2.4 Interpretations of the Derivative—Student Notes HH6ed

An Alternative Notation for the Derivative and Interpreting its Meaning

Given that $y = f(x)$, the derivative can be written as $f'(x)$ or $\dfrac{dy}{dx}$.

The second notation was introduced by Wilhelm Gottfried Leibniz, a German mathematician. The letter d stands for "small difference in . . ." so literally the notation $\dfrac{dy}{dx}$ can be thought of as

$$\frac{\text{Small difference in } y\text{-values}}{\text{Small difference in } x\text{-values}}$$

We say "the derivative of *y* with respect to *x*."

Example 1: Use the definition of derivative to find a formula for $\dfrac{dy}{dx}$ algebraically given $f(x) = x^2 - x$.

If we want to indicate that you should find the derivative at $x = 2$, you write $f'(2)$ or $\left.\dfrac{dy}{dx}\right|_{x=2}$

Example 2: Suppose $s = f(t)$ gives the distance, in meters, of a body from a fixed point as a function of time t, in seconds.

a. Describe the following in real-world terms: $\left.\dfrac{ds}{dt}\right|_{t=2}$

b. What are the units associated with this quantity?

c. What is the common term for $\dfrac{ds}{dt}$?

d. What is the real-world meaning of $f'(2) = 10$? Use units in your answer.

Example 3: The cost, C, in dollars, of building a house A ft^2 in area is given by the function $C = f(A)$.

a. What is the real world meaning of $f(2000) = 195{,}000$? Use units in your answer.

b. What is in the independent variable? Dependent variable?

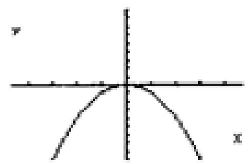
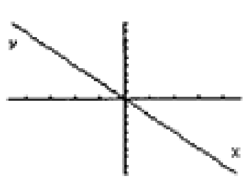
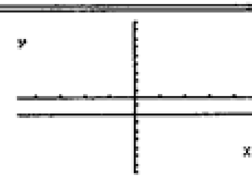
c. What is the sign of $f'(A)$? Why?

d. Rewrite $f'(A)$ in Leibniz's notation.

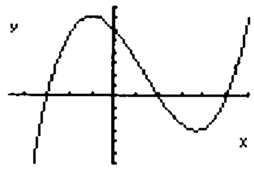
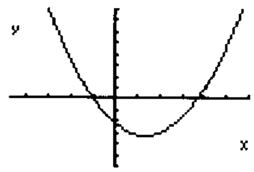
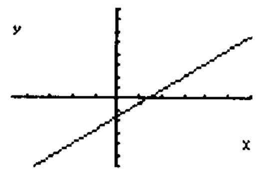
e. What are the units of $f'(2000)$?

f. What is the real world meaning of $f'(2000) = 150$?

Analyze the graph of $y = -x^2$, using the first and second derivative graphs.

Graph of f(x)	***Graph of f '(x)***	***The value of the slope of tangent line***	***Graph of f ''(x)***	***Observations***
$y = f(x) = -x^2$  • *for x <0 f is increasing* • *for x > 0 f is decreasing* • *f is always concave down*	 • *for x < 0 f '(x) is positive.* • *for x > 0 f '(x) is negative.* • *f '(x) is always decreasing*	*f '(-2) = 4* *f '(-1) = 2* *f '(0) = 0* *f '(1) = -2* *f'(2) = -4* *f '(x) is always decreasing*	 *f ''(x) < 0 for all x.*	On the same interval: • *f(x) is concave down* • *f '(x) is decreasing* • *f ''(x) < 0*

Observe the relationship between the graphs of *f(x), f '(x) and f ''(x).*

Graph of f(x)	***Graph of f'(x)***	***Graph of f''(x)***	***Observations***
 *f(x) changes concavity*	 *f '(x) decreases then increases*	 *f ''(x) is negative then positive*	• *Where f ''(x) changes from negative to positive, f(x) changes concavity.* • *Where f ''(x) = 0, f(x) changes concavity.* • *Where f '(x) changes from decreasing to increasing, f ''(x) = 0.*

Conclusions:

The First Derivative:

- If $f'(x) > 0$ on an interval, then $f(x)$ is ________________________ over that interval.
- If $f'(x) < 0$ on an interval, then $f(x)$ is ________________________ over that interval.
- If $f'(x) = 0$ on an interval, then $f(x)$ has a ________________________ at x which is

  either a ________________________ when ___________ changes sign

  or a ___________________________ when ____________ does not change sign.

The Second Derivative:

- If $f''(x) > 0$ on an interval, then $f'(x)$ is _________________________ and

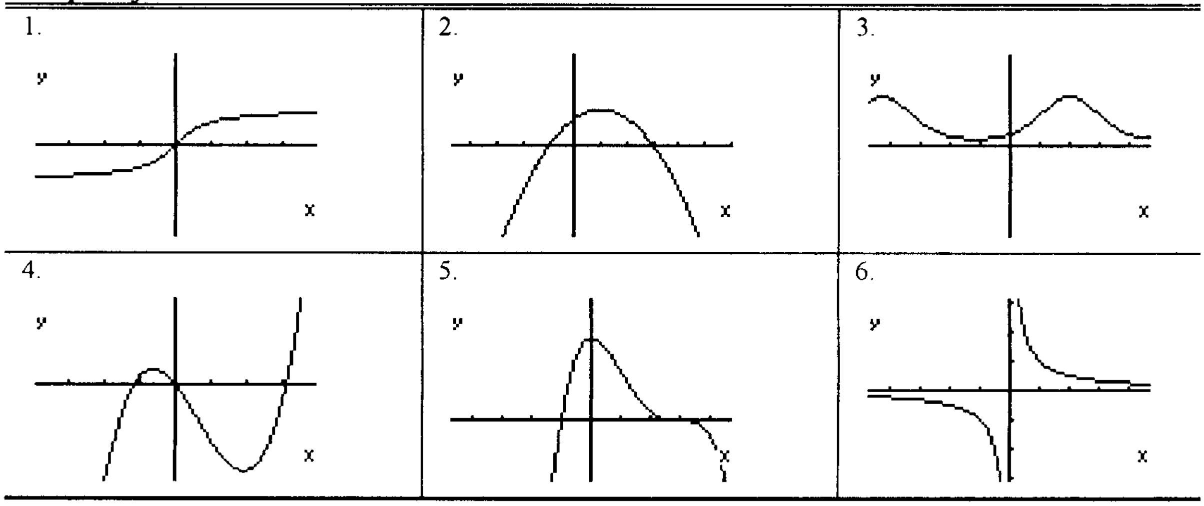
  $f(x)$ is __________________________ over that interval.
- If $f''(x) < 0$ on an interval, then $f'(x)$ is _________________________ and

  $f(x)$ is __________________________ over that interval.
- If $f''(x) = 0$ on an interval, then $f(x)$ sometimes has a ________________________________ at

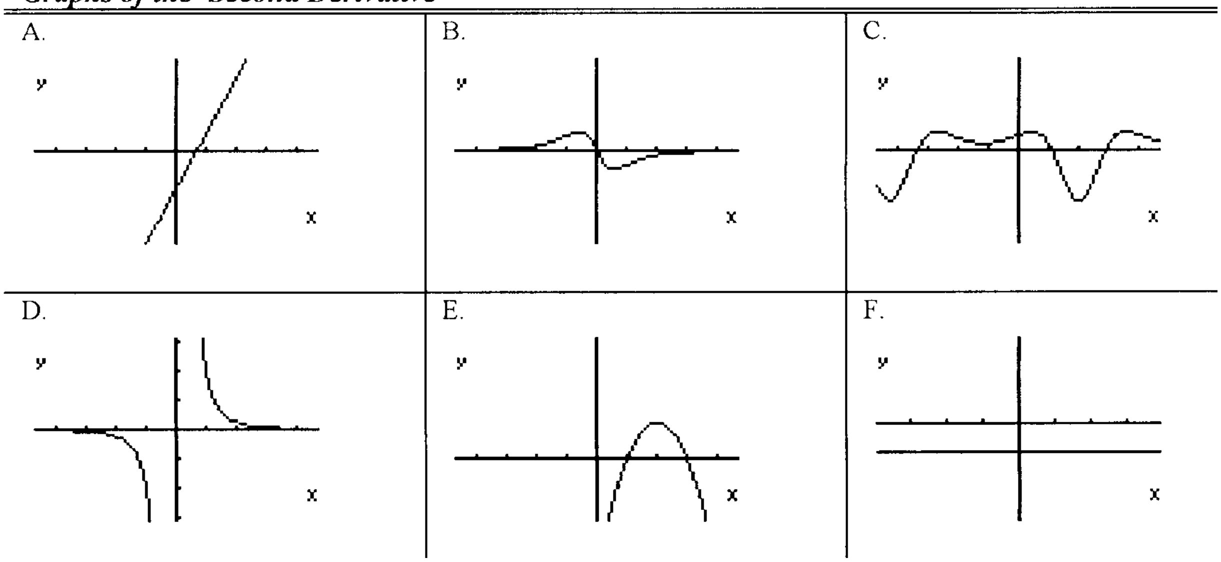
  that value of x, but only if ______________ changes sign.

Match the graph of each function below 1 - 6, with the graph of its second derivative A - F.

Graphs of the Function



Graphs of the Second Derivative



Determine which of the functions graphed below is

a) increasing at an increasing rate,
b) increasing at a decreasing rate,
c) decreasing at an increasing rate or
d) decreasing at a decreasing rate.

Explain why you have chosen each.

Graphs of Increasing and Decreasing functions