A

AREAS & VOLUMES (11 MC & 1 FRQ) **AP REVIEW** Key

Multiple Choice

Identify the choice that best completes the statement or answers the question.

NON-CALCULATOR: # 1, 2, 6, 8 & CALCULATOR ACTIVE: # 3, 4, 5, 7, 9, 10, 11

NC 1. The area of the region enclosed by the graph of $y = x^2 + 1$ and the line $y = 5$ is:

a. $\frac{14}{3}$ b. $\frac{16}{3}$ c. $\frac{28}{3}$ $\boxed{\text{d. } \frac{32}{3}}$ e. 8π

NC 2. What is the area of the region between the graphs of $y = x^2$ and $y = -x$ from $x = 0$ to $x = 2$?

a. $\frac{2}{3}$ b. $\frac{8}{3}$ c. 4 $\boxed{\text{d. } \frac{14}{3}}$ e. $\frac{16}{3}$

C 3. Let R be the region enclosed by the graph of $y = 1 + \ln(\cos^4 x)$ the x-axis, the lines $x = -\frac{2}{3}$ and $x = \frac{2}{3}$. The closest integer approximation of the area of R is:

a. 0 $\boxed{\text{b. } 1}$ c. 2 d. 3 e. 4

C 4. What is the area of the region in the first quadrant enclosed by the graphs of $y = \cos x$, $y = x$ and the y-axis?

a. 0.127 b. 0.385 $\boxed{\text{c. } 0.400}$ d. 0.600 e. 0.947

C 5. If $0 \le k < \frac{\pi}{2}$ and the area under the curve $y = \cos x$ from $x = k$ to $x = \frac{\pi}{2}$ is 0.1, then $k =$ _____.

a. 1.471 b. 1.414 c. 1.277 $\boxed{\text{d. } 1.120}$ e. 0.436

NC 6. If the region enclosed by the y-axis, the line $y = 2$, and the curve $y = \sqrt{x}$ is revolved about the y-axis, the volume of the solid generated is:

$\boxed{\text{a. } \frac{32\pi}{5}}$ b. $\frac{16\pi}{3}$ c. $\frac{16\pi}{5}$ d. $\frac{8\pi}{3}$ e. π

C 7. The base of a solid S is the region enclosed by the graph of $y = \sqrt{\ln x}$, the line $x = e$, and the x-axis. If the cross sections of S perpendicular to the x-axis are squares, then the volume of S is:

a. $\frac{1}{2}$ b. $\frac{2}{3}$ $\boxed{\text{c. } 1}$ d. 2 e. $\frac{1}{3}(e^3 - 1)$

NC 8. The base of a solid is the region in the first quadrant enclosed by the graph of y = 2 – x2 and the coordinate axes. If every cross section of the solid perpendicular to the y-axis is a square, the volume of the solid is given by:

a. $\pi\int_0^2 (2-y)^2\,dy$ $\boxed{\text{b. } \int_0^2 (2-y)\,dy}$ c. $\pi\int_0^{\sqrt{2}} (2-x^2)^2\,dx$ d. $\int_0^{\sqrt{2}} (2-x^2)^2\,dx$ e. $\int_0^{\sqrt{2}} (2-x^2)\,dx$

C 9. The region bounded by the graph of $y = 2x - x^2$ and the x-axis is the base of a solid. For this solid, each cross section perpendicular to the x-axis is an equilateral triangle. What is the volume of the solid?

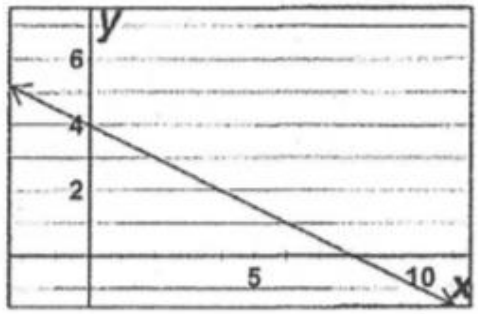
a. 1.333 b. 1.067 c. 0.577 $\boxed{\text{d. } 0.462}$ e. 0.267

10. The base of a solid is a region in the first quadrant bounded by the x-axis, the y-axis, and the line $x + 2y = 8$, as shown in the figure at the right. If cross sections of the solid perpendicular to the x-axis are semi-circles, what is the volume of the solid?

C

a. 12.566 b. 14.661 $\boxed{\text{c. } 16.755}$

d. 67.021 e. 134.041

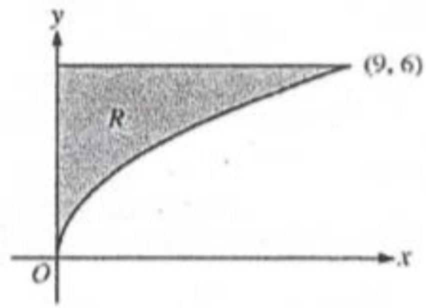


11. The base of a solid is the region in the first quadrant bounded by the y-axis, the graph of $y = \tan^{-1} x$, the horizontal line $y = 3$, and the vertical line $x = 1$. For this solid, each cross section perpendicular to the x-axis is a square. What is the volume of the solid?

C

a. 2.561 $\boxed{\text{b. } 6.612}$ c. 8.046 d. 8.755 e. 20.773

FRQ – NON-CALCULATOR



Let R be the region in the first quadrant bounded by the graph of $y = 2\sqrt{x}$, the horizontal line $y = 6$, and the y-axis, as shown in the figure above.

(a) Find the area of R. $A = 18$

$V = \pi \int_0^9 (7 - 2\sqrt{x})^2 - (1)^2 \, dx$

(b) Write, but do not evaluate, an integral expression that gives the volume of the solid generated when R is rotated about the horizontal line $y = 7$.

(c) Region R is the base of a solid. For each y, where $0 \le y \le 6$, the cross section of the solid taken perpendicular to the y-axis is a rectangle whose height is 3 times the length of its base in region R. Write, but do not evaluate, an integral expression that gives the volume of the solid. $V = \frac{3}{16} \int_0^6 y^2 \, dy$

B

AVERAGE & INSTANTANEOUS RATES OF CHANGE (5MC & 1FRQ) AP REVIEW

Multiple Choice

Identify the choice that best completes the statement or answers the question.

NON-CALCULATOR

1. What is the instantaneous rate of change at $x = 2$ of the function f given by $f(x) = \frac{x^2-2}{x-1}$?

NC

(A) -2 (B) $\frac{1}{6}$ (C) $\frac{1}{2}$ $\boxed{\text{(D) } 2}$ (E) 6

CALCULATOR

C

2. Let f be a function given by $f(x) = 3e^{2x}$ and let g be the function given by $g(x) = 6x^3$. At what value of x do the graphs of f and g have parallel tangent lines?

(A) -0.701 (B) -0.567 $\boxed{\text{(C) } -0.391}$ (D) -0.302 (E) -0.258

3. Let f be the function defined by $f(x) = x + \ln x$. What is the value of c for which the instantaneous rate of change of f at $x = c$ is the same as the average rate of change of f over $[1, 4]$?

C

(A) 0.456 (B) 1.244 $\boxed{\text{(C) } 2.164}$ (D) 2.342 (E) 2.452

4. Let f be a function given by $f(x) = 2e^{4x^2}$. For what value of x is the slope of the line tangent to the graph of f at $(x, f(x))$ equal to 3?

C

$\boxed{\text{(A) } 0.168}$ (B) 0.276 (C) 0.318 (D) 0.342 (E) 0.551

5. Let $f(x) = \sqrt{x}$. If the rate of change of f at $x = c$ is twice its rate of change at $x = 1$, then $c =$

$\boxed{\text{(A) } \frac{1}{4}}$ (B) 1 (C) 4 (D) $\frac{1}{\sqrt{2}}$ (E) $\frac{1}{2\sqrt{2}}$

FRQ – CALCULATOR ACTIVE

For $0 \le t \le 31$, the rate of change of the number of mosquitoes on Tropical Island at time t days is modeled by $R(t) = 5\sqrt{t}\cos\left(\frac{t}{5}\right)$ mosquitoes per day. There are 1000 mosquitoes on Tropical Island at time $t = 0$.

(a) Show that the number of mosquitoes is increasing at time $t = 6$.

(b) At time $t = 6$, is the number of mosquitoes increasing at an increasing rate, or is the number of mosquitoes increasing at a decreasing rate? Give a reason for your answer.

(c) According to the model, how many mosquitoes will be on the island at time $t = 31$? Round your answer to the nearest whole number.

(d) To the nearest whole number, what is the maximum number of mosquitoes for $0 \le t \le 31$? Show the analysis that leads to your conclusion.

a) $R(6) = 4.438 > 0$

b) $R'(6) = -1.913 < 0$ ∴ Mosquitos increase at decreasing rate.

c) $1000 + \int_0^{31} R(t)\,dt = 964.335$ ∴ 964 mosquitos at $t = 31$

d) $R(t) = 0$ at $t = 0, 2.5\pi, 7.5\pi$

$R(t) > 0$ on $t \in (0, 2.5\pi)$ $(7.5\pi, 31)$ + − +

$R(t) < 0$ on $t \in (2.5\pi, 7.5\pi)$

MAX occurs @ $t = 2.5\pi$ and $t = 31$

$1000 + \int_0^{2.5\pi} R(t)\,dt = 1039.357$ mosquitos at $t = 2.5\pi$

∴ MAX 1039 mosquitos @ $t = 2.5\pi$

KEY

FRQ – NON-CALCULATOR

t (seconds)	0	10	40	60
$B(t)$ (meters)	100	136	9	49
$v(t)$ (meters per second)	2.0	2.3	2.5	4.6

Ben rides a unicycle back and forth along a straight east-west track. The twice-differentiable function B models Ben's position on the track, measured in meters from the western end of the track, at time t, measured in seconds from the start of the ride. The table above gives values for $B(t)$ and Ben's velocity, $v(t)$, measured in meters per second, at selected times t.

(a) Use the data in the table to approximate Ben's acceleration at time $t = 5$ seconds. Indicate units of measure. $0.03 \frac{m}{sec^2}$

(b) Using correct units, interpret the meaning of $\int_0^{60}|v(t)|\,dt$ in the context of this problem. Approximate $\int_0^{60}|v(t)|\,dt$ using a left Riemann sum with the subintervals indicated by the data in the table.

The total distance in meters, Ben rides over the time interval $t=0$ to 60 seconds.

139 meters $= \int_0^{60}|v(t)|\,dt$

(c) For $40 \le t \le 60$, must there be a time t when Ben's velocity is 2 meters per second? Justify your answer.

$\frac{B(60)-B(40)}{20}$

$\frac{49-9}{20} = 2$

$\therefore$ MVT guarantees $c \in (40,60)$ such that $v(t)=2$ $\boxed{C}$

(d) A light is directly above the western end of the track. Ben rides so that at time t, the distance $L(t)$ between Ben and the light satisfies $(L(t))^2 = 12^2 + (B(t))^2$. At what rate is the distance between Ben and the light changing at time $t = 40$?

$2L(t)L'(t) = 2 \cdot B(t) \cdot B'(t)$

$L'(40) = \frac{3}{2}$ m/sec

$\boxed{C}$ **BIG THEOREMS** (11MC & 1FRQ) **AP REVIEW**

Multiple Choice

Identify the choice that best completes the statement or answers the question.

____ 1. The function f is continuous on the closed interval [0, 2] and has values that are given in the table ~~above~~. The equation $f(x) = \frac{1}{2}$ must have at least two solutions in the interval [0, 2] if $k=$

x	0	1	2
$f(x)$	1	k	2

a. 0
b. $\frac{1}{2}$
c. 1
d. 2
e. 3

____ 2. Let f be a function that is differentiable on the open interval (1, 10). If $f(2) = -5$, $f(5) = 5$, and $f(9) = -5$, which of the following must be true?

I. f has at least 2 zeros.
II. The graph of f has at least one horizontal tangent.
III. For some c, $2 < c < 5$, $f(c) = 3$.

a. None
b. I only
c. I and II only
d. I and III only
e. I, II and III

N

___ 3. The function f is continuous for $-2 \le x \le 1$ and differentiable for $-2 < x < 1$. If $f(-2) = -5$ and $f(1) = 4$, which of the following statements could be false?

NC

a. There exists c, where $-2 < c < 1$, such that $f(c) = 0$.
b. There exists c, where $-2 < c < 1$, such that $f'(c) = 0$.
c. There exists c, where $-2 < c < 1$, such that $f(c) = 3$.
d. There exists c, where $-2 < c < 1$, such that $f'(c) = 3$.
e. There exists c, where $-2 \le c \le 1$ such that $f(c) \ge f(x)$ for all x on the closed interval $-2 \le x \le 1$.

___ 4. If f is continuous for $a \le x \le b$ and differentiable for $a < x < b$, which of the following could be false?

NC

a. $f'(c) = \dfrac{f(b)-f(a)}{b-a}$ for some c such that $a < c < b$.
b. $f'(c) = 0$ for come c such that $a < c < b$.
c. f has a minimum value on $a \le x \le b$.
d. f has a maximum value on $a \le x \le b$.
e. $\int_a^b f(x)dx$ exists.

___ 5. The function f is continuous and differentiable on the closed interval [0,4]. The table ~~above~~ gives selected values of f on this interval. Which of the following statements must be true?

NC

x	0	1	2	3	4
$f(x)$	2	3	4	3	2

a. The minimum value of f on [0, 4] is 2.
b. The maximum value of f on [0, 4] is 4.
c. $f(x) > 0$ for $0 < x < 4$
d. $f'(x) < 0$ for $2 < x < 4$
e. There exists c, with $0 < c < 4$, for which $f'(c) = 0$.

___ 6. Let $f(x) = \int_0^{x^2} \sin t\,dt$. At how many points in the closed interval $[0, \sqrt{\pi}]$ does the instantaneous rate of change of f equal the average rate of change of f on that interval?

C

a. Zero b. One c. Two d. Three e. Four

___ 7. Let f be the function defined by $f(x) = x + \ln x$. What is the value of c for which the instantaneous rate of change of f at $x = c$ is the same as the average rate of change of f over [1,4]?

C

a. 0.456 b. 1.244 c. 2.164 d. 2.342 e. 2.452

___ 8. If f is the antiderivative of $\dfrac{x^2}{1+x^5}$ such that $f(1) = 0$, then $f(4) =$

C

a. -0.012 b. 0 c. 0.016 d. 0.376 e. 0.629

___ 9. If $F(x) = \int_0^x \sqrt{t^3+1}\,dt$, then $F'(2) =$

NC

a. -3 b. -2 c. 2 d. 3 e. 18

___ 10. $\dfrac{d}{dx}\left(\int_0^{x^2} \sin\left(t^3\right)dt\right) =$

a. $-\cos\left(x^6\right)$ b. $\sin\left(x^3\right)$ c. $\sin\left(x^6\right)$ d. $2x\sin\left(x^3\right)$ e. $2x\sin\left(x^6\right)$

____ 11. $\lim_{x \to 1} \frac{\int_1^x e^{t^2}\, dt}{x^2 - 1}$ is L'Hopital's Rule

a. 0 b. 1 $\boxed{\text{c. } \frac{e}{2}}$ d. e e. nonexistent

FRQ – NON-CALCULATOR

t (sec)	0	15	25	30	35	50	60
$v(t)$ (ft/sec)	−20	−30	−20	−14	−10	0	10
$a(t)$ (ft/sec^2)	1	5	2	1	2	4	2

6. A car travels on a straight track. During the time interval $0 \le t \le 60$ seconds, the car's velocity v, measured in feet per second, and acceleration a, measured in feet per second per second, are continuous functions. The table above shows selected values of these functions.

(a) Using appropriate units, explain the meaning of $\int_{30}^{60} |v(t)|\, dt$ in terms of the car's motion. Approximate $\int_{30}^{60} |v(t)|\, dt$ using a trapezoidal approximation with the three subintervals determined by the table.

The distance the car travels in feet from time t = 30 to t = 60 seconds.

≈ 185 ft. (Show work)

(b) Using appropriate units, explain the meaning of $\int_0^{30} a(t)\, dt$ in terms of the car's motion. Find the exact value of $\int_0^{30} a(t)\, dt$. $= \int_0^{30} v'(t)\, dt = v(30) - v(0) = -14 + 20 = 6$ ft/sec

car's change in velocity in ft/sec from time 0 to 30 seconds

(c) For $0 < t < 60$, must there be a time t when $v(t) = -5$? Justify your answer. Yes $v(35) = -10 < -5 < 0 = v(50)$ IVT guarantees $t \in (35, 50)$ such that $v(t) = -5$

(d) For $0 < t < 60$, must there be a time t when $a(t) = 0$? Justify your answer. Yes $v(0) = v(25)$ & MVT guarantees $t \in (0, 25)$ such that $a(t) = v'(t) = 0$

D

DIFFERENTIAL EQUATIONS (7 MC & 1 FRQ) **AP REVIEW**

Multiple Choice

Identify the choice that best completes the statement or answers the question.

____ 1. Population y grows according to the equation $\frac{dy}{dt} = ky$ where k is a constant and t is measured in years. If the population doubles every 10 years, then the value of k is:

C

$\boxed{\text{a. } 0.069}$ b. 0.200 c. 0.301 d. 3.322 e. 5.000

____ 2. If $\frac{dy}{dx} = ky$ and k is a nonzero constant, then y could be:

MC

a. $2e^{kty}$ $\boxed{\text{b. } 2e^{kt}}$ c. $e^{kt} + 3$ d. $kty + 5$ e. $\frac{1}{2}ky^2 + \frac{1}{2}$

____ 3. If $\frac{dy}{dx} = \sin x \cos^2 x$ and if $y = 0$ when $x = \frac{\pi}{2}$, what is the value of y when $x = 0$?

NC

a. -1 $\boxed{\text{b. } -\frac{1}{3}}$ c. 0 d. $\frac{1}{3}$ e. 1

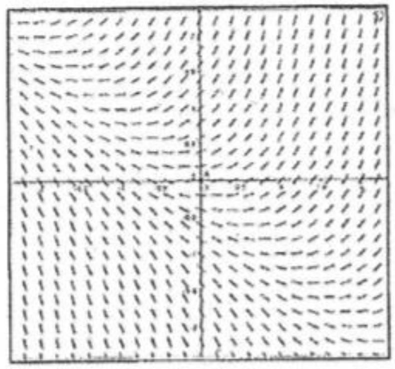
____ 4. The rate of change of the volume, V, of water in a tank with respect to time, t, is directly proportional to the square root of the volume. Which of the following is a differential equation that describes this relationship?

$\boxed{\text{a. } V(t)=k\sqrt{t}}$ b. $V(t)=k\sqrt{V}$ c. $\frac{dV}{dt}=k\sqrt{t}$ d. $\frac{dV}{dt}=\frac{k}{\sqrt{V}}$ e. $\frac{dV}{dt}=k\sqrt{V}$

____ 5. A curve has slope $2x+3$ at each point (x, y) on the curve. Which of the following is an equation for this curve if it passes through the point $(1, 2)$?

a. $y=5x-3$ b. $y=x^2+1$ c. $y=x^2+3x$ $\boxed{\text{d. } y=x^2+3x-2}$ e. $y=x^2+3x-3$

____ 6. The slope field shown at right is for which of the following differential equations?



a. $\frac{dy}{dx}=1+x$

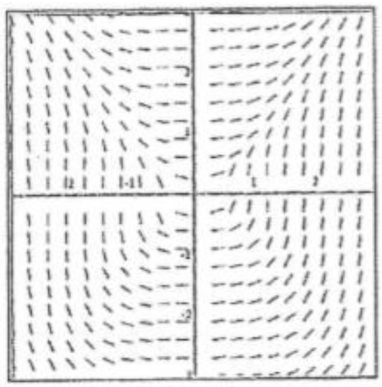
b. $\frac{dy}{dx}=x^2$

$\boxed{\text{c. } \frac{dy}{dx}=x+y}$

d. $\frac{dy}{dx}=\frac{x}{y}$

e. $\frac{dy}{dx}=\ln y$

____ 7. The slope field shown at right is for which of the following differential equations?



a. $\frac{dy}{dx}=\frac{x}{y}$

b. $\frac{dy}{dx}=\frac{x^2}{y^2}$

c. $\frac{dy}{dx}=\frac{x^3}{y}$

→ d. $\frac{dy}{dx}=\frac{x^2}{y}$

→ $\boxed{\text{e. } \frac{dy}{dx}=\frac{x^3}{y^2}}$

FRQ – NON-CALCULATOR

6. Solutions to the differential equation $\frac{dy}{dx}=xy^3$ also satisfy $\frac{d^2y}{dx^2}=y^3\left(1+3x^2y^2\right)$. Let $y=f(x)$ be a particular solution to the differential equation $\frac{dy}{dx}=xy^3$ with $f(1)=2$.

(a) Write an equation for the line tangent to the graph of $y=f(x)$ at $x=1$. $y=8(x-1)+2$

(b) Use the tangent line equation from part (a) to approximate $f(1.1)$. Given that $f(x)>0$ for $1<x<1.1$, is the approximation for $f(1.1)$ greater than or less than $f(1.1)$? Explain your reasoning.

$f(1.1)\approx 2.8$

$f(x)>0$ on $(1, 1.1)$ and $\frac{d^2y}{dx^2}>0$ on $(1, 1.1)$ so f is ccup. $\therefore 2.8<f(1.1)$ underestimate

(c) Find the particular solution $y=f(x)$ with initial condition $f(1)=2$.

$\int\frac{dy}{y^3}=\int x\,dx$

Solve

$y=\frac{2}{\sqrt{5-4x^2}}$

M

$\boxed{D}$

for $-\frac{\sqrt{5}}{2}<x<\frac{\sqrt{5}}{2}$