



# NATIONAL MATH + SCIENCE INITIATIVE

## AP Calculus

## Particle Motion

## Student Handout

**2016-2017 EDITION**

Use the following link or scan the QR code to complete the evaluation for the Study Session [https://www.surveymonkey.com/r/S\\_SSS](https://www.surveymonkey.com/r/S_SSS)





## Particle Motion

### Students should be able to:

- Understand the relationship between a particle's position, velocity, and acceleration
- Determine displacement of a particle and its total distance traveled using graphical and analytical methods
- Determine if speed of a particle is increasing or decreasing based on its velocity and acceleration
- Determine average velocity given position or velocity
- Determine direction of a particle based on its velocity
- Determine if velocity is increasing or decreasing based on acceleration
- Determine final position given an initial position and velocity function
- Determine final velocity given an initial velocity and acceleration function



## Multiple Choice

1. (calculator not allowed)

A particle moves along the  $x$ -axis so that at time  $t \geq 0$  its position is given by  $x(t) = 2t^3 - 21t^2 + 72t - 53$ . At what time  $t$  is the particle at rest?

- (A)  $t = 1$  only
- (B)  $t = 3$  only
- (C)  $t = \frac{7}{2}$  only
- (D)  $t = 3$  and  $t = \frac{7}{2}$
- (E)  $t = 3$  and  $t = 4$

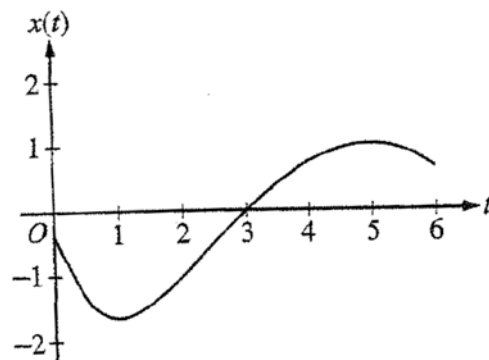
2. (calculator not allowed)

For time  $t \geq 0$ , the velocity of a particle moving along the  $x$ -axis is given by

$v(t) = (t - 5)(t - 2)^2$ . At what values of  $t$  is the acceleration of the particle equal to 0?

- (A) 2 only
- (B) 4 only
- (C) 2 and 4
- (D) 2 and 5

3. (calculator not allowed)



A particle moves along a straight line. The graph of the particle's position  $x(t)$  at time  $t$  is shown above for  $0 < t < 6$ . The graph has horizontal tangents at  $t = 1$  and  $t = 5$  and a point of inflection at  $t = 2$ . For what values of  $t$  is the velocity of the particle increasing?

- (A)  $0 < t < 2$
- (B)  $1 < t < 5$
- (C)  $2 < t < 6$
- (D)  $3 < t < 5$  only
- (E)  $1 < t < 2$  and  $5 < t < 6$

4. (calculator not allowed)

A particle moves along the  $x$ -axis with velocity given by  $v(t) = 3t^2 + 6t$  for time  $t \geq 0$ . If the particle is at position  $x = 2$  at time  $t = 0$ , what is the position of the particle at time  $t = 1$ ?

- (A) 4
- (B) 6
- (C) 0
- (D) 11
- (E) 12

5. (calculator not allowed)

The velocity of a particle moving on a line at time  $t$  is  $v = 3t^{\frac{1}{2}} + 5t^{\frac{3}{2}}$  meters per second. How many meters did the particle travel from  $t = 0$  to  $t = 4$ ?

- (A) 32
- (B) 40
- (C) 64
- (D) 80
- (E) 184

6. (calculator not allowed)

If the position of a particle on the  $x$ -axis at time  $t$  is  $-5t^2$ , then the average velocity of the particle for  $0 \leq t \leq 3$  is

- (A)  $-45$
- (B)  $-30$
- (C)  $-15$
- (D)  $-10$
- (E)  $-5$

7. (calculator not allowed)

A particle travels in a straight line with a constant acceleration of 3 meters per second per second. If the velocity of the particle is 10 meters per second at time 2 seconds, how far does the particle travel during the time interval when its velocity increases from 4 meters per second to 10 meters per second?

- (A) 20 m
- (B) 14 m
- (C) 7 m
- (D) 6 m
- (E) 3 m

8. (calculator not allowed)

A particle moves along the  $x$ -axis. The velocity of the particle at time  $t$  is  $6t - t^2$ . What is the total distance traveled by the particle from time  $t = 0$  to  $t = 3$ ?

- (A) 3
- (B) 6
- (C) 9
- (D) 18
- (E) 27

9. (calculator not allowed)

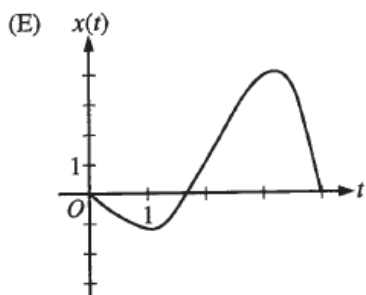
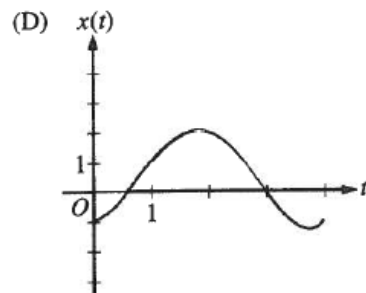
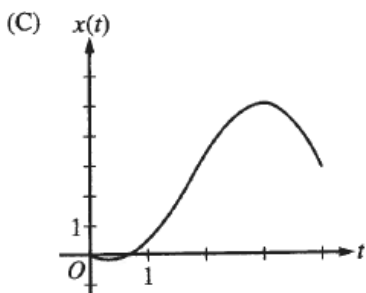
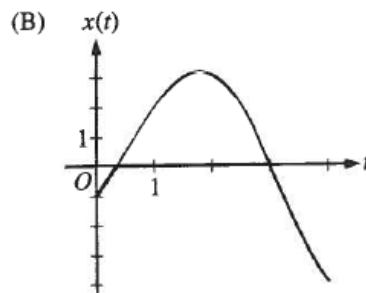
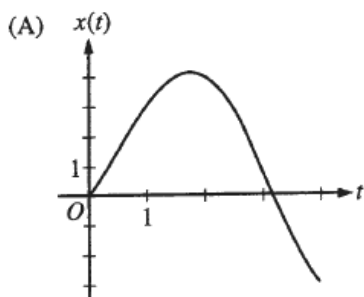
A particle moves along the  $x$ -axis with its position at time  $t$  given by  $x(t) = (t - a)(t - b)$ , where  $a$  and  $b$  are constants and  $a \neq b$ . For which of the following values of  $t$  is the particle at rest?

- (A)  $t = ab$
- (B)  $t = \frac{a + b}{2}$
- (C)  $t = a + b$
- (D)  $t = 2(a + b)$
- (E)  $t = a$  and  $t = b$

10. (calculator not allowed)

The table gives selected values of the velocity,  $v(t)$ , of a particle moving along the  $x$ -axis. At time  $t = 0$ , the particle is at the origin. Which of the following could be the graph of the position,  $x(t)$ , of the particle for  $0 \leq t \leq 4$ ?

$t$	0	1	2	3	4
$v(t)$	-1	2	3	0	-4



11. (calculator allowed)

A particle moves along the  $x$ -axis so that any time  $t \geq 0$ , its velocity is given by  $v(t) = 3 + 4.1 \cos(0.9t)$ . What is the acceleration of the particle at time  $t = 4$ ?

- (A) -2.016
- (B) -0.677
- (C) 1.633
- (D) 1.814
- (E) 2.978



12. (calculator allowed)

A particle moves along the  $x$ -axis so that any time  $t > 0$ , its acceleration is given by  $a(t) = \ln(1 + 2^t)$ . If the velocity of the particle is 2 at time  $t = 1$ , then the velocity of the particle at time  $t = 2$  is

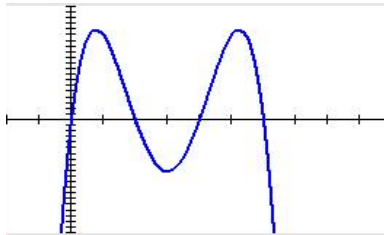
- (A) 0.462
- (B) 1.609
- (C) 2.555
- (D) 2.886
- (E) 3.346

13. (calculator allowed)

The velocity, in ft/sec, of a particle moving along the  $x$ -axis is given by the function  $v(t) = e^t + te^t$ . What is the average velocity of the particle from time  $t = 0$  to time  $t = 3$ ?

- (A) 20.086 ft/sec
- (B) 26.447 ft/sec
- (C) 32.809 ft/sec
- (D) 40.671 ft/sec
- (E) 79.342 ft/sec

14. (calculator allowed)



A particle moves along the  $x$ -axis over the time interval  $0 \leq t \leq 6$ . The graph of the velocity of the particle is shown above. Over the time interval  $0 \leq t \leq 6$ , the particle's displacement is 4 and the particle travels a total distance of 16. What is the value of  $\int_2^4 v(t) dt$ ?

- (A) -12
- (B) -6
- (C) 6
- (D) 12

## Free Response

15. (calculator allowed)

A particle moves along a straight line. For  $0 \leq t \leq 5$ , the velocity of the particle is given by  $v(t) = -2 + (t^2 + 3t)^{6/5} - t^3$ , and the position of the particle is given by  $s(t)$ . It is known that  $s(0) = 10$ .

- (a) Find all values of  $t$  in the interval  $2 \leq t \leq 4$  for which the speed of the particle is 2.
- (b) Write an expression involving an integral that gives the position  $s(t)$ . Use this expression to find the position of the particle at time  $t = 5$ .
- (c) Find all times  $t$  in the interval  $0 \leq t \leq 5$  at which the particle changes direction. Justify your answer.
- (d) Is the speed of the particle increasing or decreasing at time  $t = 4$ ? Give a reason for your answer.

16. (calculator allowed)

A particle moves along the  $y$ -axis with velocity given by  $v(t) = t \sin(t^2)$  for  $t \geq 0$ .

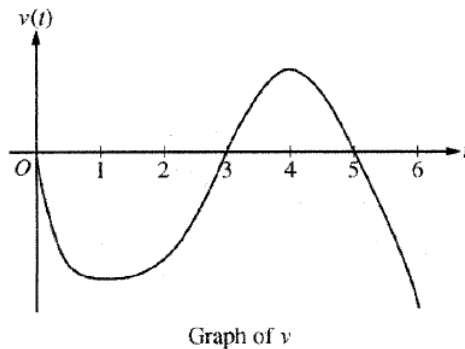
- (a) In which direction (up or down) is the particle moving at time  $t = 1.5$ ? Why?
- (b) Find the acceleration of the particle at time  $t = 1.5$ ? Is the velocity of the particle increasing at time  $t = 1.5$ ?
- (c) Given that  $y(t)$  is the position of the particle at time  $t$  and that  $y(0) = 3$ , find  $y(2)$ .
- (d) Find the total distance traveled by the particle from  $t = 0$  to  $t = 2$ .

17. (calculator allowed)

A particle moves along the  $x$ -axis so that its velocity  $v$  at time  $t$ ,  $0 \leq t \leq 5$ , is given by  $v(t) = \ln(t^2 - 3t + 3)$ . The particle is at position  $x = 8$  at time  $t = 0$ .

- (a) Find the acceleration of the particle at time  $t = 4$ .
- (b) Find the times  $t$  in the open interval  $0 < t < 5$  at which the particle changes direction. During which time intervals,  $0 < t < 5$ , does the particle travel left?
- (c) Find the position of the particle at time  $t = 2$ .
- (d) Find the average speed of the particle over the interval  $0 \leq t \leq 2$ .

18. (calculator not allowed)



A particle moves along the  $x$ -axis so that the velocity at time,  $t$ , for  $0 \leq t \leq 6$ , is given by a differentiable function  $v$  whose graph is shown above. The velocity is 0 at  $t = 0$ ,  $t = 3$ , and  $t = 5$ , and the graph has horizontal tangents at  $t = 1$  and  $t = 4$ . The areas of the regions bounded by the  $t$ -axis and the graph of  $v$  on the intervals  $[0, 3]$ ,  $[3, 5]$ , and  $[5, 6]$  are 8, 3, and 2, respectively. At time  $t = 0$ , the particle is at  $x = -2$ .

- (a) For  $0 \leq t \leq 6$ , find both the time and the position of the particle when the particle is farthest to the left. Justify your answer.
- (b) For how many values of  $t$ , where  $0 \leq t \leq 6$ , is the particle at  $x = -8$ ? Explain your reasoning.
- (c) On the interval  $2 < t < 3$ , is the speed of the particle increasing or decreasing? Give a reason for your answer.
- (e) During what time intervals, if any, is the acceleration of the particle negative? Justify your answer.

19.

Ben rides a unicycle back and forth along a straight east-west track. The twice-differentiable function  $B$  models Ben's position on the track, measured in meters from the western end of the track, at time  $t$ , measured in seconds from the start of the ride. The table gives values for  $B(t)$  and Ben's velocity,  $v(t)$ , measured in meters per second, at selected times  $t$ .

$t$ (seconds)	0	10	40	60
$B(t)$ (meters)	100	136	9	49
$v(t)$ $\left(\frac{m}{\text{sec}}\right)$	2.0	2.3	2.5	4.6

(a) Use the data in the table to approximate Ben's acceleration at time  $t = 5$  seconds. Indicate units of measure.

(b) Using correct units, interpret the meaning of  $\int_0^{60} |v(t)| dt$  in the context of this problem.

Approximate  $\int_0^{60} |v(t)| dt$  using a left Riemann sum with the subintervals indicated by the data in the table.

(c) For  $40 \leq t \leq 60$ , must there be a time  $t$  when Ben's velocity is 2 meters per second? Justify your answer.

## Particle Motion Reference Guide

Typically, if a particle is moving along the x-axis at any time,  $t$ ,  $x(t)$  represents the position of the particle; along the y-axis,  $y(t)$  is often used; along another straight line,  $s(t)$  is often used. In addition,  $v(t)$  is typically used to represent the velocity of the particle. In these types of particle motion problems,

- Position:  $x(t)$  or  $s(t)$   
Velocity:  $v(t) = s'(t)$   
Acceleration:  $a(t) = v'(t) = s''(t)$
- “Initially” means when time  $t = 0$ .
- “At the origin” means  $x(t) = 0$ .
- “At rest” means velocity  $v(t) = 0$ .
- Average velocity of the particle is  $\frac{\Delta s}{\Delta t} = \frac{s(t_2) - s(t_1)}{t_2 - t_1}$  when the position function is given;  
Average velocity of the particle is  $\frac{1}{t_2 - t_1} \int_{t_1}^{t_2} v(t) dt$  when the velocity function is given.
- Average acceleration of the particle is  $\frac{\Delta v}{\Delta t} = \frac{v(t_2) - v(t_1)}{t_2 - t_1}$  when the velocity function is given; Average acceleration of the particle is  $\frac{1}{t_2 - t_1} \int_{t_1}^{t_2} a(t) dt$  when the acceleration function is given.
- If the velocity of the particle is positive, then the particle is moving to the right (or up).
- If the velocity of the particle is negative, then the particle is moving to the left (or down).
- If the acceleration of the particle is positive, then the velocity is increasing.
- If the acceleration of the particle is negative, then the velocity is decreasing.
- Speed is the absolute value of velocity.
- If the velocity and acceleration have the same sign (both positive or both negative), then speed is increasing.
- If the velocity and acceleration are opposite in sign (one is positive and the other is negative), then speed is decreasing.
- To determine total distance traveled over a time interval, you must calculate the sum of the absolute values of the differences in position between all resting points or calculate the area under the absolute value of the velocity curve,  $\int_{t_1}^{t_2} |v(t)| dt$ .
- Displacement can be determined using  $displacement = \int_{t_1}^{t_2} v(t) dt$
- To determine the final position of a particle use  $s(t_2) = s(t_1) + \int_{t_1}^{t_2} v(t) dt$
- To determine the final velocity of a particle use  $v(t_2) = v(t_1) + \int_{t_1}^{t_2} a(t) dt$