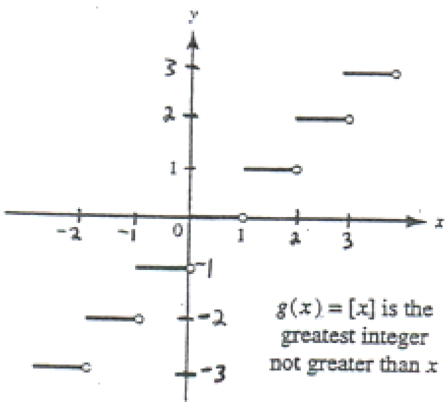
1.8 Limits--Student Notes HH6ed

Part 1: Definition of Limits

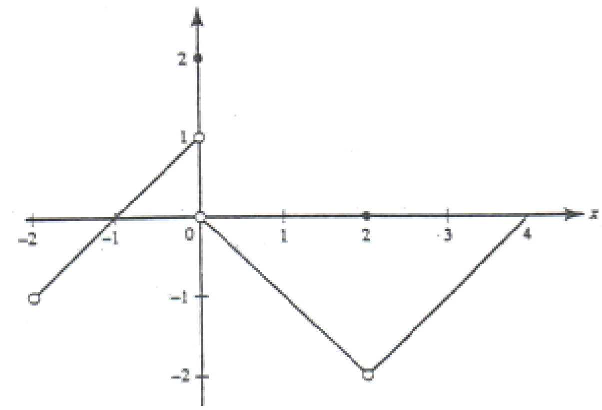
The number L is the *limit of the function f(x)* as x approaches c if, as the values of x get arbitrarily close (but not equal) to c, the values of $f(x)$ approach (or equal) L. We write $\lim_{x\to c} f(x) = L$.

In order for $\lim_{x\to c} f(x)$ to exist, the values of f must tend to the same number L as x approaches c from either the left or the right. We write $\lim_{x\to c-} f(x)$ for the *left-hand limit* of f at c (as x approaches c through values *less* than c), and $\lim_{x\to c+} f(x)$ for the *right-hand limit* of f at c (as x approaches c through values *greater* than c),

Example 1:



Example 2:



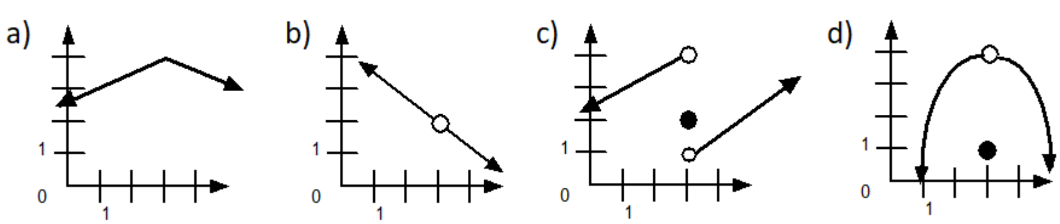
Find the following for Example 1:

1. $\lim_{x\to 0.6} x$
2. $\lim_{x\to -2.9} x$
3. $\lim_{x\to 2^-} x$
4. $\lim_{x\to 2^+} x$
5. $\lim_{x\to 2} x$

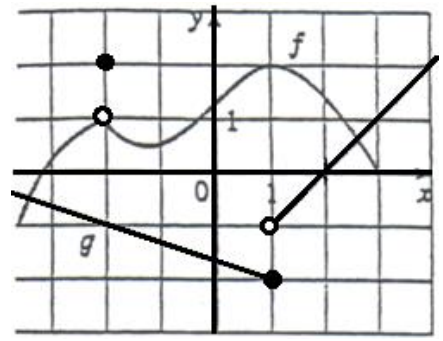
Find the following for Example 2:

6. $\lim_{x\to 3} f(x)$
7. $\lim_{x\to 2} f(x)$
8. $\lim_{x\to 0^-} f(x)$
9. $\lim_{x\to 0^+} f(x)$
10. $\lim_{x\to 0} f(x)$

Example 3: Given the graphs of each function, state whether or not $\lim_{x\to 3} f(x)$ exists and, if it does, give its value.



Use the graphs of *f* and *g* in the figure at the right to evaluate the limits, if they exist.



11. $\lim_{x \to -2}\left[f(x)+5g(x)\right]$

12. $\lim_{x \to 1}\left[f(x)\cdot g(x)\right]$

13. $\lim_{x \to 2}\left[\dfrac{f(x)}{g(x)}\right]$

14. $\lim_{x \to 3}\left[g^2(x)\right]$

Part 2: FOUR Methods for Solving Limits Analytically

Direct Substitution

15. $\lim_{x \to 5}(x-7)$

16. $\lim_{x \to 5}(x^2-3x+2)$

17. $\lim_{x \to 5}\left(e^x+\pi x\right)$

Factoring and Cancellation

18. $\lim_{x \to 3}\dfrac{x^2-x-6}{x-3}$

19. $\lim_{x \to -2}\dfrac{x+2}{x^2-4}$

20. $\lim_{x \to 3}\dfrac{1}{x-3}$

Multiply by the Conjugate

21. $\lim_{x \to 0}\dfrac{1-\sqrt{1+x}}{x}$

22. $\lim_{x \to 3}\dfrac{3-x}{\sqrt{1+x}-2}$

Simplify Complex Fractions (multiply by 1 as a fraction equal to LCD/LCD)

23. $\lim_{x \to 4}\dfrac{\left(\frac{1}{x}-\frac{1}{4}\right)}{(x-4)}$

24. $\lim_{x \to 6}\dfrac{\left(\frac{1}{x-5}-1\right)}{(x-6)}$

*Mixed Practice

25. $\lim_{x \to 2}(5x^2-3x+1)$

26. $\lim_{x \to 0}(x\cdot\cos(2x))$

27. $\lim_{x \to 3}\dfrac{x^2-9}{x-3}$

28. $\lim_{x \to 1}\dfrac{x^2-1}{1-x^2}$

29. $\lim_{\Delta x \to 0}\dfrac{(3+\Delta x)^2-3^2}{\Delta x}$

30. $\lim_{a \to 0}\dfrac{\left(\frac{1}{a-x}-\frac{1}{x}\right)}{(a)}$

Part 3: Limits at Infinity

To find $\lim_{x\to\infty} \frac{f(x)}{g(x)}$, where $f(x)$ and $g(x)$ are polynomials in x, we can divide both numerator and denominator by the highest power of c that occurs and use the fact that $\lim_{x\to\infty} \frac{1}{x} = 0$.

Examples:

31. $\lim_{x\to\infty} \frac{3-x}{4+x+x^2}$

32. $\lim_{x\to\infty} \frac{4x^4+5x+1}{37x^3-9}$

33. $\lim_{x\to\infty} \frac{x^3-4x^2+7}{3-6x-2x^3}$

Rational Function Theorem: Given $\lim_{x\to\infty} \frac{f(x)}{g(x)}$ (also applies when $x \to -\infty$)

i. when the degree of $f(x) < g(x)$ ____________________

ii. when the degree of $f(x) = g(x)$ ____________________

iii. when the degree of $f(x) > g(x)$ ____________________

*Practice:

34. $\lim_{x\to\infty} \frac{2x+1}{x-2}$

35. $\lim_{x\to-\infty} \frac{x^2+2x-3}{2x^3}$

36. $\lim_{x\to-\infty} \frac{x}{x^2+3}$

37. $\lim_{x\to\infty} \frac{13-2x}{3x+2}$

38. $\lim_{x\to-\infty} \frac{x^3-5}{1-x}$

39. $\lim_{x\to\infty} \frac{x^3-4x^4+12}{2x^4-1}$

40. Complete the table

Function	Limit as $x \to 0$	Limit as $x \to +\infty$	Limit as $x \to -\infty$
1. $f(x) = \lvert x \rvert$			
2. $f(x) = \dfrac{\lvert x \rvert}{x}$			
3. $f(x) = 6$			
4. $f(x) = \cos x$			
5. $f(x) = \dfrac{1}{x}$			
6. $f(x) = \dfrac{x+4}{x-2}$			
7. $f(x) = \dfrac{x-2}{x^2-4}$			
8. $f(x) = \dfrac{x^2-4}{x-2}$			

41. Sketch the graph of a function f that satisfies all of the following conditions:

a. Its domain is the interval [0, 4]
b. $f(0) = f(1) = f(2) = f(3) = f(4) = 1$
c. $\lim_{x \to 1} f(x) = 2$
d. $\lim_{x \to 2} f(x) = 1$
e. $\lim_{x \to 3} f(x) = 2$

42. Let $f(x) = \dfrac{2x^2 + x + k}{x^2 - 2x - 3}$ and $g(x) = \dfrac{5x^2 + 12x + k}{x^2 - 2x - 3}$.

Find the number *k* so that each limit exists?

Evaluate each limit and justify your answer.

a. $\lim_{x \to 3} f(x)$

b. $\lim_{x \to -1} f(x)$

c. $\lim_{x \to \infty} f(x)$

d. $\lim_{x \to 3} g(x)$

e. $\lim_{x \to -1} g(x)$

f. $\lim_{x \to \infty} g(x)$

43. Sketch the graph of a function satisfying the following conditions.

Graph A

1) $f(x)$ is an EVEN function (symmetry to the y-axis).
2) $f(x)$ has a single root at $x = 2$, double root at $x = 4$.
3) $f(1) = 6$
4) $\lim_{x \to 1} f(x) = -3$
5) $\lim_{x \to 0} f(x) = -7$
6) $\lim_{x \to 6^-} f(x) = \infty$
7) $\lim_{x \to 6^+} f(x) = -\infty$
8) $\lim_{x \to \infty} f(x) = 0$

Graph B

1) $f(x)$ is an ODD function (symmetry to the origin).
2) $f(x)$ has a single root at $x = 0$, triple root at $x = 4$.
3) $f(2) = -3$
4) $\lim_{x \to 2} f(x) = 4$
5) $\lim_{x \to 6} f(x) = -\infty$
6) $\lim_{x \to \infty} f(x) = \infty$
7) $f(x)$ has an oblique asymptote