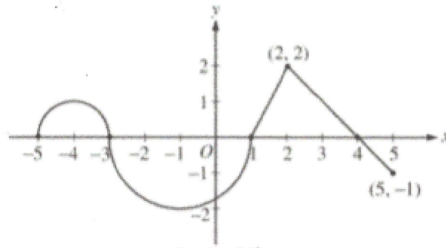
8 January 2018 Monday Night HW ANSWERS

SEMESTER EXAM FRQ REVIEW

FRQ: Justifying Extrema & Points of Inflection

Graphically:

HW. MON **#1** Let f be a function defined on the closed interval $-5 \le x \le 5$ with $f(1) = 3$. The graph of f', the derivative of f, consists of two semicircles and two line segments, as shown.



Graph of f'

(a) For $-5 \le x \le 5$, find all x at which the graph of f has a relative maximum. Justify your answer.

(b) For $-5 \le x \le 5$, find all x at which the graph of f has a point of inflection. Justify your answer.

(c) Find all intervals on which the graph of f is concave up and also has a positive slope. Justify your answer.

① Graphically

a) f has a rel. max at $x = -3, 4$ b/c $f'(x)$ changes signs ⊕ to ⊖.

b) f has a point of inflection at $x = -4, -1, 2$ b/c $f''(x)$ equals zero or is undefined and $f''(x)$ changes signs from ⊕ to ⊖ at $x = -4, 2$ since $f'(x)$ changes from inc to dec. $f''(x)$ changes signs from ⊖ to ⊕ at $x = 1$ since $f'(x)$ changes from dec to inc.

c) f is concave up when f' is increasing & ∴ $f''(x) > 0$. on $(-5, -4)$ $(-1, 2)$

f is has a positive slope when $f' > 0$ on $(-5, -3)$ $(1, 4)$

∴ f is concave up with a positive slope on $(-5, -4)$ $(1, 2)$.

FRQ: Implicit Differentiation

HW. MON **#4** Consider the curve given by $x^2 + 4y^2 = 7 + 3xy$

(a) Show that $\frac{dy}{dx} = \frac{3y - 2x}{8y - 3x}$.

(b) Show that there is a point P with x-coordinate 3 at which the line tangent to the curve at P is horizontal. Find the y-coordinate of P.

(c) Find the value of $\frac{d^2y}{dx^2}$ at the point P found in part (b). Does the curve have a local maximum, a local minimum, or neither at the point P? Justify your answer.

(4) Implicit Differentiation.

$$x^2 + 4y^2 = 7 + 3xy \longrightarrow \text{Point } P(3, ?)$$

a) $$2x + 8y\frac{dy}{dx} = 3\left(y + x\frac{dy}{dx}\right)$$

$$2x - 3y = (3x - 8y)\frac{dy}{dx}$$

$$\frac{dy}{dx} = \frac{2x-3y}{3x-8y} = \frac{3y-2x}{8y-3x} \checkmark$$

$$9 + 4y^2 = 7 + 9y$$
$$4y^2 - 9y + 2 = 0$$
$$(4y-1)(y-2) = 0$$
$$y = \frac{1}{4} \quad y = 2$$

Test to see which yields $\frac{dy}{dx} = 0$

b) Point $P(3, \square)$
at P there is a horizontal tangent

$$P(3, \tfrac{1}{4}) \quad \text{or} \quad P(3,2)$$

$$\left.\frac{dy}{dx}\right|_{(3,\frac{1}{4})} = \frac{2(3) - 3(\frac{1}{4})}{3(3) - 8(\frac{1}{4})}$$

$$\frac{6 - \frac{3}{4}}{9-2} \neq 0$$

NSPE.

$$\left.\frac{dy}{dx}\right|_{(3,2)} = \frac{2(3) - 3(2)}{3(3) - 8(2)} = \frac{0}{-7} = 0$$

$\therefore$ P(3,2) on the graph has a horizontal tangent.

Here is an alternate strategy on (b) that requires less work.

$$\frac{dy}{dx} = 0 \text{ when } \frac{2x-3y}{3x-8y} = 0$$

$$\Rightarrow \quad 2x - 3y = 0 \Big|_{x=3}$$

$$6 - 3y = 0$$

$$y = 2 \quad \therefore P(3,2)$$

c) $\frac{d^2y}{dx^2}$ at $P(3,2)$

$$\frac{dy}{dx} = \frac{2x-3y}{3x-8y} \qquad \left.\frac{dy}{dx}\right|_{(3,2)} = 0$$

$$\frac{d^2y}{dx^2} = \frac{(3x-8y)\left(2-3\frac{dy}{dx}\right) - (2x-3y)\left(3-8\frac{dy}{dx}\right)}{(3x-8y)^2}$$

$$\left.\frac{d^2y}{dx^2}\right|_{(3,2)} = \frac{\big((3)(3)-8(2)\big)(2) - \cancel{\big(2(3)-3(2)\big)(3)}}{\big(3(3)-8(2)\big)^2}$$

$$\frac{d^2y}{dx^2} \quad \frac{(-7)(2)}{(-7)^2} = \frac{2}{-7} < 0 \quad \therefore$$ the graph is concave down @ P(3,2) since $\left.\frac{d^2y}{dx^2}\right|_{(3,2)} < 0$ & P(3,2) is a local MAX.