

Calculator Skills for the AP Exam

Using a graphing calculator on the AP Calculus exam

Students are expected to know how to use their graphing calculators on the AP Calculus exams proficiently to accomplish the following four capabilities:

- Plot the graph of a function within an arbitrary viewing window
- Find the zeros of functions (solve equations numerically)
- Numerically calculate the derivative of a function
- Numerically calculate the value of a definite integral

Multiple Choice

1. A particle moves along the x -axis. The velocity of the particle at time t is given by $v(t)$, and the acceleration of the particle at time t is given by $a(t)$. Which of the following gives the average velocity of the particle from time $t = 0$ to the time $t = 8$?

(A) $\frac{a(8) - a(0)}{8}$

(B) $\frac{1}{8} \int_0^8 v(t) dt$

(C) $\frac{1}{2} \int_0^8 |v(t)| dt$

(D) $\frac{1}{2} \int_0^8 v(t) dt$

(E) $\frac{v(0) - v(8)}{2}$

2. Let $f'(x) = \frac{\ln x}{e^x} - \cos x$ for $1 < x < 6$. On what intervals is f concave down?

(A) (1.481, 4.726)

(B) (1, 3.105)

(C) (3.105, 6)

(D) (1, 4.726)

(E) $(1, 1.481) \cup (4.726, 6)$

3. Water is pumped into a tank at a rate of $r(t) = 30(1 - e^{-0.16t})$ gallons per minute, where t is the number of minutes since the pump was turned on. If the tank contained 800 gallons of water when the pump was turned on, how much water, to the nearest gallon, is in the tank after 20 minutes?

(A) 380 gallons

(B) 420 gallons

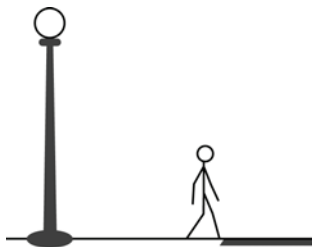
(C) 829 gallons

(D) 1220 gallons

(E) 1376 gallons

4. Two particles start at the origin and move along the x -axis. For $0 \leq t \leq 4$, their respective position functions are given to be $x_1 = \ln(t^2)$ and $x_2 = (t-5)^2$. For what value of t does the acceleration of x_1 equal the velocity of x_2 ?
- (A) 1.060
(B) 0.470
(C) 3.960
(D) 2.039
(E) None

5. The average value of the function $f(x) = \cos(x^2)$ on the closed interval $[0, 2]$ is
- (A) 0.231
(B) 0.461
(C) 0.780
(D) 0.977
(E) 1.253



6. A person whose height is 6 feet is walking away from the base of a streetlight along a straight path at a rate of 4 feet per second. If the height of the streetlight is 15 feet, what is the rate at which the person's shadow is lengthening?
- (A) 1.5 ft/sec
(B) 2.667 ft/sec
(C) 3.75 ft/sec
(D) 6 ft/sec
(E) 10 ft/sec
7. If c satisfies the conclusion of the Mean Value Theorem for $f(x) = \sin^{-1} x$ on the interval $0 \leq x \leq 1$, then c is
- (A) 0.500
(B) 0.771
(C) 0.785
(D) 1
(E) 1.186

8. A particle moves along a line so that its acceleration for $t \geq 0$ is given by $a(t) = \frac{t+3}{\sqrt{t^3+1}}$. If the particle's velocity at $t = 0$ is 5, what is the velocity of the particle at $t = 3$?
- (A) 0.713
(B) 1.134
(C) 6.134
(D) 6.710
(E) 11.710
9. Given $f(x) = \int_1^x \frac{t^2}{(e^t+1)(\sin t)} dt$; $f''(1) =$
- (A) 0
(B) 0.200
(C) 0.320
(D) 0.341
(E) does not exist
10. The area of the region bounded by the curves $y = e^{-x}$, $y = \ln x$, and the line $x = 1$ is
- (A) 0.042
(B) 0.054
(C) 0.096
(D) 0.728
(E) 1.686
11. Let $g(x)$ be the function given by $g(x) = \int_0^x e^t(t^2 - 1) dt$. Which of the following must be true?
- I. g is decreasing on $(0,1)$
II. g is decreasing on $(1,2)$
III. $g(2) > 0$
- (A) I only
(B) II only
(C) III only
(D) I and III only
(E) I, II, and III

12. Let R be the region in the first quadrant enclosed by the graphs $y = x^4 + 1$ and $y = x + 16$. The volume of the solid generated by revolving R about the x -axis is
- (A) 25.616
 - (B) 80.475
 - (C) 507.539
 - (D) 1594.480
 - (E) 3188.959

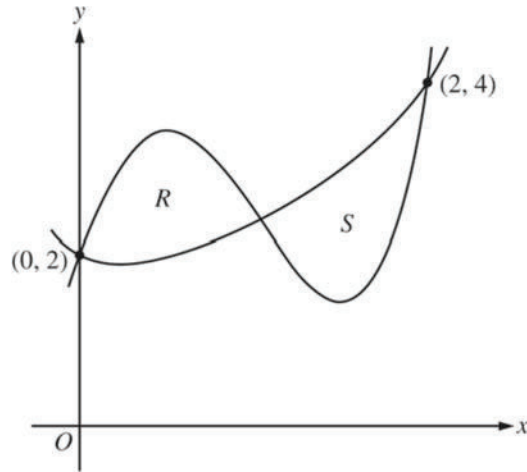
Free Response Questions (all are AP and calculator allowed)

13.

t (minutes)	0	4	9	15	20
$W(t)$ (degrees Fahrenheit)	55.0	57.1	61.8	67.9	71.0

The temperature of water in a tub at time t is modeled by a strictly increasing, twice-differentiable function W , where $W(t)$ is measured in degrees Fahrenheit and t is measured in minutes. At time $t = 0$, the temperature of the water is 55°F . The water is heated for 30 minutes, beginning at time $t = 0$. Values of $W(t)$ at selected times t for the first 20 minutes are given in the table above.

- (a) Use the data in the table to estimate $W'(12)$. Show the computations that lead to your answer. Using correct units, interpret the meaning of your answer in the context of this problem.
- (b) Use the data in the table to evaluate $\int_0^{20} W'(t) dt$. Using correct units, interpret the meaning of $\int_0^{20} W'(t) dt$ in the context of this problem.
- (c) For $0 \leq t \leq 20$, the average temperature of the water in the tub is $\frac{1}{20} \int_0^{20} W(t) dt$. Use a left Riemann sum with the four subintervals indicated by the data in the table to approximate $\frac{1}{20} \int_0^{20} W(t) dt$. Does this approximation overestimate or underestimate the average temperature of the water over these 20 minutes? Explain your reasoning.
- (d) For $20 \leq t \leq 25$, the function W that models the water temperature has first derivative given by $W'(t) = 0.4\sqrt{t} \cos(0.06t)$. Based on the model, what is the temperature of the water at time $t = 25$?



14. Let f and g be the functions defined by $f(x) = 1 + x + e^{x^2-2x}$ and $g(x) = x^4 - 6.5x^2 + 6x + 2$.

Let R and S be the two regions enclosed by the graphs of f and g shown in the figure above.

(a) Find the sum of the areas of regions R and S .

(b) Region S is the base of a solid whose cross sections perpendicular to the x -axis are squares. Find the volume of the solid.

(c) Let h be the vertical distance between the graphs of f and g in region S . Find the rate at which h changes with respect to x when $x = 1.8$.

15. Grass clippings are placed in a bin, where they decompose. For $0 \leq t \leq 30$, the amount of grass clippings remaining in the bin is modeled by $A(t) = 6.687(0.931)^t$, where $A(t)$ is measured in pounds and t is measured in days.
- (a) Find the average rate of change of $A(t)$ over the interval $0 \leq t \leq 30$. Indicate units of measure.
- (b) Find the value of $A'(15)$. Using correct units, interpret the meaning of the value in the context of the problem.
- (c) Find the time t for which the amount of grass clippings in the bin is equal to the average amount of grass clippings in the bin over the interval $0 \leq t \leq 30$.
- (d) For $t > 30$, $L(t)$, the linear approximation to A at $t = 30$, is a better model for the amount of grass clippings remaining in the bin. Use $L(t)$ to predict the time at which there will be 0.5 pound of grass clippings remaining in the bin. Show the work that leads to your answer.

AP Calculus Exam Format

The AP exam has four parts:

1. 30 question multiple choice test *without* calculators (60 min)
2. 15 question multiple choice test *with* calculators (45 min) Note: only six or seven of these questions actually require the use of the calculator. The others are conceptual questions where the calculator has no role.
3. 2 question free response test *with* calculators (30 min)
4. 4 question free response test *without* calculators (60 min)

After the completion of the first free response portion, students will receive the "non-calculator free response" test, but will still be able to work on the original 2 free response questions. However, they will no longer have access to their graphing calculators for those questions.

A student's ability to use a graphing calculator effectively will be tested on the calculator active portions of the AP Exam. Students should know when and how to properly use their graphing calculators on the calculator active questions.

- **One very common mistake that hurts a student's score is trying to work out a complex problem by hand when a calculator is available!**
- **Students need to avoid preliminary rounding and use the storing feature of their calculator to aid in the accuracy of answers.**
- **When possible, refer to the names of functions given in the problem (such as $f(x)$ or $g(x)$) to avoid making careless errors when setting up solutions.**

IMPORTANT NOTE: Graphing calculators are a valuable tool for numeric calculations and to understand the behavior of a graph but CANNOT be used as justification on free response questions. An appropriate mathematical justification and/or explanation is necessary. Also, when using a graphing calculator on the free response questions, avoid writing down calculator syntax on the exam. Credit will not be awarded for simply writing down what is "typed" into the calculator. Proper calculus notation must be used.

Calculator skills that are required or beneficial for the AP exam:

- Determine the point of intersection
- Determine the zero of a function
- Store and recall values
- Determine points of extrema
- Calculate numerical derivatives
- Calculate definite integrals
- Graph derivative(s) given a function