☆ WED CLASS

1. A ball is dropped from the top of the Empire State building to the ground below. The height, y, of the ball above the ground (in feet) is given as a function of time, t, in seconds by $y(t) = 1250 - 16t^2$
   a. Find the velocity of the ball at time t. What is the sign of the velocity? Why is this to be expected?
   b. Show that the acceleration of the ball is a constant. What is the value and sign of this constant?
   c. When does the ball hit the ground, and how fast is it going at that time? Give your answer in feet/sec.

① $y(t) = 1250 - 16t^2$ (t sec, $y(t)$ feet)

a) velocity $y'(t) = -32t$ ft/sec.

velocity is negative b/c the height of the object is decreasing over time.

b) acceleration $y''(t) = -32$ ft/sec/sec $= -32 \frac{ft}{sec^2}$

This is the acceleration due to gravity on earth.

c) $y(t) = 0 = 1250 - 16t^2$

$16t^2 = 1250$

$t^2 = \frac{625}{8} \therefore t = \pm\frac{25}{2\sqrt{2}}$ exclude $t<0$ seconds.

So $t = +\frac{25}{2\sqrt{2}} = \frac{25\sqrt{2}}{4}$ seconds 8.838834765 seconds

or $\underline{8.839 \text{ sec}}$

$\underline{8.838 \text{ sec}}$

$v(t) = y'(t)$ @ $t = 8.838$ sec

$v(8.838) \approx -282.8427125$ ft/sec. $-282.842 \frac{ft}{sec}$ $-282.843 \frac{ft}{sec}$

When the ball hits the ground its velocity is -282.842 ft/sec.

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2. The temperature, H, in degrees Fahrenheit ($^{\circ}F$), of a can of soda that is put into a refrigerator to cool is given as a function of time, t, in hours by $H(t) = 40 + 30e^{-2t}$.

   a. Find the rate at which the temperature of the soda is changing in ($^{\circ}F/hour$).

   b. What is the sign of $\frac{dH}{dt}$? Why?

   c. When, for $t \geq 0$, is the magnitude of $\frac{dH}{dt}$ largest? In terms of the can of soda, why is this?

② $H(t) = 40 + 30e^{-2t}$

a) $H'(t) = -60e^{-2t}$ $^{\circ}F/hr.$

b) $\frac{dH}{dt} < 0$ b/c the soda temperature is decreasing

c) max. $\frac{dH}{dt} \longrightarrow H''(t) = 120e^{-2t} = 0$ never

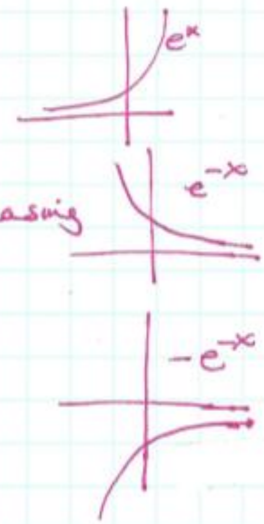
so check endpoints

$t = 0 \quad \frac{dH}{dt} = -60$

$t = \infty \quad \frac{dH}{dt} = -0$

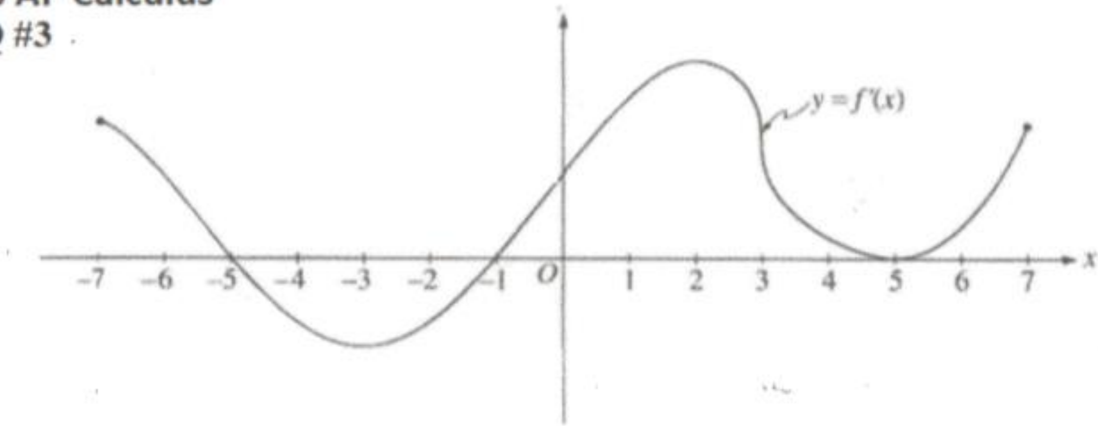
$\therefore \frac{dH}{dt}$ has largest magnitude at $t = 0$ when $\frac{dH}{dt} = -60\ ^{\circ}F/hr.$

When the soda is placed in the refridgerator it is cooling exponentially & has the greatest rate of change initially & then the rate of change is not as great as time passes.



2000 AP Calculus
FRQ #3

☆
WED CLASS



3. The figure above shows the graph of f', the derivative of the function f, for $-7 \le x \le 7$. The graph of f' has horizontal tangent lines at $x = -3$, $x = 2$, and $x = 5$, and a vertical tangent line at $x = 3$.

   (a) Find all values of x, for $-7 < x < 7$, at which f attains a relative minimum. Justify your answer.

   (b) Find all values of x, for $-7 < x < 7$, at which f attains a relative maximum. Justify your answer.

   (c) Find all values of x, for $-7 < x < 7$, at which $f''(x) < 0$.

   (d) At what value of x, for $-7 \le x \le 7$, does f attain its absolute maximum? Justify your answer.

AP 2000 FRQ #3

a) f has a relative minimum when $f'(x)$ changes signs from ⊖ to ⊕ which occurs at $\underline{x = -1}$.

b) f has a relative maximum when $f'(x)$ changes signs from ⊕ to ⊖ which occurs at $\underline{x = -5}$.

c) $f''(x) < 0$ when $f'(x)$ is decreasing which occurs when $x \in (-7, -3)\ (2, 5)$

d) f has an <u>absolute maximum</u> either at a relative maximum $x = -5$ or at an endpoint $x = -7$ or $x = 7$.] STATE CANDIDATES

Because $f(x)$ is increasing on $(-1, 7)$ the absolute maximum occurs at $x = 7$.

$f(-5) > f(-7)$ b/c $f(x)$ is increasing on $(-7, -5)$.

$f(7) > f(-5)$ b/c although $f(x)$ decreases on $(-5, -1)$ $f(x)$ decreases by only a small amount compared to the increase of $f(x)$ on $(-1, 7)$.
so $f(7)$ is the ABSOLUTE MAX.

H.W.
WED

3. Interpret what each of the following sentences says about the derivative of the given function. In each case, be sure to indicate what the function and any variables are. Then, sketch the graph of a function which satisfies the properties indicated in the sentence.

   a. The price of a product decreases as more of it is produced.

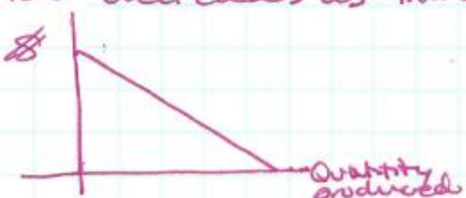
   b. The child's temperature has been rising the last three hours, but not as rapidly since we gave her an antibiotic an hour ago.

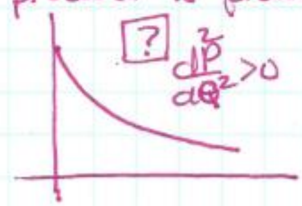
   c. The cost of health insurance is rising at an ever increasing rate.

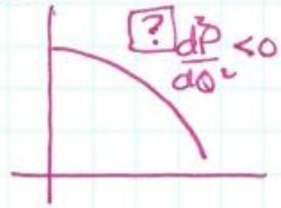
   d. The car is gradually slowing to a stop.

(3) a) price decreases as more product is produced

$\frac{dP}{dQ} < 0$



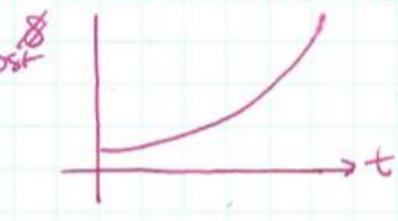




b) temperature rising but not as rapidly since antibiotic given

$\frac{dT}{dt} > 0 \quad \frac{d^2T}{dt^2} < 0.$

c) Cost of health insurance rising at increasing rate.



$\frac{dC}{dt} > 0 \quad \frac{d^2C}{dt^2} > 0$

d). Car gradually slowing to a stop.

$\frac{dD}{dt} < 0 \quad \frac{d^2D}{dt^2} > 0$

H.W.
WBD

4. Water is flowing into a large spherical tank at a constant rate. Let $V(t)$ be the volume of water in the tank and $H(t)$ be the height of the water level at time t.

   a. Give the physical interpretation of $\frac{dV}{dt}$ and $\frac{dH}{dt}$.

   b. Is $\frac{dV}{dt}$ positive, negative or zero when the tank is one quarter full? Justify your answer.

   c. Is $\frac{dH}{dt}$ positive, negative or zero when the tank is one quarter full? Justify your answer.

   d. Which of $\frac{dV}{dt}$ and $\frac{dH}{dt}$ is constant? Explain your answer.

(4) $V(t)$ = volume of H_2O in tank
$H(t)$ = height of H_2O in tank.
Spherical tank.

a) $\frac{dV}{dt}$ is the rate at which the volume is changing at a given time t. in $\frac{\text{cubic units}}{\text{time}}$.

b) $\frac{dV}{dt} > 0$ since the volume of water in the tank is increasing when the tank is one quarter full.

c) $\frac{dH}{dt} > 0$ since the height of water in the tank is increasing when the tank is one quarter full.

d) $\frac{dV}{dt}$ is constant if the water is flowing into the tank at a constant rate.

$\frac{dH}{dt}$ is changing as water flows in b/c the diameter at each depth is changing.
dH/dt will be smallest at ½ full.

x	-2	$-2<x<-1$	-1	$-1<x<1$	1	$1<x<3$	3
$f(x)$	12	Positive	8	Positive	2	Positive	7
$f'(x)$	-5	Negative	0	Negative	0	Positive	$\frac{1}{2}$
$g(x)$	-1	Negative	0	Positive	3	Positive	1
$g'(x)$	2	Positive	$\frac{3}{2}$	Positive	0	Negative	-2

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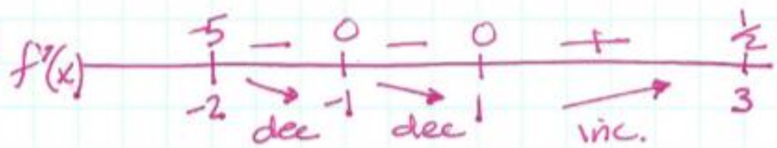
(3) The twice-differentiable functions f and g are defined for all real numbers x. Values of f, f', g, and g' for various values of x are given in the table to the right.

a. Find the x-coordinate of each relative minimum of f on the interval [-2, 3]. Justify your answers.

b. The function h is defined by $h(x) = \ln(f(x))$. Find $h'(3)$. Show the computations that lead to your answer.

(3)



a) f(x) has a relative minimum at $x=1$ (1,2) $f(1)=2$
b/c $f'(x)$ changes signs from ⊖ to ⊕.

b) $h(x) = \ln(f(x))$

$h'(x) = \frac{1}{f(x)} \cdot f'(x)$

$h'(3) = \frac{f'(3)}{f(3)} = \frac{\frac{1}{2}}{7} = \frac{1}{14}$