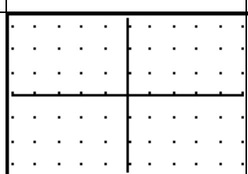
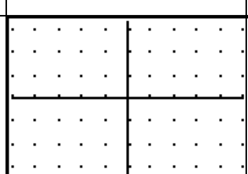
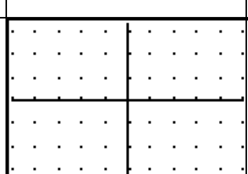
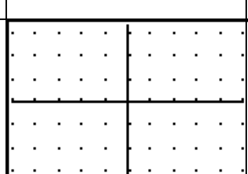
1.7 Continuous Functions – Student Notes

Objective: To understand a continuous function by investigating the relationship between an intuitive sense of continuity and its mathematical definition.

What is Continuity?

1. Which of the following situations could be classified as examples of continuous quantities?
   - Electricity used in a house during one month.
   - The amount of change in your pocket over one school day.
   - The height of the Ocean City tides over one year.
   - The temperature inside your oven during Thanksgiving Day.

2. Use **ZOOM 4: Decimal** window & the table on your calculator to investigate the different types of behaviors near and at $x = 0$. Consider the domain of the function as you investigate the behavior.

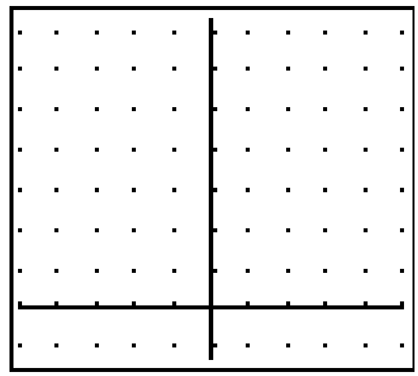
$y = f(x)$	Graph	Find $f(0)$.	Find $\lim_{x \to 0} f(x) =$	Is $f(x)$ continuous at $x = 0$?
$f(x) = x$		$f(0) = 0$	$\lim_{x \to 0} f(x) = 0$	$f(x)$ is continous at $x = 0$ *because* $f(x)$ has no holes, asymptotes, or jumps.
$f(x) = \frac{x^2}{x}$				
$f(x) = \frac{1}{x}$				
$f(x) = \frac{\lvert x \rvert}{x}$				
$f(x) = x^{\frac{2}{3}}$				
$f(x) = \frac{\sin(x)}{x}$				

3. Write a definition of continuity at a point $x = c$ in terms of limits and the definition of the function at the point where $x = c$.

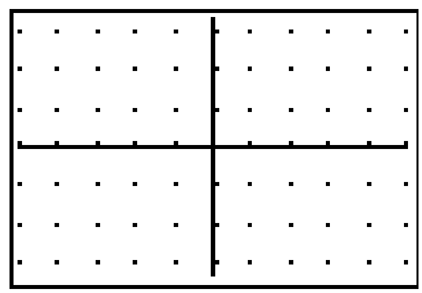
Mathematical Definition of CONTINUITY	Describe 3 types of DISCONTINUITY?
1.______________________________	1.______________________________
2.______________________________	2.______________________________
3.______________________________	3.______________________________

4. Sketch the graph of $f(x) = \begin{cases} 2x+4, & x \in (-\infty, -1] \\ x^2, & x \in (-1, \infty) \end{cases}$.

Is $f(x)$ continuous at $x = -1$? Explain.



5. Sketch the graph of $f(x) = \begin{cases} -1, & x \in (-\infty, -2] \\ 2, & x \in (-2, 0) \\ x, & x \in [0, \infty) \end{cases}$.



Use the graph of $f(x)$ to determine continuity at:

a) $x = -2$? Explain.	b) $x = 0$? Explain.	c) $x = 1$? Explain.

6. Sketch the graph of a function *f* that satisfies all of the following conditions:

- The domain of *f* is $x \in [-3, 3]$.
- $f(-3) = f(-1) = f(1) = f(3) = 2$
- It is discontinuous at -1 and 1.
- *f* is continuous on the open interval $x \in (-1, 1)$.

7. Let $f(x)=\begin{cases} ax^2+2 & if \quad x<-1 \\ x & if \quad x\geq -1 \end{cases}$ What value of a makes f continuous at $x=-1$?

8. Suppose that the function f is continuous at and that f is defined by the rule

$$f(x)=\begin{cases} kx^2+2 & if \quad x<5 \\ 4x+7 & if \quad x\geq 5 \end{cases}$$

a. Find k.
b. *Find $\lim_{x\to 5} f(x)$.

*This means find the y-value that the function gets close to as x-value gets closer and closer to $x=5$.

9. Given two functions $f(x)$ and $h(x)$ such that $f(x)=x^3-3x^2-4x+12$ and $h(x)=\begin{cases} \dfrac{f(x)}{x-3} & for \quad x\neq 3 \\ p & for \quad x=3 \end{cases}$

a. Find all zeros of the function *f*.
b. Find the value of *p* so that the function *h* is continuous at *x* = 3. Justify your answer.
c. Using the value of *p* found in (b), determine whether *h* is an even function. Justify your answer.

(This problem is from the 1976 AP AB Calculus Exam)

10. Given the function f defined by $f(x)=\dfrac{2x-2}{x^2+x-2}$

a. For what values of x is f(x) discontinuous?
b. At each point of discontinuity found in part (a), determine whether f(x) has a limit and, if so, give the value of the limit.
c. Write an equation for each vertical and horizontal asymptote. Justify your answer.
d. Draw a detailed and complete graph of f(x) using the information you found in parts a, b, & c.

(This problem is from the 1978 AP AB Calculus Exam)