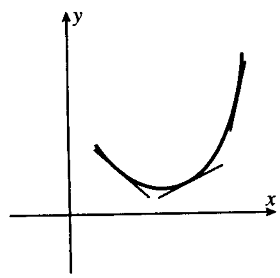
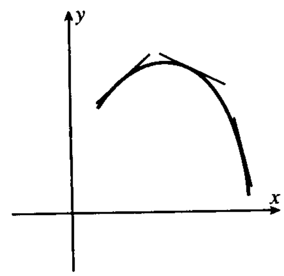
A function $f(x)$ is *concave upward* on an interval $I$ if $f(x)$ lies above all tangent lines to $f(x)$ in $I$.
A function $f(x)$ is *concave downward* on an interval $I$ if $f(x)$ lies below all tangent lines to $f(x)$ in $I$.



concave upward

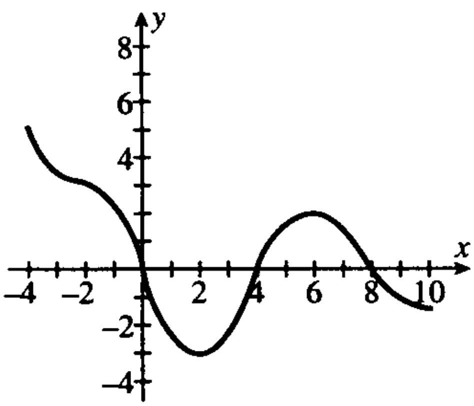


concave downward

The test for concavity involves the second derivative: If $f(x)$ is twice differentiable on an interval $I$ (meaning $f''(x)$ exists for all $x$ on the interval $I$ ) then

a. If $f''(x) > 0$ for all $x$ on the interval $I$, then $f$ is *concave upward* on $I$.
b. If $f''(x) < 0$ for all $x$ on the interval $I$, then $f$ is *concave downward* on $I$.

Example 1: Use the graph below to answer true or false to each.




**a)** $f''(x) > 0$ for $x \in (2, 4)$

**b)** $f''(x) < 0$ for $x \in (-4, -2)$

**c)** $f''(6) = 0$

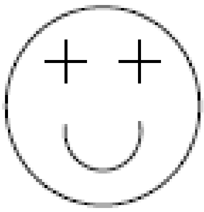
**d)** $f''(2) > 0$

**e)** $f$ is concave upward on $(0, 2)$

The concavity test can be remembered with the following pictures … keep in mind these are NOT to be used for justification.

$f''$ positive

$f$ Concave UP




$f''$ negative

$f$ Concave DOWN

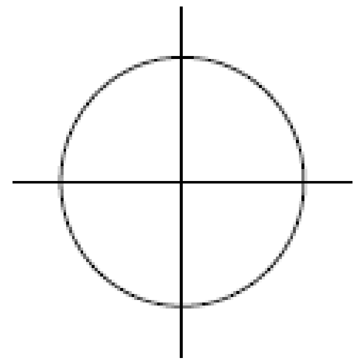

***Example***: Label each quadrant below with one of the following descriptions:

i) Increasing and Concave Up

ii) Increasing and Concave Down

iii) Decreasing and Concave Up

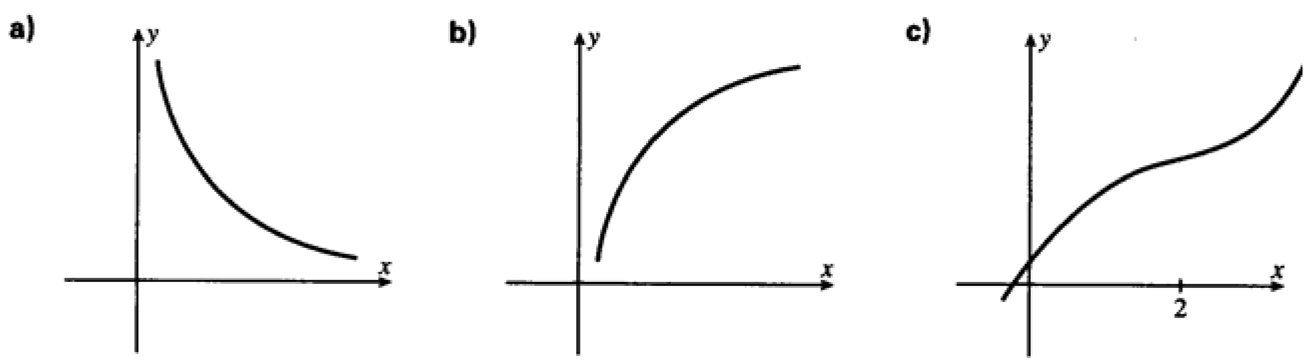
iv) Decreasing and Concave Down.



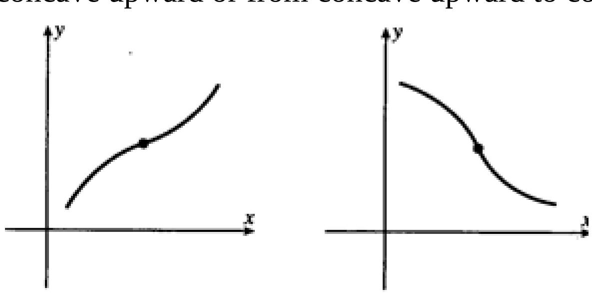
***Points of Inflection***

**A point of inflection is a point on the graph where the concavity changes.**

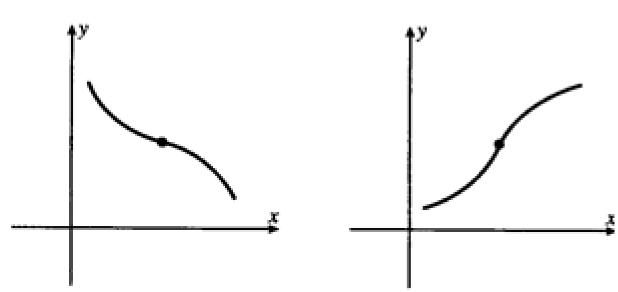
Example 2: The graph of a function $f$ is given. What can be said about $f'(x)$ and $f''(x)$ for each (i.e., positive/negative/where)?




A *point of inflection* for $f$ is a point on the graph of $f$ where concavity changes from concave downward to concave upward or from concave upward to concave downward.




Concave downward to concave upward




Concave upward to concave downward

Example 3: Sketch a graph of a function having all of the following properties.

$f(-1) = 4, f(0) = 2, f(2) = 1, f(3) = 0$
$f'(x) \leq 0$ for $x < 3$ and
$f'(x) \geq 0$ for $x > 3$.
$f''(x) < 0$ for $0 < x < 2$ and
$f''(x) \geq 0$ elsewhere.