☆ **Algebraically:**

THU CLASS **#3** Suppose that the function f has a continuous second derivative for all x, and that $f(0)=2,\ f'(0)=-3$, and $f''(0)=0$. Let g be a function whose derivative is given by $g'(x)=e^{-2x}\left(3f(x)+2f'(x)\right)$ for all x.

(a) Write an equation of the tangent line to the graph of f at the point where $x=0$.

(b) Is there sufficient information to determine whether or not the graph of the f has a point of inflection when $x=0$? Justify your answer.

(c) Given that $g(0)=4$, write an equation of the line tangent to the graph of g at the point where $x=0$.

(d) Show that $g''(x)=e^{-2x}\left(-6f(x)-f'(x)+2f''(x)\right)$. Does g have a local maximum at $x=0$? Justify your answer.

③ Algebraically $\quad g'(x)=e^{-2x}\left(3f(x)+2f'(x)\right)$

$f(0)=2 \quad f'(0)=-3 \quad f''(0)=0$

☆ a) tangent line to f @ x=0 $\quad y=-3(x-0)+2$

$y=-3x+2$

b) does f have inflection pt @ x=0? (0,2) is a possible Inf pt. but there is not enough information to guarantee a pt of inf. b/c we don't know if $f''(x)$ changes signs at $x=0$.

c) $g(0)=4 \quad g'(0)=e^0\left(3f(0)+2f'(0)\right)$

$=1\left(3(2)+2(-3)\right)$

$=1(6-6)=0$

$y=4$ is the tangent line to $g(x)$ at $x=0$ b/c tangent line is horizontal

d) $g''(x)=(e^{-2x})\left(-6f(x)-f'(x)+2f''(x)\right)$ ← SHOW THIS

$g'(x)=(e^{-2x})\left(3f(x)+2f'(x)\right)$ TAKE DERIVATIVE USE PRODUCT RULE.

$g''(x)=\left(-2e^{-2x}\right)\left(3f(x)+2f'(x)\right)+\left(e^{-2x}\right)\left(3f'(x)+2f''(x)\right)$

$=e^{-2x}\left(-6f(x)-4f'(x)+3f'(x)+2f''(x)\right)$

$g''(x)=(e^{-2x})\left(-6f(x)-f'(x)+2f''(x)\right)$ ✓ DONE. ✓

2nd Deriv Test for Local Max? → $g''(0)=(e^0)\left(-6f(0)-f'(0)+2f''(0)\right)$

$=1\left(-6(2)-(-3)+\cancel{2(0)}\right)$

$=-12+3$

$g''(0)=-9<0$ $\quad$ B/c $g''(0)<0$ $g(x)$ is concave down & $g(0)$ is a Local max by the 2nd Derivative Test.

☆ CLASS THU

#6 Consider the curve defined by $x^2 + xy + y^2 = 27$

(a) Write an expression for the slope of the curve at any point (x, y).

(b) Determine whether the lines tangent to the curve at the $x-$ intercepts of the curve are parallel. Show the analysis that leads to your conclusion.

(c) Find the points on the curve where the lines tangent to the curve are vertical.

6 $\quad x^2 + xy + y^2 = 27$

a) $\frac{dy}{dx} = ?$ $\quad 2x + y + x\frac{dy}{dx} + 2y\frac{dy}{dx} = 0$

$$(x+2y)\frac{dy}{dx} = -(2x+y)$$

$$\frac{dy}{dx} = \frac{-(2x+y)}{(x+2y)}$$

b) x intercepts of the curve occur when $y=0$.

$x^2 = 27$

$x = \pm 3\sqrt{3}$

$$\left.\frac{dy}{dx}\right|_{(3\sqrt{3},0)} = \frac{-(6\sqrt{3}+0)}{(3\sqrt{3}+0)} = -2 \qquad \left.\frac{dy}{dx}\right|_{(-3\sqrt{3},0)} = \frac{-(-6\sqrt{3}+0)}{(-3\sqrt{3}+0)} = -2$$

The tangent lines to the curve at the x-intercepts are parallel b/c their slopes are both -2.

c) tangent lines are vertical when $\frac{dy}{dx}$ is undefined

$\therefore$ when $(x+2y)=0$ as long as $-(2x+y) \neq 0$ too.

$\therefore x = -2y$ or $y = \frac{-x}{2}$

$\Downarrow$

$(-2y)^2 + (-2y)y + y^2 = 27$

$4y^2 - 2y^2 + y^2 = 27$

$3y^2 = 27$

$y^2 = 9$

$y = \pm 3$

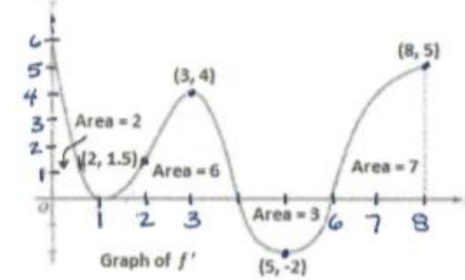
$y = +3 \rightarrow x = -6 \therefore (-6, 3)$
$y = -3 \rightarrow x = +6 \therefore (6, -3)$ $\Big\}$ Points on the curve where tangent lines are vertical.

HW 2. The figure to the right shows the graph of f', the twice-differentiable function f, on the closed interval $0 \le x \le 8$.

THU

The graph of f' has horizontal tangent lines at $x = 1$, $x = 3$, and $x = 5$.

The function is defined for all real numbers and satisfies $f(8) = 4$.



a. Find all values of x on the open interval $0 < x < 8$ for which the function f has a local minimum. Justify your answer.

b. On what open intervals contained in $0 < x < 8$ is the graph of f both concave down and increasing? Explain your reasoning.

c. The function g is defined by $g(x) = (f(x))^3$. If $f(3) = \frac{5}{2}$, find the slope of the line tangent to the graph of g at $x = 3$.

Derivatives f, f', f''

② a) local min @ $x = 6$ b/c f' changes signs ⊖ to ⊕.

b) f is concave down on $(0,1)$ $(3,5)$ b/c f' decreasing
f is increasing on $(0,4)$, $(6,8)$ b/c $f'(x) > 0$.
∴ f is ccd & inc on $(0,1), (3,4)$.

c) $g(x) = (f(x))^3$ $\quad f(3) = \frac{5}{2}$ $\quad g(3) = (f(3))^3 = (4)^3 = 64$

$g'(x) = 3(f(x))^2 \cdot f'(x)$

$g'(3) = 3(f(3))^2 \cdot f'(3)$ from graph.

$g'(3) = 3\left(\frac{5}{2}\right)^2 (4)$

$g'(3) = 75$

$\boxed{y = 75(x-3) + 64}$