INTEGRATION TECHNIQUES (11 MC & 1 FRQ) **AP REVIEW**

Multiple Choice

Identify the choice that best completes the statement or answers the question.

____ 1. $\int_0^{\frac{\pi}{4}} \sin x\,dx =$

a. $-\frac{\sqrt{2}}{2}$ b. $\frac{\sqrt{2}}{2}$ c. $-\frac{\sqrt{2}}{2}-1$ d. $-\frac{\sqrt{2}}{2}+1$ e. $\frac{\sqrt{2}}{2}-1$

____ 2. $\int_0^1 e^{-4x}\,dx =$

a. $\frac{-e^{-4}}{4}$
b. $-4e^{-4}$
c. $e^{-4}-1$
d. $\frac{1}{4}-\frac{e^{-4}}{4}$
e. $4-4e^{-4}$

____ 3. $\frac{1}{2}\int e^{\frac{t}{2}}\,dt =$

a. $e^{-t}+C$
b. $e^{-\frac{t}{2}}+C$
c. $e^{\frac{t}{2}}+C$
d. $2e^{\frac{t}{2}}+C$
e. $e^{t}+C$

____ 4. $\int x^2\cos\left(x^3\right)dx =$

a. $-\frac{1}{3}\sin\left(x^3\right)+C$
b. $\frac{1}{3}\sin\left(x^3\right)+C$
c. $-\frac{x^3}{3}\sin\left(x^3\right)+C$
d. $\frac{x^3}{3}\sin\left(x^3\right)+C$
e. $\frac{x^3}{3}\sin\left(\frac{x^4}{4}\right)+C$

____ 5. Using the substitution $u=2x+1$, $\int_0^2 \sqrt{2x+1}\,dx$ is equivalent to

a. $\frac{1}{2}\int_{\frac{-1}{2}}^{\frac{1}{2}} \sqrt{u}\,du$
b. $\frac{1}{2}\int_0^2 \sqrt{u}\,du$
c. $\frac{1}{2}\int_1^5 \sqrt{u}\,du$
d. $\int_0^2 \sqrt{u}\,du$
e. $\int_1^5 \sqrt{u}\,du$

6. $\int_1^e \left(\frac{x^2-1}{x}\right)dx =$

a. $e-\frac{1}{e}$
b. e^2-e
c. $\frac{e^2}{2}-e+\frac{1}{2}$
d. e^2-2
e. $\frac{e^2}{2}-\frac{3}{2}$

7. What are all values of k for which $\int_{-3}^{k} x^2\,dx = 0$?

a. -3
b. 0
c. 3
d. -3 and
e. 3-3, 0, and 3

8. $\int_1^2 \left(4x^3-6x\right)dx =$

a. 2
b. 4
c. 6
d. 36
e. 42

9. What is the average value of $y = x^2\sqrt{x^3+1}$ on the interval $[0, 2]$?

a. $\frac{26}{9}$ b. $\frac{52}{9}$ c. $\frac{26}{3}$ d. $\frac{52}{3}$ e. 24

10. If f is the antiderivative of $\frac{x^2}{1+x^3}$ such that $f(1) = 0$, then $f(4) =$

Repeat

a. -0.012
b. 0
c. 0.016
d. 0.376
e. 0.629

11. Let $F(x)$ be an antiderivative of $\frac{(\ln x)^3}{x}$. If $F(1) = 0$, then $F(9) =$

a. 0.048
b. 0.144
c. 5.827
d. 23.308
e. 1,640.250

FRQ – NON-CALCULATOR

6. Let f be the function satisfying $f'(x) = x\sqrt{f(x)}$ for all real numbers x, where $f(3) = 25$.

(a) Find $f''(3)$. $= \frac{19}{2}$

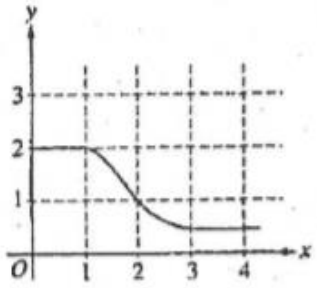
(b) Write an expression for $y = f(x)$ by solving the differential equation $\frac{dy}{dx} = x\sqrt{y}$ with the initial condition $f(3) = 25$.

$y = +\frac{1}{16}(x^2+11)^2$

INTERPRETING GRAPHS (16 MC : 2 FRQ) AP REVIEW

Multiple Choice

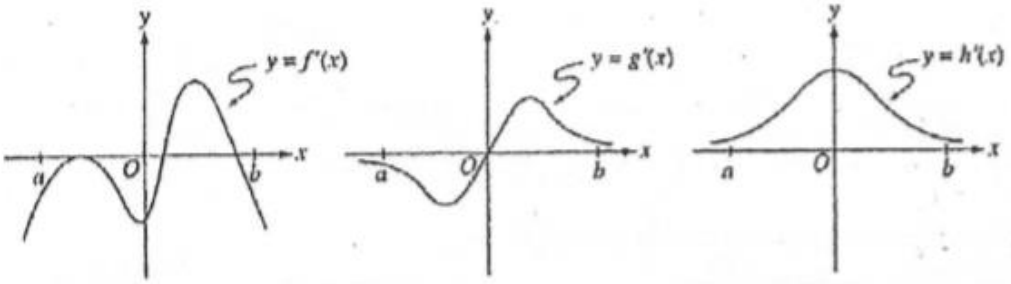
Identify the choice that best completes the statement or answers the question.

1. 

The graph of f is shown in the figure.

If $\int_1^3 f(x)dx = 2.3$ and $F'(x) = f(x)$, then $F(3) - F(0) =$

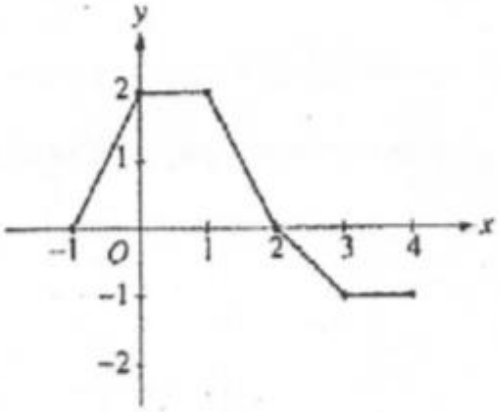
a. 0.3 b. 1.3 c. 3.3 d. 4.3 e. 5.3

2. 

The graphs of the derivatives of the functions f, g, and h are shown above. Which of the functions f, g, or h have a relative maximum on the open interval $a < x < b$?

a. f only b. g only c. h only d. f and g only e. f, g, and h

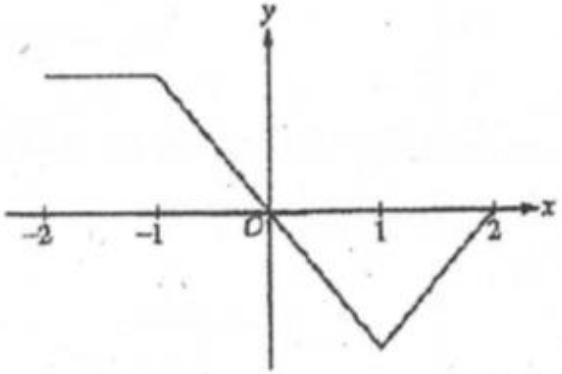
3. The graph of a piecewise-linear function f, for $-1 \le x \le 4$, is shown



What is the value of $\int_{-1}^{4} f(x)dx$?

a. 1
b. 2.5
c. 4
d. 5.5
e. 8

4. The graph of f', the derivative of the function f, is shown



Graph of f'

Which of the following statements is true about f?

a. f is decreasing for $-1 \le x \le 1$.
b. f is increasing for $-2 \le x \le 0$.
c. f is increasing for $1 \le x \le 2$.
d. f has a local minimum at $x = 0$.
e. f is not differentiable at $x = -1$ and $x = 1$.

5. The first derivative of the function f is given by $f'(x) = \frac{\cos^2 x}{x} - \frac{1}{5}$. How many critical values does f have on the open interval (0, 10)?

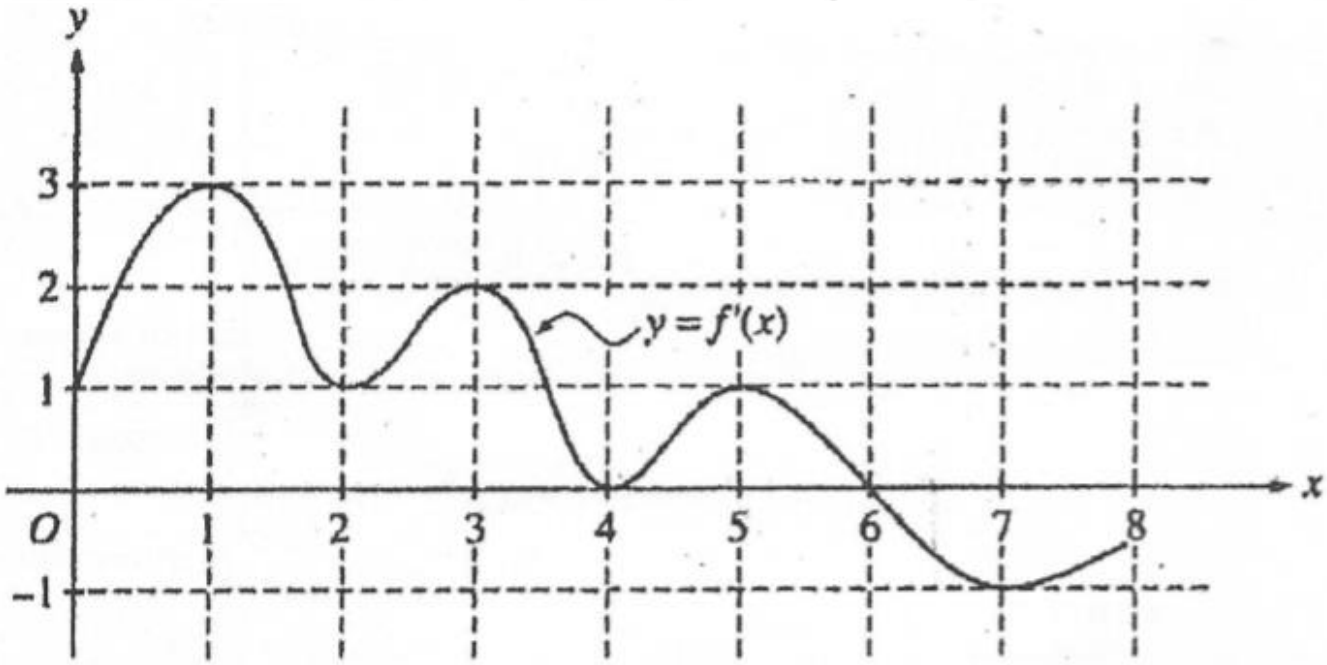
KEY

a. One b. Three c. Four d. Five e. Seven

6. The function f has first derivative given by $f'(x) = \frac{\sqrt{x}}{1+x+x^3}$ What is the x-coordinate ofthe inflection point of the graph of f?

a. 1.008 b. 0.473 c. 0 d. -0.278 e. The graph of f has no inflection point.

Questions 7-9 refer to the graph and the information given below.



The function f is defined on the closed interval [0, 8]. The graph of its derivative f is shown above.

7. The point (3, 5) is on the graph of $y = f(x)$. An equation of the line tangent to the graph of f at (3, 5) is

a. $y = 2$ b. $y = 5$ c. $y - 5 = 2(x - 3)$ d. $y + 5 = 2(x - 3)$ e. $y + 5 = 2(x + 3)$

8. How many points of inflection does the graph of f have?

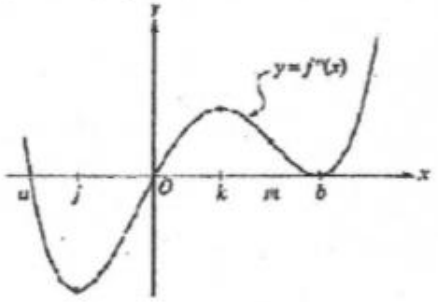
a. Two b. Three c. Four d. Five e. Six

9. At what value of x does the absolute minimum of f occur?

a. 0 b. 2 c. 4 d. 6 e. 8

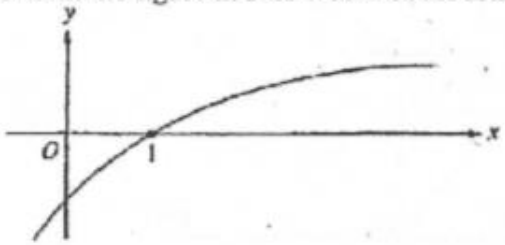
10. The second derivative of the function f is given by $f''(x) = x(x-a)(x-b)^2$ The graph of f'' is shown ~~above~~. For what values of x does the graph of f have a point of inflection?

a. 0 and a only
b. 0 and m only
c. b and j only
d. 0, a, and b
e. b, j, and k

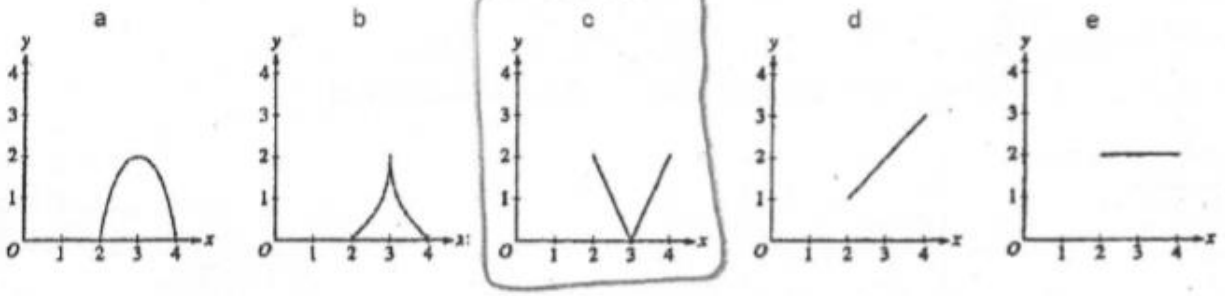


11. The graph of a twice-differentiable function f is shown In the figure ~~above~~. Which of the following is true?

a. $f(1) < f'(1) < f''(1)$
b. $f(1) < f''(1) < f'(1)$
c. $f'(1) < f(1) < f''(1)$
d. $f''(1) < f(1) < f'(1)$
e. $f''(1) < f'(1) < f(1)$



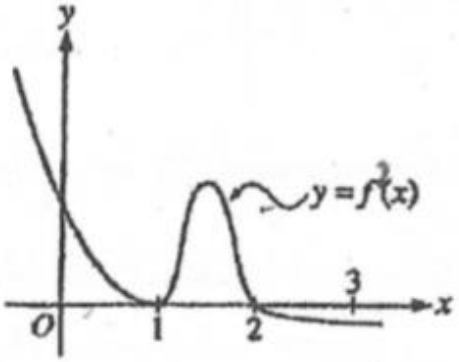
12. On the closed interval [2, 4], which of the following could be the graph of a function f with the property that $\frac{1}{4-2}\int_2^4 f(t)dt = 1$?



13. The graph of f', the derivative of the function f, is shown ~~above~~. If $f(0) = 0$, which of the following must be true?

I. $f(0) > f(1)$
II. $f(2) > f(1)$
III. $f(1) > f(3)$

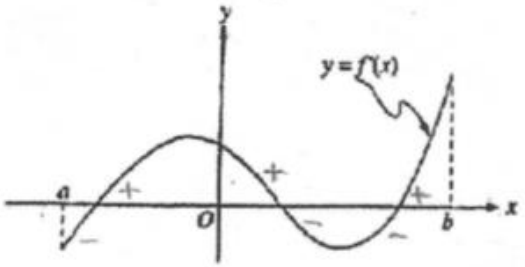
a. I only
b. II only
c. III only
d. I and II only
e. II and III only

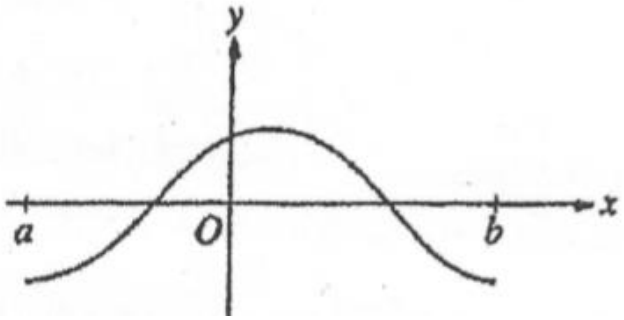


KEY

14. The graph of f', the derivative of f, is shown in the figure ~~above~~. Which of the following describes all relative extrema of f on the open interval (a, b) ?

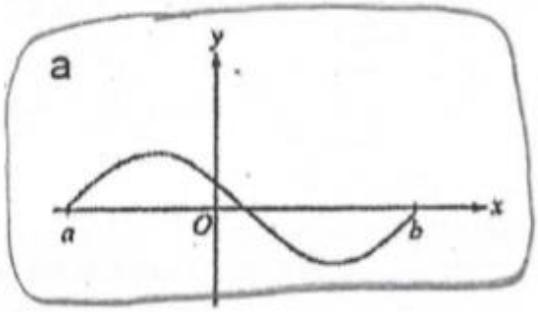
a. One relative maximum and two relative minima
b. Two relative maxima and one relative minimum
c. Three relative maxima and one relative minimum
d. One relative maximum and three relative minima
e. Three relative maxima and two relative minima



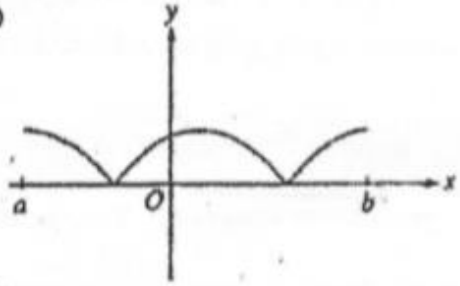


15. The graph of f is shown in the figure above. Which of the following could be the graph of the derivative of f?

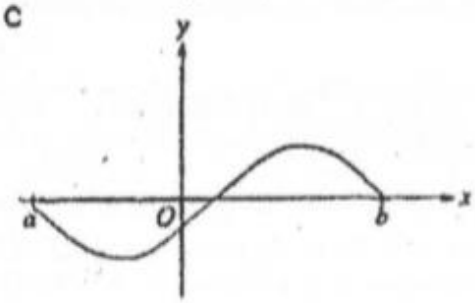
a



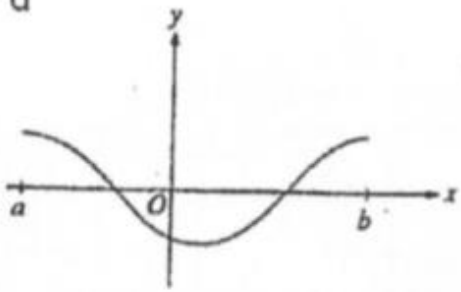
b



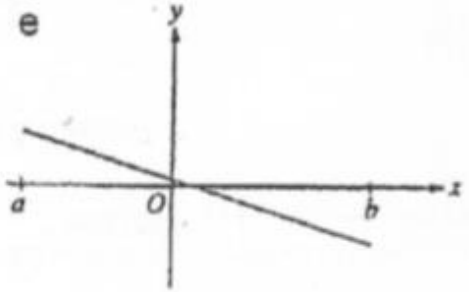
c



d

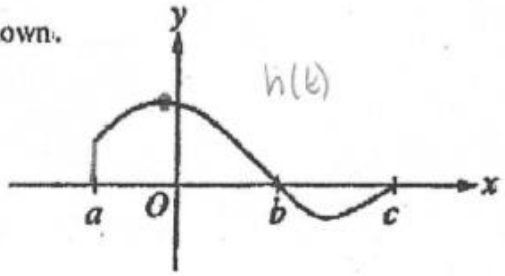


e

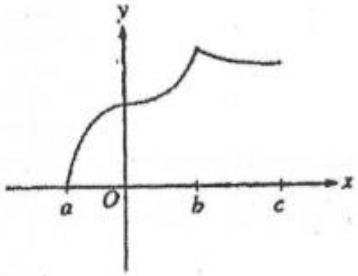


16. Let $f(x) = \int_a^x h(t)\,dt$, where h has the graph shown.

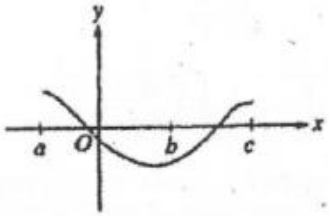
Which of the following could be the graph of f?



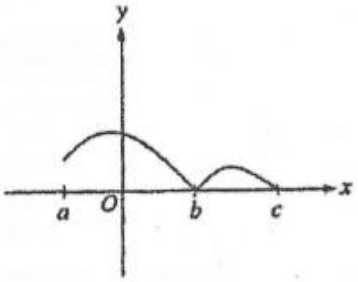
a



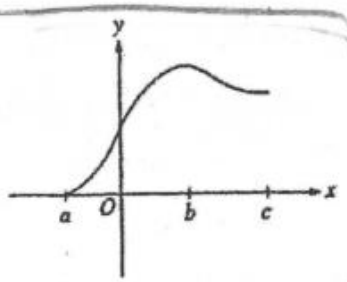
d



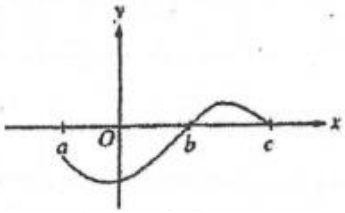
b



e

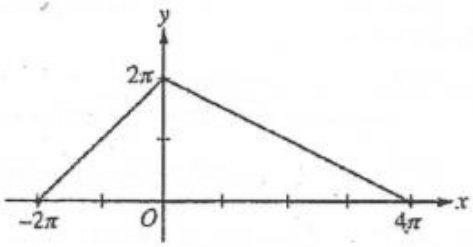


c



FRQ – NON-CALCULATOR

Let g be the piecewise-linear function defined on $[-2\pi, 4\pi]$ whose graph is given above, and let $f(x) = g(x) - \cos\left(\frac{x}{2}\right)$.



Graph of g

(a) Find $\int_{-2\pi}^{4\pi} f(x)\,dx$. Show the computations that lead to your answer. $= 6\pi^2 - \left[2\sin\left(\frac{x}{2}\right)\right]_{-2\pi}^{4\pi} = 6\pi^2$

(b) Find all x-values in the open interval $(-2\pi, 4\pi)$ for which f has a critical point.

b) $f'(x) = g'(x) + \frac{1}{2}\sin\left(\frac{x}{2}\right) = \begin{cases} 1 + \frac{1}{2}\sin\left(\frac{x}{2}\right) & (-2\pi, 0) \\ -\frac{1}{2} + \frac{1}{2}\sin\left(\frac{x}{2}\right) & (0, 4\pi) \end{cases}$

$f'(x)$ dne @ $x = 0$

for $(-2\pi, 0)$, $f'(x) \neq 0$

for $(0, 4\pi)$, $f'(x) = 0$ @ $x = \pi$

$\therefore$ f has Critical pt @ $x = 0, \pi$

(c) Let $h(x) = \int_0^{3x} g(t)\,dt$. Find $h'\left(-\frac{\pi}{3}\right)$.

$h'(x) = g(3x) \cdot 3$

$h'\left(-\frac{\pi}{3}\right) = 3\,g(-\pi) = 3\pi$

FRQ – NON-CALCULATOR

The continuous function f is defined on the interval $-4 \le x \le 3$. The graph of f consists of two quarter circles and one line segment, as shown in the figure above.

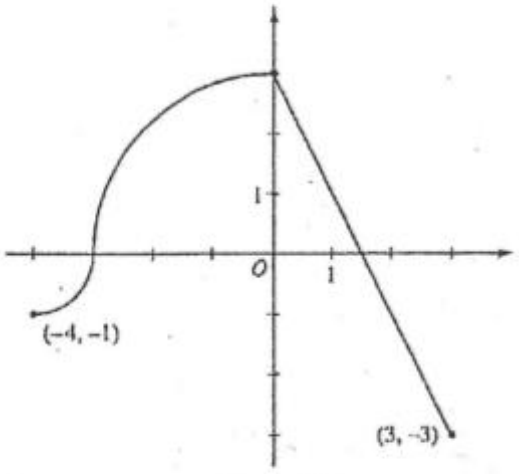
Let $g(x) = 2x + \int_0^x f(t)\,dt$.

(a) Find $g(-3)$. Find $g'(x)$ and evaluate $g'(-3)$.

(b) Determine the x-coordinate of the point at which g has an absolute maximum on the interval $-4 \le x \le 3$. Justify your answer.

(c) Find all values of x on the interval $-4 < x < 3$ for which the graph of g has a point of inflection. Give a reason for your answer.

(d) Find the average rate of change of f on the interval $-4 \le x \le 3$. There is no point c, $-4 < c < 3$, for which $f'(c)$ is equal to that average rate of change. Explain why this statement does not contradict the Mean Value Theorem.



Graph of f

FRQ

a) $g(-3) = 2(-3) + \int_0^{-3} f(t)\,dt = -6 - \frac{9\pi}{4}$

$g'(x) = 2 + f(x)$

$g'(-3) = 2 + f(-3) = 2$

b) $g'(x) = 0$ when $f(x) = -2$ $\therefore x = \frac{5}{2}$

$g'(x) > 0$ for $(-4, \frac{5}{2})$

$g'(x) < 0$ for $(\frac{5}{2}, 3)$

$\therefore$ ABS max @ $x = \frac{5}{2}$

d) Avg Rt of Change

$\frac{f(3) - f(-4)}{3 + 4} = -\frac{2}{7}$

MVT can only be applied to a differentiable function on the interval $(-4, 3)$ but $f(x)$ is not differentiable at $x = -3$ or $x = 0$.

c) $g''(x) = f'(x)$ changes signs at $x = 0$ $\therefore g(x)$ has point of inflection at $x = 0$

G

PARTICLE MOTION (12 MC & 1 FRQ) **AP REVIEW**

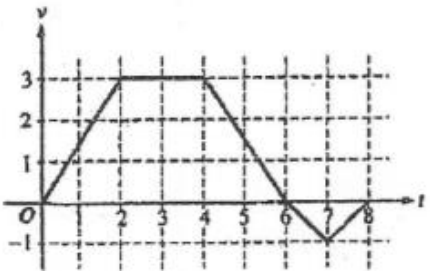
Multiple Choice

Identify the choice that best completes the statement or answers the question.

NON-CALCULATOR #1-6 & Calculator Active #7-12

Questions 1 - 2 refer to the following situation.

A bug begins to crawl up a vertical wire at time $t = 0$. The velocity v of the bug at time t, $0 \le t \le 8$, is given by the function whose graph is shown.



1. At what value of t does the bug change direction?

NC
a. 2
b. 4
c. 6
d. 7
e. 8

2. What is the total distance the bug traveled from $t = 0$ to $t = 8$?

MC
a. 14
b. 13
c. 11
d. 8
e. 6

3. A particle moves along the x-axis so that its position at time t is given by $x(t) = t^3 - 6t + 5$. For what value of t is the velocity of the particle zero?

NC
a. 1
b. 2
c. 3
d. 4
e. 5

4. The maximum acceleration attained on the interval $0 \le t \le 3$ by the particle whose velocity is given by $v(t) = t^3 - 3t^2 + 12t + 4$ is

NC
a. 9
b. 12
c. 14
d. 21
e. 40

5. A particle moves along the x-axis so that its acceleration at any time t is $a(t) = 2t - 7$. If the initial velocity of the particle is 6, at what time t during the interval $0 \le t \le 4$ is the particle farthest to the right?

NC
a. 0
b. 1
c. 2
d. 3
e. 4

6. A particle moves along the x-axis so that at time $t \ge 0$ its position is given by $x(t) = 2t^3 - 21t^2 + 72t - 53$. At what time t is the particle at rest?

NC
a. $t = 1$ only
b. $t = 3$ only
c. $t = \frac{7}{2}$ only
d. $t = 3$ and $t = \frac{7}{2}$
e. $t = 3$ and $t = 4$

Calculator Active #7-12

7. The data for the acceleration $a(t)$ of a car from 0 to 6 seconds are given in the table ~~above~~. If the velocity at $t = 0$ is 11 feet per second, the approximate value of the velocity at $t = 6$, computed using a left-hand Riemann sum with three subintervals of equal length, is

C
a. 26 ft/sec
b. 30 ft/sec
c. 37 ft/sec
d. 39 ft/sec
e. 41 ft/sec

t (sec)	0	2	4	6
$a(t)$ (ft/sec^2)	5	2	8	3

8. At time $t \geq 0$, the acceleration of a particle moving on the x-axis is $a(t) = t + \sin t$. At $t = 0$, the velocity of the particle is -2. For what value of t will the velocity of the particle be zero?

C

a. 1.02
b. 1.48
c. 1.85
d. 2.81
e. 3.14

9. Aparticle moves along the x-axis so that at any time $t \geq 0$, its velocity is given by $v(t) = 3 + 4.1\cos(0.9t)$. What is the acceleration of the particle at time $t = 4$?

C

a. -2.016
b. -0.677
c. 1.633
d. 1.814
e. 2.978

10. The position of an object attached to a spring is given by $y(t) = \frac{1}{6}\cos(5t) - \frac{1}{4}\sin(5t)$, where t is time in seconds. In the first 4 seconds, how many times is the velocity of the object equal to 0?

C

a. Zero
b. Three
c. Five
d. Six
e. Seven

11. A particle moves along the x-axis so that at any time $t \geq 0$, its velocity is given by $v(t) = \cos\left(2 - t^2\right)$. The position of the particle is 3 at time $t = 0$. What is the position of the particle when its velocity is first equal to 0?

E

a. 0.411
b. 1.310
c. 2.816
d. 3.091
e. 3.411

12. The height h, in meters, of an object at time t is given by $h(t) = 24t + 24t^{\frac{3}{2}} - 16t^2$. What is the height of the object at the instant when it reaches its maximum upward velocity?

C

a. 2.545 meters
b. 10.263 meters
c. 34.125 meters
d. 54.889 meters
e. 89.005 meters

FRQ

a) $v(5.5) = -0.45337 < 0$
$a(5.5) = -1.35851 < 0$
$\therefore$ speed of particle is increasing since $v < 0$ & $a < 0$.

b) Avg Vel $= \frac{1}{6}\int_0^6 v(t)\,dt = 1.949$

c) Total Distance $= \int_0^6 |v(t)|\,dt = 12.573$

d) $v(t) = 0$ @ $t = 5.19552$ & $v(t)$ changes signs at $t = 5.19532$ from negative to positive
$x(5.19532) = 2 + \int_0^{5.19522} v(t)\,dt = 14.134$ or $= 14.135$

FRQ

Question 1

For $0 \le t \le 6$, a particle is moving along the x-axis. The particle's position, $x(t)$, is not explicitly given. The velocity of the particle is given by $v(t) = 2\sin\left(e^{t/4}\right) + 1$. The acceleration of the particle is given by $a(t) = \frac{1}{2}e^{t/4}\cos\left(e^{t/4}\right)$ and $x(0) = 2$.

(a) Is the speed of the particle increasing or decreasing at time $t = 5.5$? Give a reason for your answer.

(b) Find the average velocity of the particle for the time period $0 \le t \le 6$.

(c) Find the total distance traveled by the particle from time $t = 0$ to $t = 6$.

(d) For $0 \le t \le 6$, the particle changes direction exactly once. Find the position of the particle at that time.

4 RATES & ACCUMULATIONS (6 MC & 1 FRQ) AP REVIEW

Multiple Choice

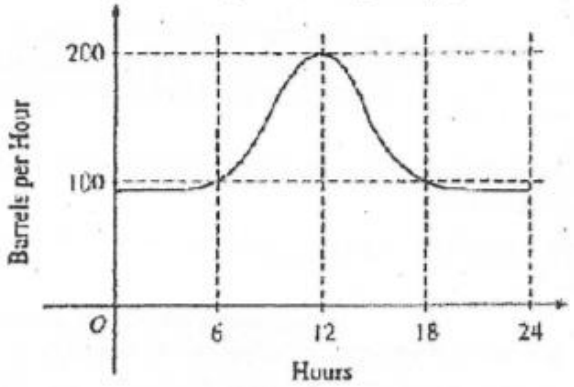
Identify the choice that best completes the statement or answers the question.

NC

____ 2. The flow of oil, in barrels per hour, through a pipeline on July 9 is given by the graph shown ~~above~~. Of the following, which best approximates the total number of barrels of oil that passed through the pipeline that day?

a. 500 b. 600 c. 2,400

d. 3,000 e. 4,800



C

4. Insects destroyed a crop at the rate of $\frac{100e^{-0.1t}}{2-e^{-3t}}$ tons per day, where time t is measured in days. To the nearest ton, how many tons did the insects destroy during the time interval $7 \le t \le 14$?

a. 125 b. 100 c. 88 d. 50 e. 12

5. The rate of change of the altitude of a hot-air balloon is given by $r(t) = t^3 - 4t^2 + 6$ for $0 \le t \le 8$. Which of the
C following expressions gives the change in altitude of the balloon during the time the altitude is decreasing?

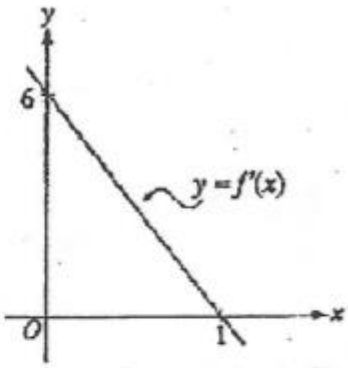
a. $\int_{1.372}^{3.514} r(t)dt$ b. $\int_{0}^{8} r(t)dt$ c. $\int_{0}^{2.667} r(t)dt$ d. $\int_{1.372}^{3.514} r'(t)dt$ e. $\int_{0}^{2.667} r'(t)dt$

6. A pizza, heated to a temperature of 350 degrees Fahrenheit (°F), is taken out of an oven and placed in a 75°F
C room at time $t = 0$ minutes. The temperature of the pizza is changing at a rate of $-110e^{-0.4t}$ degrees Fahrenheit per minute. To the nearest degree, what is the temperature of the pizza at time $t = 5$ minutes?

a. 112°F b. 119°F c. 147°F d. 238°F e. 335°F

7. The graph of f', the derivative of f, is the line shown in the figure
NC If $f(0) = 5$, then $f(1) =$

a. 0
b. 3
c. 6
d. 8
e. 11



8. A particle moves along the x-axis so that at any time $t > 0$, its acceleration is given by $a(t) = \ln\left(1 + 2^t\right)$. If the
C velocity of the particle is 2 at time $t = 1$, then the velocity of the particle at time $t = 2$ is

a. 0.462 b. 1.690 c. 2.555 d. 2.886 e. 3.346

FRQ – CALCULATOR ACTIVE

2 A 12,000-liter tank of water is filled to capacity. At time $t = 0$, water begins to drain out of the tank at a rate
C modeled by $r(t)$, measured in liters per hour, where r is given by the piecewise-defined function

$$r(t) = \begin{cases} \frac{600t}{t+3} & \text{for } 0 \le t \le 5 \\ 1000e^{-0.2t} & \text{for } t > 5 \end{cases}$$

(a) Is r continuous at $t = 5$? Show the work that leads to your answer.

(b) Find the average rate at which water is draining from the tank between time $t = 0$ and time $t = 8$ hours.

(c) Find $r'(3)$. Using correct units, explain the meaning of that value in the context of this problem.

(d) Write, but do not solve, an equation involving an integral to find the time A when the amount of water in the tank is 9000 liters.

FRQ

a) $\lim_{t \to 5^-} \left(\frac{600t}{t+3} \right) = 375 = r(5)$

$\lim_{t \to 5^+} \left(1000 e^{-.2t} \right) = 367.879$

B/C Lft Limit $\neq$ Rt Limit $r(t)$ is NOT continuous at $t = 5$.

b) $\frac{1}{8} \int_0^8 r(t)\, dt = \frac{1}{8} \left[\int_0^5 \frac{600t}{t+3} dt + \int_5^8 1000 e^{-.2t} dt \right]$

$= 258.052$ or 258.053

c) $r'(3) = 50$ The rate at which water is draining out of the tank at time $t = 3$ hrs is increasing at 50 Litres $/ hr^2$.

d) $12000 - \int_0^A r(t)\, dt = 9000$ ...