CALC

TUE CLASS

AP Practice Questions

Focus on Derivative Rules – Calculator Active – except for question #3.

1. On a certain workday, the rate, in tons per hour, at which unprocessed gravel arrives at a gravel processing plant is modeled by $G(t) = 90 + 45\cos\left(\frac{t^2}{18}\right)$, where t is measured in hours and $0 \le t \le 8$. At the beginning of the workday ($t = 0$), the plant has 500 tons of unprocessed gravel. During the hours of operation, $0 \le t \le 8$, the plant processes gravel at a constant rate of 100 tons per hour.

   a. Find $G'(5)$. Using correct units, interpret your answer in the context of the problem.

AP Practice Questions.

① $G(t) = 90 + 45\cos\left(\frac{t^2}{18}\right) \leftarrow Y_1$

a) $G'(5) = -24.587$

-24.588

$\frac{d}{dx}(Y_1)\Big|_{x=5} = -24.58750937$

CALC

$G(t)$ is the rate in tons/hr. at which unprocessed gravel arrives at the plant.

$G'(5) = -24.588 \text{ tons/hr}^2$

The rate at which unprocessed gravel arrives at the plant is decreasing at a rate of 24.588 tons per hour per hour. at time t = 5 hours.

t	0	1	2	3	4	5	6
$C(t)$ (ounces)	0	5.3	8.8	11.2	12.8	13.8	14.5

CALC

TUE CLASS

2. Hot water is dripping through a coffeemaker, filling a large cup with coffee. The amount of coffee in the cup at time t, $0 \le t \le 6$, is given by a differentiable function C, where t is measured in minutes. Selected values of $C(t)$, measured in ounces, are given in the table above.

   a. **Use the data in the table to approximate $C'(3.5)$. Show the computations that lead to your answer, and indicate units of measure.**

   b. **The amount of coffee in the cup, in ounces, is modeled by $B(t) = 16 - 16e^{-0.4t}$. Using the model, find the rate at which the amount of coffee in the cup is changing when $t = 5$.**

② a) Using data in table approximate $C'(3.5) \approx \dfrac{C(4) - C(3)}{4-3}$

$= \dfrac{12.8 - 11.2}{1} = 1.6 \dfrac{\text{ounces}}{\text{minute}}$

CALC

b) $B(t) = 16 - 16e^{-0.4t}$

$B'(t) = (16)(0.4)\,e^{-0.4t}$

$B'(5) = 6.4\,e^{-.4(5)} = 0.8661458127$

or $\left.\dfrac{d}{dx}\left(16 - 16e^{-.4x}\right)\right|_{x=5}$

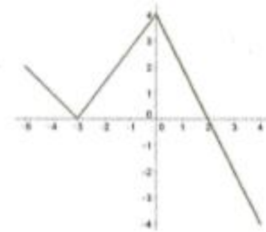
$= 0.866145836$

$B'(5) = 0.866 \dfrac{\text{ounces}}{\text{minute}}$

NON CALC

TUE CLASS

3. The graph of the derivative of f is defined on the closed interval [-5, 4]. The graph of f' consists of three line segments and is shown to the right.



a. On what open intervals contained in $-5 < x < 4$ is the graph of f both increasing and concave down? Give a reason for your answer.

b. The function h is defined by $h(x) = \frac{f(x)}{5x}$. Find $h'(3)$ if $f(3) = 6$.

c. The function p is defined by $p(x) = f(x^2 - x)$. Find the slope of the line tangent to the graph of p at the point where $x = -1$.

non calc.

3) a) f is increasing on $(-5, 2)$ b/c $f' > 0$

f is concave down on $(-5,-3)$, $(0,4)$ b/c $f'' < 0$ since f' is decreasing.

$\therefore$ f is increasing & concave down on $(-5,-3)$, $(0,2)$

b) $h(x) = \frac{f(x)}{5x}$ $\quad h'(x) = \frac{5x f'(x) - 5f(x)}{25x^2} = \frac{x f'(x) - f(x)}{5x^2}$

$h'(3) = \frac{(3)f'(3) - f(3)}{5(9)} = \frac{3(-2) - (6)}{45}$

$h'(3) = \frac{-6-6}{45} = \frac{-12}{45} = \frac{-4}{15}$

c) $p(x) = f(x^2 - x)$

$p'(x) = f'(x^2 - x) \cdot (2x - 1)$

$p'(-1) = f'(2)(-3) = 0$

Algebraically:

HW TUE **#2** Consider the differentiable function f having domain all positive real numbers, and for which it is known that $f'(x) = (4-x)x^{-3}$ for $x > 0$

(a) Find the x-coordinate of the critical point of f. Determine whether the point is a relative maximum, a relative minimum, or neither for the function f. Justify your answer.

(b) Find all intervals on which the graph of f is concave down. Justify your answer.

② Algebraically. $f'(x) = (4-x)(x^{-3}) = \frac{4-x}{x^3} = \frac{-(x-4)}{x^3}$

a) Crit. points occur when $f'(x) = 0$ or undefined

$\therefore$ CP. $x = 4, 0$.

$x = 4$ is a rel max b/c

$f'(x)$ changes signs ⊕ to ⊖.

$x = 0$ is a relative min. b/c

$f'(x)$ changes signs ⊖ to ⊕.

b) f is concave down when $f''(x) < 0$ which occurs on $(-\infty, 6)$.

$f''(x) = (-1)(x^{-3}) + (4-x)(-3)(x^{-4})$

$f''(x) = \frac{-x + (4-x)(-3)}{x^4} = \frac{-x - 12 + 3x}{x^4} = \frac{2x-12}{x^4}$

$f''(x) = \frac{2(x-6)}{x^4}$

HW TUE **#5** Consider the curve given by $-8x^2 + 5xy + y^3 = -149$.

(a) Find $\frac{dy}{dx}$.

(b) Write an equation for the line tangent to the curve at the point $(4, -1)$.

(c) There is a number k so that the point $(4.2, k)$ is on the curve. Using the tangent line found in part (b), approximate the value of k.

(d) Write an equation that can be solved to find the actual value of k so that the point $(4.2, k)$ is on the curve.

(e) Solve the equation found in part (d) for the value of k.

⑤ $-8x^2 + 5xy + y^3 = -149$

a) $\boxed{\text{Find } \frac{dy}{dx}}$ $\rightarrow$ $-16x + 5\left(y + x\frac{dy}{dx}\right) + 3y^2\frac{dy}{dx} = 0$

$$(5x + 3y^2)\frac{dy}{dx} = 16x - 5y$$

$$\frac{dy}{dx} = \frac{16x - 5y}{5x + 3y^2}$$

b) tangent line at $(4,-1)$

$$\left.\frac{dy}{dx}\right|_{(4,-1)} = \frac{16(4) - 5(-1)}{5(4) - 3(-1)^2} = \frac{64 + 5}{20 + 3} = \frac{69}{23} = 3$$

$$y = 3(x-4) - 1$$

c) $(4.2, k)$ is on the curve

Use tangent line to approximate k.

$$y = 3(4.2-4) - 1 = 3(.2) - 1 = {}^{-}0.4$$

$$k \approx {}^{-}0.4$$

d) $-8(4)^2 + 5(4)y + y^3 = -149$

$-128 + 20y + y^3 = -149$

$y^3 + 20y + 21 = 0$ → this is the equation that will find the actual y-value when $x = 4$.

e) $y = -1$

By inspection

$y = -1$

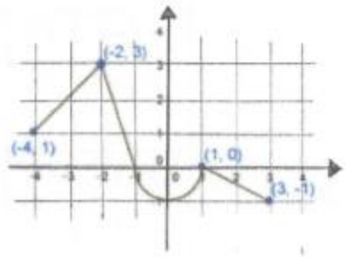
or use calculator to find $y = -1$

The tangent line estimated -0.4.
The actual value on graph is -1.

Derivative Applications: f, f', f"

HW
TUES

1. The graph of g' consists of three line segments and a semicircle centered at the origin is shown to the right.



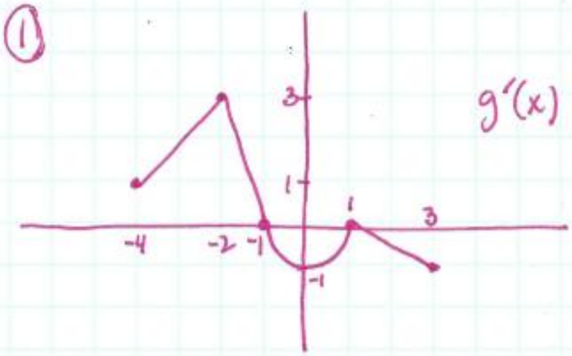
   a. For each of $g'(3)$ and $g''(3)$, find the value or explain why it does not exist.

   b. Find the x-coordinate of each point at which the graph of g has a horizontal tangent line. For each of these points, determine whether g has relative minimum, relative maximum, or neither a minimum nor a maximum at the point. Justify your answer.

   c. For $-4 < x < 3$, find all values of x for which the graph of g has a point of inflection. Explain your reasoning.

Derivative Applications f, f', f''

①



a) $g'(3) = -1$

$g''(3) = -\frac{1}{2}$ "on the left."

b) g has a horizontal tangent at $x = -1$ & 1 b/c $g'(x) = 0$

$g(-1)$ is a relative max b/c $g'(x)$ changes signs ⊕ to ⊖

$g(1)$ is neither a max nor min b/c $g'(x)$ does not change signs.

c) $g(x)$ has possible inflection points when $g''(x) = 0$ or is undefined.

∴ possible inf. points are $x = -2, 0, 1$.

B/c $g'(x)$ changes from increasing to decreasing at $x = -2, 1$ $g''(x)$ changes ⊕ to ⊖ so these are inflection pts.

B/c $g'(x)$ changes from decreasing to increasing at $x = 0$ $g''(x)$ changes ⊖ to ⊕ so this is also an inf. pt.

Inf pts @ $x = -2, 0, 1$.