. $(0.1)^2C$
e. $(0.1)^2\pi C$

____ 4. If the base $b$ of a triangle is increasing at a rate of 3 inches per minute while its height $h$ is decreasing at a rate of 3 inches per minute, which of the following must be true about the area $A$ of the triangle?

a. $A$ is always increasing.
b. $A$ is always decreasing.
c. $A$ is decreasing only when $b < h$.
d. $A$ is decreasing only when $b > h$.
e. $A$ remains constant.

____ 5. The radius of a circle is increasing at a constant rate of 0.2 meters per second. What is the rate of increase in the area of the circle at the instant when the circumference of the circle is $20\pi$ meters?

a. $0.04\pi$ m²/sec
b. $0.4\pi$ m²/sec
c. $4\pi$ m²/sec
d. $20\pi$ m²/sec
e. $100\pi$ m²/sec

3. Oil is leaking from a pipeline on the surface of a lake and forms an oil slick whose volume increases at a constant rate of 2000 cubic centimeters per minute. The oil slick takes the form of a right circular cylinder with both its radius and height changing with time. (Note: The volume $V$ of a right circular cylinder with radius $r$ and height $h$ is given by $V = \pi r^2 h$.)

   (a) At the instant when the radius of the oil slick is 100 centimeters and the height is 0.5 centimeter, the radius is increasing at the rate of 2.5 centimeters per minute. At this instant, what is the rate of change of the height of the oil slick with respect to time, in centimeters per minute?

   (b) A recovery device arrives on the scene and begins removing oil. The rate at which oil is removed is $R(t) = 400\sqrt{t}$ cubic centimeters per minute, where $t$ is the time in minutes since the device began working. Oil continues to leak at the rate of 2000 cubic centimeters per minute. Find the time $t$ when the oil slick reaches its maximum volume. Justify your answer.

   (c) By the time the recovery device began removing oil, 60,000 cubic centimeters of oil had already leaked. Write, but do not evaluate, an expression involving an integral that gives the volume of oil at the time found in part (b).

FRQ

a) $r = 100$ cm

$h = .5$ cm

$\frac{dV}{dt} = 2000 \frac{cm^3}{min}$

$\frac{dr}{dt} = 2.5 \frac{cm}{min}$

$$\frac{dV}{dt} = 2\pi r \frac{dr}{dt} \cdot h + \pi r^2 \frac{dh}{dt}$$

$$2000 = 2\pi(100)(2.5)(.5) + \pi(100)^2 \frac{dh}{dt}$$

$$\frac{dh}{dt} = 0.038 \text{ or } 0.039 \ \frac{cm}{min}$$

b) $\frac{dV}{dt} = 2000 - R(t) = 0$ when $R(t) = 2000$, $t = 25$ min

$\frac{dV}{dt}$ changes signs from $+$ to $-$ at $t = 25$ min

$\therefore$ oil slick reaches MAX volume 25 minutes after the recovery device begins working.

c) The volume of oil, in $cm^3$, in the slick at time $t = 25$ min is

$$Vol. = 60000 + \int_0^{25} (2000 - R(t))\, dt.$$

I TABLE QUESTIONS (9 MC : 2FRQ) AP REVIEW

**Multiple Choice**

Identify the choice that best completes the statement or answers the question.

**NON-CALCULATOR #1-3 & Calculator Active #4-9**

1.

| $x$ | 2 | 5 | 7 | 8 |
|---|---|---|---|---|
| $f(x)$ | 10 | 30 | 40 | 20 |

The function $f$ is continuous on the closed interval [2, 8] and has values that are given in the table above. Using the subintervals [2, 5], [5, 7], and [7, 8], what is the trapezoidal approximation of $\int_2^8 f(x)dx$?

a. 110
b. 130
c. 160
d. 190
e. 210

2.

| $x$ | $f(x)$ | $f'(x)$ | $g(x)$ | $g'(x)$ |
|---|---|---|---|---|
| $-1$ | 6 | 5 | 3 | $-2$ |
| 1 | 3 | $-3$ | $-1$ | 2 |
| 3 | 1 | $-2$ | 2 | 3 |

The table above gives values of $f$, $f'$, $g$, and $g'$ at selected values of $x$. If $h(x) = f(g(x))$, then $h'(1) =$

a. 5
b. 6
c. 9
d. 10
e. 12

3. For all $x$ in the closed interval [2, 5], the function $f$ has a positive first derivative and a negative second derivative. Which of the following could be a table of values for $f$?

a.

| $x$ | $f(x)$ |
|---|---|
| 2 | 7 |
| 3 | 9 |
| 4 | 12 |
| 5 | 16 |

b.

| $x$ | $f(x)$ |
|---|---|
| 2 | 7 |
| 3 | 11 |
| 4 | 14 |
| 5 | 16 |

c.

| $x$ | $f(x)$ |
|---|---|
| 2 | 16 |
| 3 | 12 |
| 4 | 9 |
| 5 | 7 |

d.

| $x$ | $f(x)$ |
|---|---|
| 2 | 16 |
| 3 | 14 |
| 4 | 11 |
| 5 | 7 |

e.

| $x$ | $f(x)$ |
|---|---|
| 2 | 16 |
| 3 | 13 |
| 4 | 10 |
| 5 | 7 |

**Calculator Active #4-9**

4.

| $x$ | 0 | 0.5 | 1.0 | 1.5 | 2.0 |
|---|---|---|---|---|---|
| $f(x)$ | 3 | 3 | 5 | 8 | 13 |

A table of values for a continuous function $f$ is shown above. If four equal subintervals of [0, 2] are used, which of the following is the trapezoidal approximation of $\int_0^2 f(x)dx$ ?

a. 8
b. 12
c. 16
d. 24
e. 32

5.

| $t$ (sec) | 0 | 2 | 4 | 6 |
|---|---|---|---|---|
| $a(t)$ (ft/sec$^2$) | 5 | 2 | 8 | 3 |

The data for the acceleration $a(t)$ of a car from 0 to 6 seconds are given in the table above. If the velocity at $t = 0$ is 11 feet per second, the approximate value of the velocity at $t = 6$, computed using a left-hand Riemann sum with three subintervals of equal length, is

a. 26 ft/sec
b. 30 ft/sec
c. 37 ft/sec
d. 39 ft/sec
e. 41 ft/sec

6.

| $x$ | 2 | 5 | 10 | 14 |
|---|---|---|---|---|
| $f(x)$ | 12 | 28 | 34 | 30 |

The function $f$ is continuous on the closed interval [2, 14] and has values as shown in the table above. Using the subintervals [2, 5], [5,10], and [10, 14], what is the approximation of $\int_2^{14} f(x)dx$ found by using a right Riemann sum?

a. 296
b. 312
c. 343
d. 374
e. 390

7.

Repeat

| $x$ | 0 | 1 | 2 |
|---|---|---|---|
| $f(x)$ | 1 | $k$ | 2 |

The function $f$ is continuous on the closed interval [0, 2] and has values that are given in the table above. The equation $f(x) = \frac{1}{2}$ must have at least two solutions in the interval [0, 2] if $k=$

a. 0
b. $\frac{1}{2}$
c. 1
d. 2
e. 3

8.

| $x$ | −4 | −3 | −2 | −1 | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|---|---|---|---|
| $g'(x)$ | 2 | 3 | 0 | −3 | −2 | −1 | 0 | 3 | 2 |

The derivative $g'$ of a function $g$ is continuous and has exactly two zeros. Selected values of $g'$ are given in the table above. If the domain of $g$ is the set of all real numbers, then $g$ is decreasing on which of the following intervals?

a. $-2 \le x \le 2$ only
b. $-1 \le x \le 1$ only
c. $x \ge -2$
d. $x \ge 2$ only
e. $x \le -2$ or $x \ge 2$

9.

| $x$ | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|
| $f(x)$ | 2 | 3 | 4 | 3 | 2 |

The function $f$ is continuous and differentiable on the closed interval [0,4]. The table above gives selected values of $f$ on this interval. Which of the following statements must be true?

a. The minimum value of $f$ on [0, 4] is 2.
b. The maximum value of $f$ on [0, 4] is 4.
c. $f(x) > 0$ for $0 < x < 4$
d. $f'(x) < 0$ for $2 < x < 4$
e. There exists $c$, with $0 < c < 4$, for which $f'(c) = 0$.

**FRQ – Calculator Active**

| $t$ (minutes) | 0 | 2 | 5 | 9 | 10 |
|---|---|---|---|---|---|
| $H(t)$ (degrees Celsius) | 66 | 60 | 52 | 44 | 43 |

As a pot of tea cools, the temperature of the tea is modeled by a differentiable function $H$ for $0 \le t \le 10$, where time $t$ is measured in minutes and temperature $H(t)$ is measured in degrees Celsius. Values of $H(t)$ at selected values of time $t$ are shown in the table above.

(a) Use the data in the table to approximate the rate at which the temperature of the tea is changing at time $t = 3.5$. Show the computations that lead to your answer.

(b) Using correct units, explain the meaning of $\frac{1}{10}\int_0^{10} H(t)\,dt$ in the context of this problem. Use a trapezoidal sum with the four subintervals indicated by the table to estimate $\frac{1}{10}\int_0^{10} H(t)\,dt$.

(c) Evaluate $\int_0^{10} H'(t)\,dt$. Using correct units, explain the meaning of the expression in the context of this problem.

(d) At time $t = 0$, biscuits with temperature 100°C were removed from an oven. The temperature of the biscuits at time $t$ is modeled by a differentiable function $B$ for which it is known that $B'(t) = -13.84e^{-0.173t}$. Using the given models, at time $t = 10$, how much cooler are the biscuits than the tea?

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FRQ Pot of Tea a) $H'(3.5) \approx \frac{H(5)-H(2)}{5-2} = \frac{52-60}{3} = -2.666 \; {}^\circ\mathrm{C}/\mathrm{min}$ $-2.667$

b) $\frac{1}{10}\int_0^{10} H(t)\,dt$ is the average temperature of the tea in °Celsius over the 10 minutes.

c) $\int_0^{10} H(t)\,dt = H(10) - H(0) = 43 - 66 = -23°C$

The temperature of the tea drops 23°C from time $t=0$ to $t=10$ minutes

d) $B(10) = 100 + \int_0^{10} B'(t)\,dt = 34.18275$

$H(10) - B(10) = 8.817$ So the biscuits are 8.817 °C cooler than the tea.

**FRQ – NON-CALCULATOR**

| $x$ | $-1.5$ | $-1.0$ | $-0.5$ | $0$ | $0.5$ | $1.0$ | $1.5$ |
|---|---|---|---|---|---|---|---|
| $f(x)$ | $-1$ | $-4$ | $-6$ | $-7$ | $-6$ | $-4$ | $-1$ |
| $f'(x)$ | $-7$ | $-5$ | $-3$ | $0$ | $3$ | $5$ | $7$ |

6. Let $f$ be a function that is differentiable for all real numbers. The table above gives the values of $f$ and its derivative $f'$ for selected points $x$ in the closed interval $-1.5 \le x \le 1.5$. The second derivative of $f$ has the property that $f''(x) > 0$ for $-1.5 \le x \le 1.5$.

(a) Evaluate $\int_0^{1.5} (3f'(x) + 4)\,dx$. Show the work that leads to your answer.

(b) Write an equation of the line tangent to the graph of $f$ at the point where $x = 1$. Use this line to approximate the value of $f(1.2)$. Is this approximation greater than or less than the actual value of $f(1.2)$? Give a reason for your answer.

(c) Find a positive real number $r$ having the property that there must exist a value $c$ with $0 < c < 0.5$ and $f''(c) = r$. Give a reason for your answer.

(d) Let $g$ be the function given by $g(x) = \begin{cases} 2x^2 - x - 7 & \text{for } x < 0 \\ 2x^2 + x - 7 & \text{for } x \ge 0. \end{cases}$

The graph of $g$ passes through each of the points $(x, f(x))$ given in the table above. Is it possible that $f$ and $g$ are the same function? Give a reason for your answer.

$\boxed{\text{FRQ}}$

a) $\int_0^{1.5} (3f'(x) + 4)\,dx = 3\int_0^{1.5} f'(x)\,dx + \int_0^{1.5} 4\,dx$

$= 3f(x) + 4x \Big|_0^{1.5} = 3(-1-(-7)) + 4(1.5) = 24$

b) $y = 5(x-1) - 4$ & $f(1.2) \approx 5(.2) - 4 = -3$

The approximation is less than $f(1.2)$ b/c $f(x)$ is concave up on $x \in (1, 1.2)$

c) MVT guarantees $c \in (0, 0.5)$ such that $f''(c) = \frac{f'(\frac{1}{2}) - f'(0)}{(\frac{1}{2})}$

$f''(c) = 6 = r$

d) $\lim_{x\to 0^-} g'(x) = \lim_{x \to 0^-} (4x-1) = -1$

$\lim_{x\to 0^+} g'(x) = \lim_{x\to 0^+} (4x+1) = 1$

$\therefore$ $g'(x)$ is not continuous at $x=0$ but $f'(x)$ is continuous at $x=0$ so $f \ne g$.

$\boxed{\text{OR}}$

$g''(x) = 4$ for all $x \ne 0$ but in part (c) $f''(c) = 6$ for $c \ne 0$ so $f \ne g$.