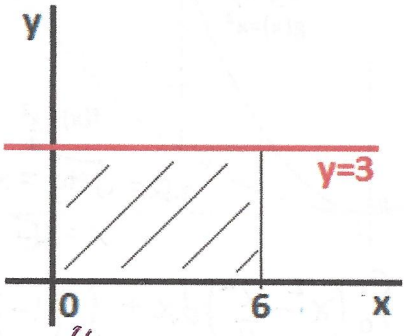
§5.4 Area between Curves

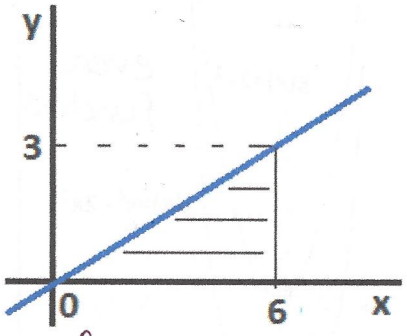
Part (a) & (b): Write and evaluate an integral for the area under the curve
Part (c): Write and evaluate an integral for the area between the curves.

1)



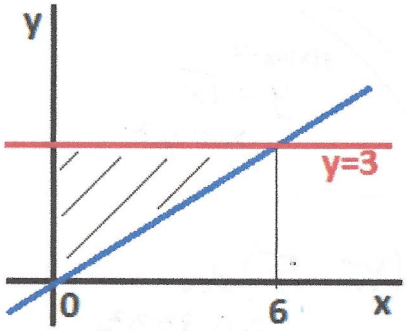
(a) $\int_0^6 3\,dx$

$= 3(6-0) = 18$



(b) $\int_0^6 \frac{1}{2}x\,dx$

$\frac{1}{2}(6)(3) = 9$

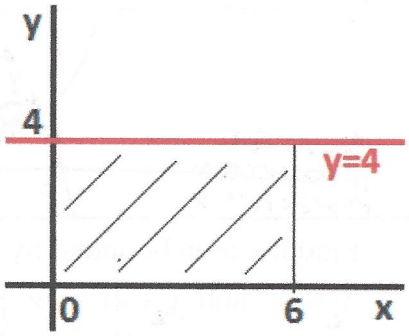


(c) $\int_0^6 (3 - \frac{1}{2}x)\,dx$

$\int_0^6 3\,dx - \int_0^6 \frac{1}{2}x\,dx$

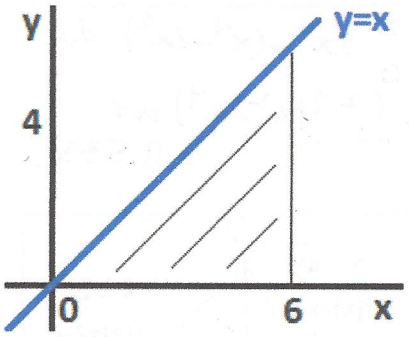
$18 - 9 = 9$

2)



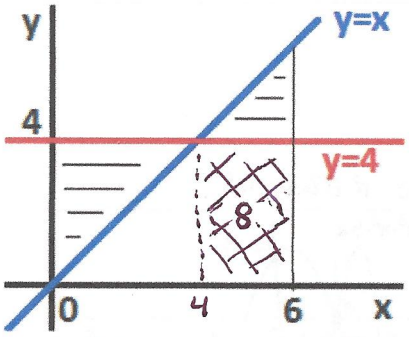
(a) $\int_0^6 4\,dx$

$4(6-0) = 24$



(b) $\int_0^6 x\,dx$

$\frac{1}{2}(6)(6) = 18$



(c) $\int_0^4 (4-x)\,dx + \int_4^6 (x-4)\,dx$

$\frac{1}{2}(4)(4) + \frac{1}{2}(2)(2)$

$8 + 2 = 10$

$18 - 8 = 10$ ✓

Example #5: Integrating with respect to y.

Find the area of the region R in the first quadrant that is bounded above by $y = \sqrt{x}$ and below by the x-axis and the line $y = x - 2$.

OPTION 1: $a < x < b$

$\int_a^b (\text{upper} - \text{lower})\,dx$

$\int_0^2 (\sqrt{x} - 0)\,dx + \int_2^4 \sqrt{x} - (x-2)\,dx$

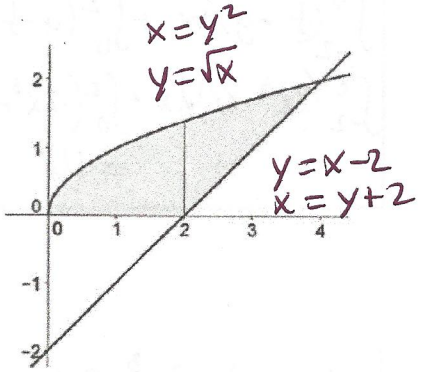
$= 3.333\ldots$

OPTION 2: $c < y < d$

$\int_c^d (\text{right} - \text{left})\,dy$

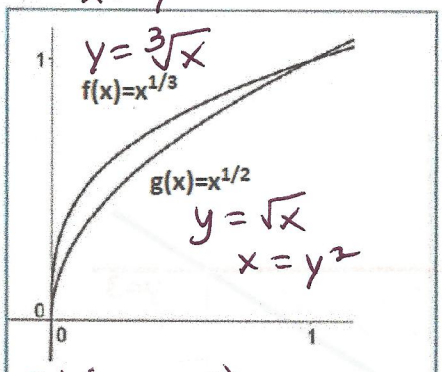
$\int_0^2 (y+2) - y^2\,dy$

$= 3.333\ldots$



Practice: Set up at least two different definite integrals to find the area of each enclosed region.

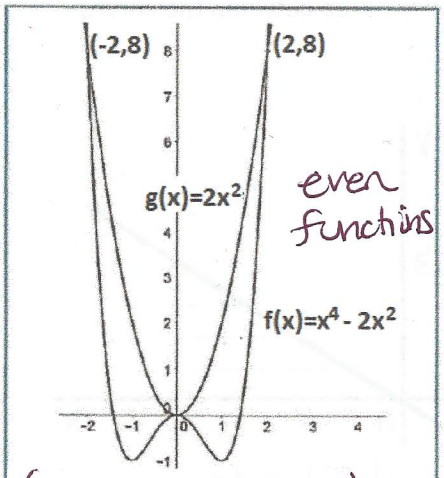
$x = y^3$



$\int_0^1 (\sqrt[3]{x} - \sqrt{x})\,dx$
$= 0.083$

OR

$\int_0^1 (y^2 - y^3)\,dy$
$= 0.083$

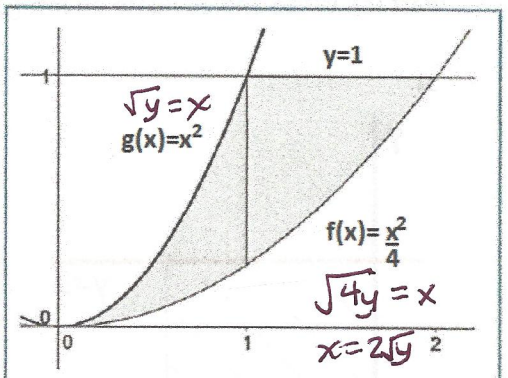


even functions

$\int_{-2}^{2} 2x^2 - (x^4 - 2x^2)\,dx$
$\int_{-2}^{2} (4x^2 - x^4)\,dx$
$= 8.533$

OR

$2\int_0^2 2x^2 - (x^4 - 2x^2)\,dx$
$2\int_0^2 (4x^2 - x^4)\,dx$
$= 8.533$

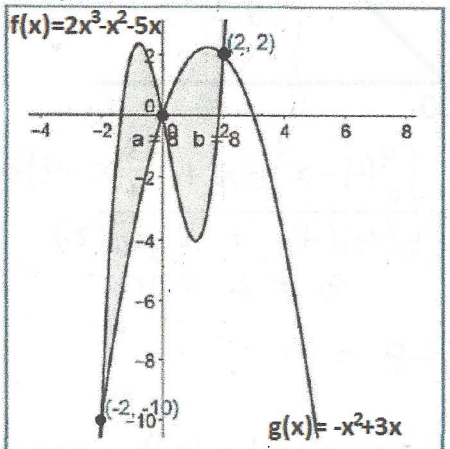


$\int_0^1 \left(x^2 - \frac{x^2}{4}\right)dx + \int_1^2 \left(1 - \frac{x^2}{4}\right)dx$
$= 0.667$

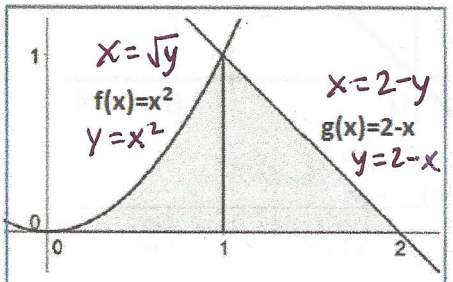
OR

$\int_0^1 (2\sqrt{y} - \sqrt{y})\,dy = 0.667$

TWO REGIONS:



$\int_{-2}^{0} (f-g)\,dx + \int_0^2 (g-f)\,dx$
$\int_{-2}^{0} (2x^3 - 8x)\,dx + \int_0^2 (-2x^3 + 8x)\,dx$
$= 16$

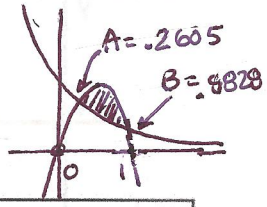


$\int_0^1 x^2\,dx + \int_1^2 (2-x)\,dx$
$= 0.833$

OR

$\int_0^1 (2 - y - \sqrt{y})\,dy$
0.833

SOLVE FOR INTERSECTION POINTS – STOP



Find the area bounded by $y_1 = e^{-x}$ and $y_2 = 4(x - x^2)$.

$y = 4x(1-x)$

$\int_A^B (4x - 4x^2 - e^{-x})\,dx$
$\int_A^B (y_2 - y_1)\,dx = 0.172$

Find the area bounded by $y = \sin(x) + 2$ and $y = 0.5$ for $6 \le x \le 10$.

$\int_6^{10} (\sin(x) + 2 - \frac{1}{2})\,dx$
$\int_6^{10} (\sin(x) + \frac{3}{2})\,dx = 7.799$

$f(x) - g(x) = 2x^3 - 8x$
$g(x) - f(x) = -2x^3 + 8x$