NATIONAL MATH + SCIENCE INITIATIVE

AP Calculus

Fundamental Theorem of Calculus

Student Handout

2016-2017 EDITION

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https://www.surveymonkey.com/r/S_SSS

Fundamental Theorem of Calculus

Students should be able to:

- Use the fundamental theorem to evaluate definite integrals.
- Use various forms of the fundamental theorem in application situations.
- Calculate the average value of a function over a particular interval.
- Use the other fundamental theorem.

Multiple Choice

1. (calculator not allowed)

   $\int_{-2}^{x}\left(4t^3-2t\right)dt=$

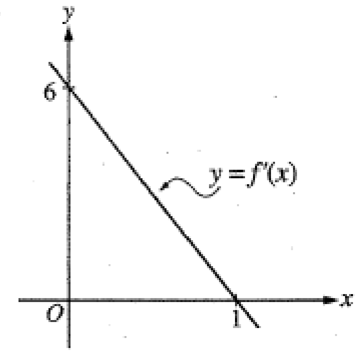
   (A) x^4-x^2
   (B) x^4-x^2-12
   (C) $4x^3-2x$
   (D) $4x^3-2x-28$

2. (calculator not allowed)

   What is the average value of y for the part of the curve $y=3x-x^2$ which is in the <u>first quadrant</u>?

   (A) −6
   (B) −2
   (C) $\frac{3}{2}$
   (D) $\frac{9}{4}$
   (E) $\frac{9}{2}$

3. (calculator not allowed)



   The graph of $f\,'$, the derivative of f, is the line shown in the figure above. If $f(0)=5$, then $f(1)=$

   (A) 0
   (B) 3
   (C) 6
   (D) 8
   (E) 11

4. (calculator not allowed)

$$\frac{d}{dx}\int_0^{x^2} \sin(t^3)dt =$$

(A) $-\cos(x^6)$
(B) $\sin(x^3)$
(C) $\sin(x^6)$
(D) $2x\sin(x^3)$
(E) $2x\sin(x^6)$

5. (calculator not allowed)

Let $f(x) = \int_{-2}^{x^2-3x} e^{t^2}\, dt$. At what value of x is $f(x)$ a minimum?

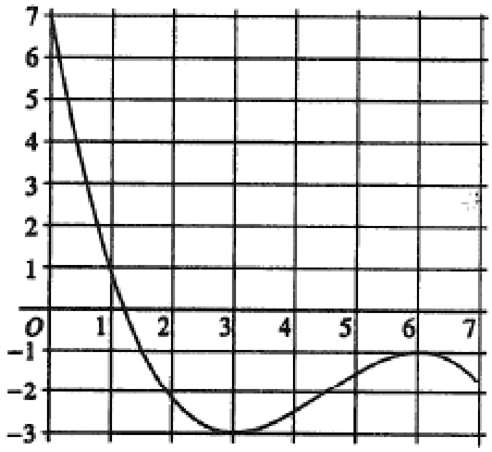
(A) For no value of x
(B) $\frac{1}{2}$
(C) $\frac{3}{2}$
(D) 2
(E) 3

6. (calculator not allowed)

$$\int_0^{\frac{\pi}{2}} \frac{\cos\theta}{\sqrt{1+\sin\theta}}\, d\theta =$$

(A) $-2\left(\sqrt{2}-1\right)$
(B) $-2\sqrt{2}$
(C) $2\sqrt{2}$
(D) $2\left(\sqrt{2}-1\right)$
(E) $2\left(\sqrt{2}+1\right)$

7. (calculator not allowed)



Graph of f

The graph of the function f shown in the figure above has horizontal tangents at $x = 3$ and $x = 6$. If $g(x) = \int_0^{2x} f(t)\,dt$, what is the value of $g'(3)$?

(A) 0
(B) −1
(C) −2
(D) −3
(E) −6

8. (calculator allowed)

Let h be the function defined by $h(x) = \frac{1}{\sqrt{x^5 + 1}}$. If g is an antiderivative of h and $g(2) = 3$, what is the value of $g(4)$?

(A) −0.020
(B) 0.152
(C) 3.031
(D) 3.152

9. (calculator not allowed)

Which of the following is an equation of the line tangent to the graph of $y = 2 - \int_1^x e^{t^2}\,dt$ at the point where $x = 1$?

(A) $y = 2$
(B) $y = 2(x-1) - e$
(C) $y = -e(x-1) + 2$
(D) $y = -2e(x-1) + 2$

10. (calculator not allowed)

If $f(x)=\int_1^{x^3}\frac{1}{1+\ln t}dt$ for $x\geq 1$, then $f'(2)=$

(A) $\frac{1}{1+\ln 2}$

(B) $\frac{12}{1+\ln 2}$

(C) $\frac{1}{1+\ln 8}$

(D) $\frac{12}{1+\ln 8}$

11. (calculator allowed)

If $f'(x)=\ln(x^2)$ and $f(5)=8$, then $f(3)=$

(A) 2.497
(B) 4.171
(C) 5.502
(D) 13.502

12. (calculator allowed)

Let g be the function given by $g(x)=\int_0^x \sin(t^2)dt$ for $-1\leq x\leq 3$. On which of the following intervals is g decreasing?

(A) $-1\leq x\leq 0$
(B) $0\leq x\leq 1.772$
(C) $1.253\leq x\leq 2.171$
(D) $1.772\leq x\leq 2.507$
(E) $2.802\leq x\leq 3$

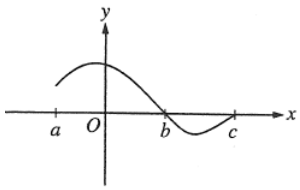
13. (calculator allowed)

If $0\leq x\leq 4$, of the following, which is the greatest value of x such that

$\int_0^x (t^2-2t)\,dt \geq \int_2^x t\,dt$?

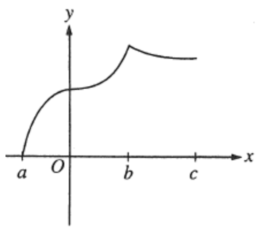
(A) 1.35
(B) 1.38
(C) 1.41
(D) 1.48
(E) 1.59

14. (calculator allowed)

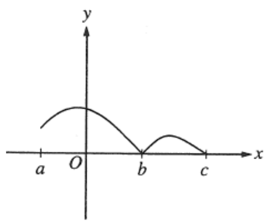


Let $f(x)=\int_a^x h(t)\,dt$, where h has the graph shown above. Which of the following could be the graph of f ?

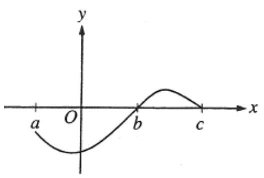
(A)



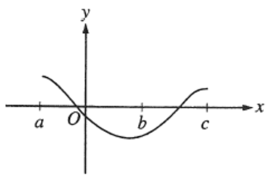
(B)



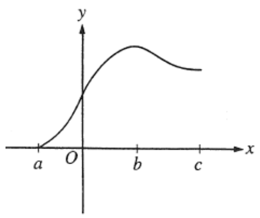
(C)



(D)



(E)



15. (calculator allowed)
A pizza, heated to a temperature of 350 degrees Fahrenheit (^oF), is taken out of an oven and placed in a 75^oF room at time $t=0$ minutes. The temperature of the pizza is changing at a rate of $-110e^{-0.4t}$ degrees Fahrenheit per minute. To the nearest degree, what is the temperature of the pizza at time $t=5$ minutes?

(A) $112°F$
(B) $119°F$
(C) $147°F$
(D) $238°F$
(E) $335°F$

16. (calculator allowed)
For all values of x, the continuous function f is positive and decreasing. Let g be the function given by $g(x)=\int_2^x f(t)\,dt$. Which of the following could be a table of values for g?

(A)

x	$g(x)$
1	–2
2	0
3	1

(B)

x	$g(x)$
1	–2
2	0
3	3

(C)

x	$g(x)$
1	1
2	0
3	–2

(D)

x	$g(x)$
1	2
2	0
3	–1

(E)

x	$g(x)$
1	3
2	0
3	2

17. (calculator allowed)
The rate of change of the altitude of a hot-air balloon is given by $r(t)=t^3-4t^2+6$ for $0\le t\le 8$. Which of the following expressions gives the change in altitude of the balloon during the time the altitude is decreasing?

(A) $\int_{1.572}^{3.514} r(t)\,dt$

(B) $\int_{0}^{8} r(t)\,dt$

(C) $\int_{0}^{2.667} r(t)\,dt$

(D) $\int_{1.572}^{3.514} r'(t)\,dt$

(E) $\int_{0}^{2.667} r'(t)\,dt$

Free Response

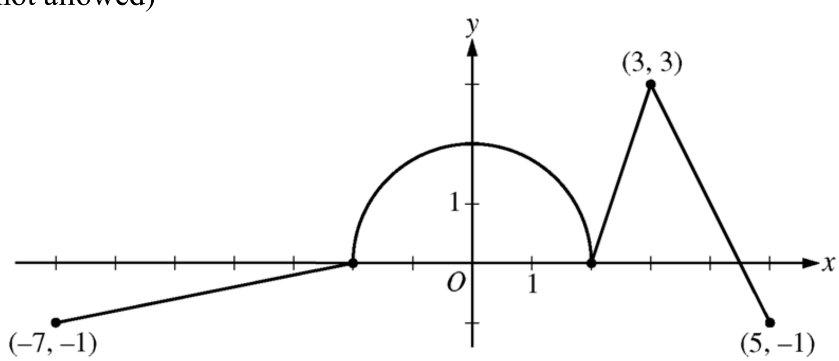
18. (calculator not allowed)

x	2	3	5	8	13
$f(x)$	1	4	–2	3	6

Let f be a function that is twice differentiable for all real numbers. The table above gives values of f for selected points in the closed interval $2 \le x \le 13$.

(b) Evaluate $\int_2^{13} (3 - 5f'(x))\,dx$. Show the work that leads to your answer.

19. (calculator not allowed)



Graph of g'

The function g is defined and differentiable on the closed interval $[-7, 5]$ and satisfies $g(0) = 5$. The graph of $y = g'(x)$, the derivative of g, consists of a semicircle and three line segments, as shown in the figure above.

(a) Find $g(3)$ and $g(-2)$.

20. (calculator allowed)
As a pot of tea cools, the temperature of the tea is modeled by a differentiable function H for $0 \le t \le 10$, where time t is measured in minutes and temperature $H(t)$ is measured in degrees Celsius. Values of $H(t)$ at selected values of time t are shown in the table.

t (minutes)	0	2	5	9	10
$H(t)$ (degrees Celsius)	66	60	52	44	43

(c) Evaluate $\int_0^{10} H'(t)\, dt$. Using correct units, explain the meaning of the expression in the context of this problem

(d) At time $t = 0$, biscuits with temperature 100°C were removed from an oven. The temperature of the biscuits at time t is modeled by a differentiable function B for which it is known that $B'(t) = -13.84e^{-0.173t}$. Using the given models, at time $t = 10$, how much cooler are the biscuits than the tea?

21. (calculator allowed)

At a certain height, a tree trunk has a circular cross section. The radius $R(t)$ of that cross section grows at a rate modeled by the function

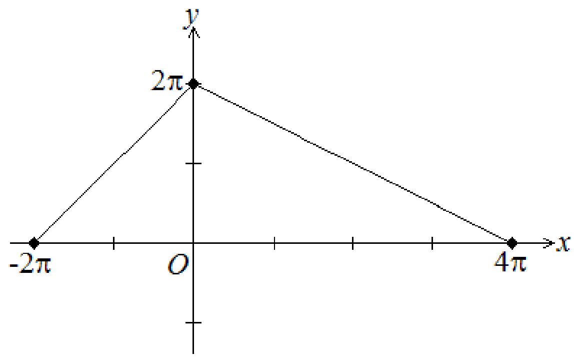
$$\frac{dR}{dt} = \frac{1}{16}\left(3 + \sin(t^2)\right) \text{ centimeters per year}$$

for $0 \le t \le 3$, where time t is measured in years. At time $t = 0$, the radius is 6 centimeters. The area of the cross section at time t is denoted by $A(t)$.

(a) Write an expression, involving an integral, for the radius $R(t)$ for $0 \le t \le 3$. Use your expression to find $R(3)$.

(c) Evaluate $\int_0^3 A'(t)\,dt$. Using appropriate units, interpret the meaning of that integral in terms of cross-sectional area.

22. (calculator not allowed)

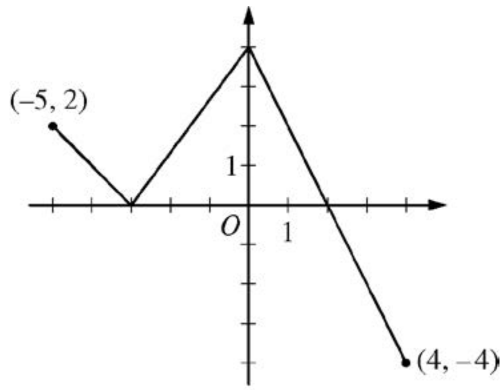


Let g be the piecewise-linear defined function on $[-2\pi, 4\pi]$ whose graph is given above, and let $f(x) = g(x) - \cos\left(\frac{x}{2}\right)$.

(a) Find $\int_{-2\pi}^{4\pi} f(x)\,dx$. Show the computations that lead to your answer.

(c) Let $h(x) = \int_0^{3x} g(t)\,dt$. Find $h'\left(-\frac{\pi}{3}\right)$.

23. (calculator not allowed)



Graph of f

The function f is defined on the closed interval $[-5, 4]$. The graph of f consists of three line segments and is shown in the figure above. Let g be the function defined by $g(x) = \int_{-3}^{x} f(t)\,dt.$

(b) On what open intervals contained in $-5 < x < 4$ is the graph of g both increasing and concave down? Give a reason for your answer.

24. (calculator not allowed)
The rate at which rainwater flows into a drainpipe is modeled by the function R, where $R(t) = 20\sin\left(\frac{t^2}{35}\right)$ cubic feet per hour, t is measured in hours, and $0 \le t \le 8$. The pipe is partially blocked, allowing water to drain out the other end of the pipe at a rate modeled by $D(t) = -0.04t^3 + 0.4t^2 + 0.96t$ cubic feet per hour, for $0 \le t \le 8$. There are 30 cubic feet of water in the pipe at time $t = 0$.

(a) How many cubic feet of rainwater flow into the pipe during the 8-hour time interval $0 \le t \le 8$?

(c) At what time $0 \le t \le 8$, is the amount of the pipe at a minimum? Justify your answer.

Fundamental Theorem of Calculus Reference Page

- Fundamental Theorem of Calculus

$$\int_a^b f(x)dx = F(b) - F(a)$$

- The Fundamental Theorem of Calculus can be written in various ways.

$$\int_a^b f'(x)dx = f(b) - f(a)$$

$$f(b) = f(a) + \int_a^b f'(x)dx$$

$$f(a) = f(b) - \int_a^b f'(x)dx$$

$$\int_a^b f''(x)dx = f'(b) - f'(a)$$

- Average value of a function over a particular interval

$$f_{avg} = \frac{\int_a^b f(x)dx}{b-a} = \frac{F(b)-F(a)}{b-a}$$

- Other Fundamental Theorem of Calculus

$$\frac{d}{dx}\left(\int_a^x f(t)dt\right) = f(x)$$

$$\frac{d}{dx}\left(\int_a^{g(x)} f(t)dt\right) = f(g(x))\,g'(x)$$

- Fundamental Theorem of Calculus in context

$$Amount = \int_{begining\ time}^{ending\ time} Rate\ dt$$

$$Current\ Amount = Initial\ Amount + \int_{time1}^{time2} \text{"}rate\ in\text{"}\ dt - \int_{time1}^{time2} \text{"}rate\ out\text{"}\ dt$$